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Inventory Model with Different Deterioration Rates for Imperfect Quality Items and Linear Demand

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Abstract

Many times it happens that units produced or ordered are not of 100% good quality. A deterministic inventory model with imperfect quality is developed when deterioration rate is different during a cycle. Here it is assumed that holding cost is time dependent. Demand is considered as linear function of time. Numerical example is taken to support the model.

Keywords: Inventory model, Varying Deterioration, Linear demand, Time varying holding cost, defective items

1. INTRODUCTION:

Deterioration of items is a general phenomenon for many inventory systems and therefore deterioration effect cannot be ignored in real life. Ghare and Schrader [3] considered inventory model with constant rate of deterioration. Covert and Philip [2] extended the model by considering variable rate of deterioration. The related work are found in (Nahmias [5], Raffat [6], Ruxian, et al [7]).Many times it happens that units produced or ordered are not of 100% good quality.

Cheng [1] developed a model of imperfect production quantity by establishing relationship between demand dependent unit production cost and imperfect production process. Salman and Jaber [8] developed an inventory model in which items received are of defective quality and after 100% screening, imperfect items are withdrawn from the inventory and sold at a discounted price. Jaggi et al. [4] developed an inventory model for deteriorating items with imperfect quality under permissible delay in payment. Patel and Patel [10] developed an EOQ model for deteriorating items with imperfect quantity items. Patel and Sheikh [9] developed an inventory model with different deterioration rates and time varying holding cost.

Generally the products are such that initially there is no deterioration. Deterioration starts after certain time and again after certain time the rate of deterioration increases with time. Here we have used such a concept and developed the deteriorating items inventory models.

In this paper we have developed an inventory model for imperfect quality items with different deterioration rates for the cycle time. Holding cost is taken as function of time. Shortages are not allowed. To illustrate the model, numerical example is provided. Sensitivity analysis for major parameters is also carried out.

2. ASSUMPTIONS AND NOTATIONS:

NOTATIONS:

The following notations are used for the development of the model:

$D(t)$: Demand rate is a linear function of time t ($a+bt$, $a>0$, $0<b<1$)

c : Purchasing cost per unit

p : Selling price per unit

d : defective items (%)

$1-d$: good items (%)

λ : Screening rate

SR : Sales revenue

A : Replenishment cost per order

z : Screening cost per unit

p_d : Price of defective items per unit

$h(t)$: Variable Holding cost ($x + yt$)

t_1 : Screening time

T : Length of inventory cycle

$I(t)$: Inventory level at any instant of time t, $0 \leq t \leq T$

Q : Order quantity

θ : Deterioration rate during $\mu_1 \leq t \leq \mu_2$, $0< \theta_1<1$

θt : Deterioration rate during , $\mu_2 \leq t \leq T$, $0< \theta_2<1$

π : Total relevant profit per unit time.

ASSUMPTIONS:

The following assumptions are considered for the development of the model.

- The demand of the product is declining as a linear function of time.
- Replenishment rate is infinite and instantaneous.
- Lead time is zero.
- Shortages are not allowed.
- The screening process and demand proceeds simultaneously but screening rate (λ) is greater than the demand rate i.e. $\lambda > (a+bt)$.
- The defective items are independent of deterioration.
- Deteriorated units can neither be repaired nor replaced during the cycle time.
- A single product is considered.
- Holding cost is time dependent.
- The screening rate (λ) is sufficiently large. In general, this assumption should be acceptable since the automatic screening machine usually takes only little time to inspect all items produced or purchased.

3. THE MATHEMATICAL MODEL AND ANALYSIS:

In the following situation, Q items are received at the beginning of the period. Each lot having a d % defective items. The nature of the inventory level is shown in the given figure, where screening process is done for all the received items at the rate of λ units per unit time which is greater than demand rate for the time period 0 to t_1 . During the screening process the demand occurs parallel to the screening process and is fulfilled from the goods which are found to be of perfect quality by screening process. The defective items are sold immediately after the screening process at time t_1 as a single batch at a discounted price. After the screening process at time t_1 the inventory level will be $I(t_1)$ and at time T, inventory level will become zero due to demand and partially due to deterioration.

Also here $t_1 = \frac{Q}{\lambda}$ (1)

and defective percentage (d) is restricted to $d \leq 1 - \frac{(x+yt)}{\lambda}$ (2)

Let I(t) be the inventory at time t ($0 \leq t \leq T$) as shown in figure.

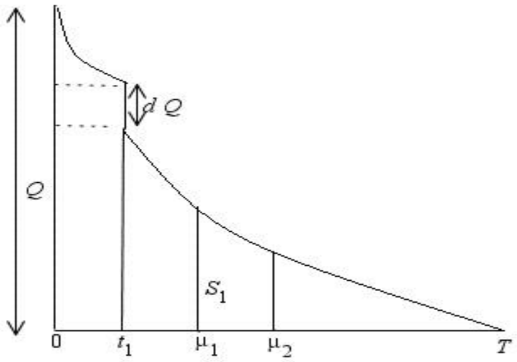


Figure 1

The differential equations which describes the instantaneous states of I(t) over the period (0, T) is given by

$$\frac{dI(t)}{dt} = -(a + bt), \qquad 0 \le t \le \mu_1 \tag{3}$$

$$\frac{dI(t)}{dt} + \theta I(t) = -(a + bt), \qquad \mu_1 \le t \le \mu_2 \tag{4}$$

$$\frac{dI(t)}{dt} + \theta t I(t) = -(a + bt), \qquad \mu_2 \le t \le T \tag{5}$$

with initial conditions $I(0) = Q$, $I(\mu_1) = S_1$ and $I(T) = 0$.
Solutions of these equations are given by

$$I(t) = Q - (at + \frac{1}{2}bt^2), \tag{6}$$

$$I(t) = \left[\begin{array}{l} a(\mu_1 - t) + \frac{1}{2}b(\mu_1^2 - t^2) + \frac{1}{2}a\theta(\mu_1^2 - t^2) \\ + \frac{1}{3}b\theta(\mu_1^3 - t^3) - a\theta t(\mu_1 - t) - \frac{1}{2}b\theta t(\mu_1^2 - t^2) \end{array}\right] + S_1\left[1 + \theta(\mu_1 - t)\right] \tag{7}$$

$$I(t) = \left[\begin{array}{l} a(T - t) + \frac{1}{2}b(T^2 - t^2) + \frac{1}{6}a\theta(T^3 - t^3) \\ + \frac{1}{8}b\theta(T^4 - t^4) - \frac{1}{2}a\theta t^2(T - t) - \frac{1}{4}b\theta t^2(T^2 - t^2) \end{array}\right]. \tag{8}$$

(by neglecting higher powers of θ)

After screening process, the number of defective items at time t_1 is dQ.
So effective inventory level during $t_1 \le t \le T$ is given by

$$I(t) = -(at + \frac{1}{2}bt^2) + Q(1-d). \tag{9}$$

From equation (6), putting $t = \mu_1$, we have

$$Q = S_1 + \left(a\mu_1 + \frac{1}{2}b\mu_1^2\right). \tag{10}$$

From equations (7) and (8), putting $t = \mu_2$, we have

$$I(\mu_2) = \begin{bmatrix} a(\mu_1 - \mu_2) + \frac{1}{2}b(\mu_1^2 - \mu_2^2) + \frac{1}{2}a\theta(\mu_1^2 - \mu_2^2) \\ + \frac{1}{3}b\theta(\mu_1^3 - \mu_2^3) - a\theta\mu_2(\mu_1 - \mu_2) - \frac{1}{2}b\theta\mu_2(\mu_1^2 - \mu_2^2) \end{bmatrix} + S_1\left[1 + \theta(\mu_1 - \mu_2)\right] \tag{11}$$

$$I(\mu_2) = \begin{bmatrix} a(T - \mu_2) + \frac{1}{2}b(T^2 - \mu_2^2) + \frac{1}{6}a\theta(T^3 - \mu_2^3) \\ + \frac{1}{8}b\theta(T^4 - \mu_2^4) - \frac{1}{2}a\theta\mu_2^2(T - \mu_2) - \frac{1}{4}b\theta\mu_2^2(T^2 - \mu_2^2) \end{bmatrix}. \tag{12}$$

So from equations (11) and (12), we get

$$S_1 = \frac{1}{\left[1 + \theta(\mu_1 - \mu_2)\right]}\begin{bmatrix} a(T - \mu_2) + \frac{1}{2}b(T^2 - \mu_2^2) + \frac{1}{6}a\theta(T^3 - \mu_2^3) \\ + \frac{1}{8}b\theta(T^4 - \mu_2^4) - \frac{1}{2}a\theta\mu_2^2(T - \mu_2) - \frac{1}{4}b\theta\mu_2^2(T^2 - \mu_2^2) \\ - a(\mu_1 - \mu_2) - \frac{1}{2}b(\mu_1^2 - \mu_2^2) - \frac{1}{2}a\theta(\mu_1^2 - \mu_2^2) \\ - \frac{1}{3}b\theta(\mu_1^3 - \mu_2^3) + a\theta\mu_2(\mu_1 - \mu_2) + \frac{1}{2}b\theta\mu_2(\mu_1^2 - \mu_2^2) \end{bmatrix}. \tag{13}$$

Putting value of S_1 from equation (13) into equation (10), we have

$$Q = \frac{1}{\left[1+\theta(\mu_1-\mu_2)\right]}\begin{bmatrix} a(T - \mu_2) + \frac{1}{2}b(T^2 - \mu_2^2) + \frac{1}{6}a\theta(T^3 - \mu_2^3) \\ +\frac{1}{8}b\theta(T^4-\mu_2^4)-\frac{1}{2}a\theta\mu_2^2(T-\mu_2)-\frac{1}{4}b\theta\mu_2^2(T^2-\mu_2^2) \\ - a(\mu_1 - \mu_2) - \frac{1}{2}b(\mu_1^2 - \mu_2^2) - \frac{1}{2}a\theta(\mu_1^2 - \mu_2^2) \\ -\frac{1}{3}b\theta(\mu_1^3-\mu_2^3)+a\theta\mu_2(\mu_1-\mu_2)+\frac{1}{2}b\theta\mu_2(\mu_1^2-\mu_2^2) \end{bmatrix} + \left(a\mu_1+\frac{1}{2}b\mu_1^2\right). \tag{14}$$

Using (14) in (6), we have

$$I(t) = \frac{1}{\left[1+\theta\left(\mu_1 - \mu_2\right)\right]}\left[\begin{array}{l} a\left(T-\mu_2\right) + \frac{1}{2}b\left(T^2-\mu_2^2\right) + \frac{1}{6}a\theta\left(T^3-\mu_2^3\right) \\ + \frac{1}{8}b\theta\left(T^4-\mu_2^4\right) - \frac{1}{2}a\theta\mu_2^2\left(T-\mu_2\right) - \frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right) \\ - a\left(\mu_1-\mu_2\right) - \frac{1}{2}b\left(\mu_1^2-\mu_2^2\right) - \frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right) \\ - \frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right) + a\theta\mu_2\left(\mu_1-\mu_2\right) + \frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right) \end{array}\right] \tag{15}$$

$$+ a\left(\mu_1 - t\right) \ \frac{1}{2}b\left(\mu_1^2 - t^2\right).$$

Similarly, using (14) in (9), we have

$$I(t_1) = \frac{(1\text{-}d)}{\left[1+\theta\left(\mu_1-\mu_2\right)\right]}\left[\begin{array}{l} a\left(T-\mu_2\right) + \frac{1}{2}b\left(T^2-\mu_2^2\right) + \frac{1}{6}a\theta\left(T^3-\mu_2^3\right) \\ + \frac{1}{8}b\theta\left(T^4-\mu_2^4\right) - \frac{1}{2}a\theta\mu_2^2\left(T-\mu_2\right) - \frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right) \\ - a\left(\mu_1-\mu_2\right) - \frac{1}{2}b\left(\mu_1^2-\mu_2^2\right) - \frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right) \\ - \frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right) + a\theta\mu_2\left(\mu_1-\mu_2\right) + \frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right) \end{array}\right] \tag{16}$$

$$+ (1\text{-}d)\left(a\mu_1 + \frac{1}{2}b\mu_1^2\right) - \left(at + \frac{1}{2}bt^2\right)$$

Similarly putting value of S_1 from equation (13) in equation (7), we have

$$I(t) = \frac{\left[1+\theta\left(\mu_1 - t\right)\right]}{\left[1+\theta\left(\mu_1 - \mu_2\right)\right]}\left[\begin{array}{l} a\left(T-\mu_2\right) + \frac{1}{2}b\left(T^2-\mu_2^2\right) + \frac{1}{6}a\theta\left(T^3-\mu_2^3\right) \\ + \frac{1}{8}b\theta\left(T^4-\mu_2^4\right) - \frac{1}{2}a\theta\mu_2^2\left(T-\mu_2\right) - \frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right) \\ - a\left(\mu_1-\mu_2\right) - \frac{1}{2}b\left(\mu_1^2-\mu_2^2\right) - \frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right) \\ - \frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right) + a\theta\mu_2\left(\mu_1-\mu_2\right) + \frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right) \end{array}\right] \tag{17}$$

$$+ \left[\begin{array}{l} a\left(\mu_1 - t\right) + \frac{1}{2}b\left(\mu_1^2 - t^2\right) + \frac{1}{2}a\theta\left(\mu_1^2 - t^2\right) \\ + \frac{1}{3}b\theta\left(\mu_1^3 - t^3\right) - a\theta t\left(\mu_1 - t\right) - \frac{1}{2}b\theta t\left(\mu_1^2 - t^2\right) \end{array}\right]$$

Based on the assumptions and descriptions of the model, the total annual relevant profit (μ), include the following elements:

(i) Ordering cost (OC) = A (18)

(ii) Purchasing cost (PC) = cQ (19)

(iii) Screening cost (SrC) = zQ (20)

(iv) $HC = \int_0^T (x+yt)I(t)dt$

$$= \int_0^{t_1}(x+yt)I(t)dt + \int_{t_1}^{\mu_1}(x+yt)I(t)dt + \int_{\mu_1}^{\mu_2}(x+yt)I(t)dt + \int_{\mu_2}^{T}(x+yt)I(t)dt$$

$$= x\left(\begin{array}{l}\dfrac{1}{\left[1+\theta\left(\mu_1-\mu_2\right)\right]}\left(\begin{array}{l}a\left(T-\mu_2\right)+\frac{1}{2}b\left(T^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(T^3-\mu_2^3\right)\\ +\frac{1}{8}b\theta\left(T^4-\mu_2^4\right)-\frac{1}{2}a\theta\mu_2^2\left(T-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right)\\ -a\left(\mu_1-\mu_2\right)-\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right)-\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right)\\ -\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2\left(\mu_1-\mu_2\right)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right)\end{array}\right)\\ +a\mu_1+\frac{1}{2}b\mu_1^2\end{array}\right)t_1$$

$$- x\left(\begin{array}{l}\dfrac{1}{\left[1+\theta\left(\mu_1-\mu_2\right)\right]}\left((1\text{-}d)\left(\begin{array}{l}a\left(T-\mu_2\right)+\frac{1}{2}b\left(T^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(T^3-\mu_2^3\right)\\ +\frac{1}{8}b\theta\left(T^4-\mu_2^4\right)-\frac{1}{2}a\theta\mu_2^2\left(T-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right)\\ -a\left(\mu_1-\mu_2\right)-\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right)-\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right)\\ -\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2\left(\mu_1-\mu_2\right)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right)\end{array}\right)\right)\\ +(1\text{-}d)\left(a\mu_1+\frac{1}{2}b\mu_1^2\right)\end{array}\right)t_1$$

$$+x\left(aT+\frac{1}{2}bT^2+\frac{1}{6}a\theta T^3+\frac{1}{8}b\theta T^4\right)T$$

$$+x\left(\begin{array}{l}\dfrac{1}{\left[1+\theta\left(\mu_1-\mu_2\right)\right]}\left((1+\theta\mu_1)\left(\begin{array}{l}a\left(T-\mu_2\right)+\frac{1}{2}b\left(T^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(T^3-\mu_2^3\right)\\+\frac{1}{8}b\theta\left(T^4-\mu_2^4\right)-\frac{1}{2}a\theta\mu_2^2\left(T-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right)\\-a\left(\mu_1-\mu_2\right)-\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right)-\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right)\\-\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2\left(\mu_1-\mu_2\right)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right)\end{array}\right)\right)\\+a\mu_1+\frac{1}{2}b\mu_1^2+\frac{1}{2}a\theta\mu_1^2+\frac{1}{3}b\theta\mu_1^3\end{array}\right)\mu_2$$

$$-\frac{1}{4}\left(\frac{1}{6}xb\theta+y\left(-\frac{1}{2}b+\frac{1}{2}a\theta\right)\right)\mu_1^4$$

$$-\frac{1}{3}\left(x\left(-\frac{1}{2}b+\frac{1}{2}a\theta\right)+y\left(\begin{array}{l}-\dfrac{1}{\left[1+\theta\left(\mu_1-\mu_2\right)\right]}\\\left(\theta\left(\begin{array}{l}a\left(T-\mu_2\right)+\frac{1}{2}b\left(T^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(T^3-\mu_2^3\right)\\+\frac{1}{8}b\theta\left(T^4-\mu_2^4\right)-\frac{1}{2}a\theta\mu_2^2\left(T-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right)\\-a\left(\mu_1-\mu_2\right)-\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right)-\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right)\\-\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2\left(\mu_1-\mu_2\right)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right)\end{array}\right)\right)\\-a-a\theta\mu_1-\frac{1}{2}b\theta\mu_1^2\end{array}\right)\right)\mu_1^3$$

$$-\frac{1}{30}yb\theta\mu_1^5+\frac{1}{48}yb\theta T^6+\frac{1}{30}yb\theta\mu_2^5$$

$$+\frac{1}{2}\left(\begin{array}{l} x\left(\begin{array}{l} -\dfrac{1}{\left[1+\theta\left(\mu_1-\mu_2\right)\right]}\left(\theta\left(\begin{array}{l} a\left(T-\mu_2\right)+\dfrac{1}{2}b\left(T^2-\mu_2^2\right)+\dfrac{1}{6}a\theta\left(T^3-\mu_2^3\right)\\ +\dfrac{1}{8}b\theta\left(T^4-\mu_2^4\right)-\dfrac{1}{2}a\theta\mu_2^2\left(T-\mu_2\right)-\dfrac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right)\\ -a\left(\mu_1-\mu_2\right)-\dfrac{1}{2}b\left(\mu_1^2-\mu_2^2\right)-\dfrac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right)\\ -\dfrac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2\left(\mu_1-\mu_2\right)+\dfrac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right) \end{array}\right)\right)\\ -a-a\theta\mu_1-\dfrac{1}{2}b\theta\mu_1^2 \end{array}\right)\\ +y\left(\begin{array}{l} \dfrac{1}{\left[1+\theta\left(\mu_1-\mu_2\right)\right]}\left(\left(1+\theta\mu_1\right)\left(\begin{array}{l} a\left(T-\mu_2\right)+\dfrac{1}{2}b\left(T^2-\mu_2^2\right)+\dfrac{1}{6}a\theta\left(T^3-\mu_2^3\right)\\ +\dfrac{1}{8}b\theta\left(T^4-\mu_2^4\right)-\dfrac{1}{2}a\theta\mu_2^2\left(T-\mu_2\right)-\dfrac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right)\\ -a\left(\mu_1-\mu_2\right)-\dfrac{1}{2}b\left(\mu_1^2-\mu_2^2\right)-\dfrac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right)\\ -\dfrac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2\left(\mu_1-\mu_2\right)+\dfrac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right) \end{array}\right)\right)\\ +a\mu_1+\dfrac{1}{2}b\mu_1^2+\dfrac{1}{2}a\theta\mu_1^2+\dfrac{1}{3}b\theta\mu_1^3 \end{array}\right) \end{array}\right)\mu_2^2$$

$$+\frac{1}{5}\left(\frac{1}{8}xb\theta+\frac{1}{3}ya\theta\right)T^5+\frac{1}{4}\left(\frac{1}{3}xa\theta+y\left(-\frac{1}{2}b-\frac{1}{2}a\theta T-\frac{1}{4}b\theta T^2\right)\right)T^4$$

$$+\frac{1}{3}\left(x\left(-\frac{1}{2}b-\frac{1}{2}a\theta T-\frac{1}{4}b\theta T^2\right)-ya\right)T^3+\frac{1}{2}\left(-xa+y\left(aT+\frac{1}{2}bT^2+\frac{1}{6}a\theta T^3+\frac{1}{8}b\theta T^4\right)\right)T^2$$

$$+\frac{1}{2}\left(-xa+y\left(\begin{array}{l} \dfrac{1}{\left[1+\theta\left(\mu_1-\mu_2\right)\right]}\left((1-d)\left(\begin{array}{l} a\left(T-\mu_2\right)+\dfrac{1}{2}b\left(T^2-\mu_2^2\right)+\dfrac{1}{6}a\theta\left(T^3-\mu_2^3\right)\\ +\dfrac{1}{8}b\theta\left(T^4-\mu_2^4\right)-\dfrac{1}{2}a\theta\mu_2^2\left(T-\mu_2\right)-\dfrac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right)\\ -a\left(\mu_1-\mu_2\right)-\dfrac{1}{2}b\left(\mu_1^2-\mu_2^2\right)-\dfrac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right)\\ -\dfrac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2\left(\mu_1-\mu_2\right)+\dfrac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right) \end{array}\right)\right)\\ +(1-d)\left(a\mu_1+\dfrac{1}{2}b\mu_1^2\right) \end{array}\right)\right)\mu_1^2$$

$$-\frac{1}{2}\left(\begin{array}{l} x\left(\begin{array}{l} -\dfrac{1}{\left[1+\theta\left(\mu_1-\mu_2\right)\right]}\theta\left(\begin{array}{l} a\left(T-\mu_2\right)+\dfrac{1}{2}b\left(T^2-\mu_2^2\right)+\dfrac{1}{6}a\theta\left(T^3-\mu_2^3\right) \\ +\dfrac{1}{8}b\theta\left(T^4-\mu_2^4\right)-\dfrac{1}{2}a\theta\mu_2^2\left(T-\mu_2\right)-\dfrac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right) \\ -a\left(\mu_1-\mu_2\right)-\dfrac{1}{2}b\left(\mu_1^2-\mu_2^2\right)-\dfrac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right) \\ -\dfrac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2\left(\mu_1-\mu_2\right)+\dfrac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right) \end{array}\right) \\ -a-a\theta\mu_1-\dfrac{1}{2}b\theta\mu_1^2 \end{array}\right) \\ +y\left(\begin{array}{l} \dfrac{1}{\left[1+\theta\left(\mu_1-\mu_2\right)\right]}\left(1+\theta\mu_1\right)\left(\begin{array}{l} a\left(T-\mu_2\right)+\dfrac{1}{2}b\left(T^2-\mu_2^2\right)+\dfrac{1}{6}a\theta\left(T^3-\mu_2^3\right) \\ +\dfrac{1}{8}b\theta\left(T^4-\mu_2^4\right)-\dfrac{1}{2}a\theta\mu_2^2\left(T-\mu_2\right)-\dfrac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right) \\ -a\left(\mu_1-\mu_2\right)-\dfrac{1}{2}b\left(\mu_1^2-\mu_2^2\right)-\dfrac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right) \\ -\dfrac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2\left(\mu_1-\mu_2\right)+\dfrac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right) \end{array}\right) \\ +a\mu_1+\dfrac{1}{2}b\mu_1^2+\dfrac{1}{2}a\theta\mu_1^2+\dfrac{1}{3}b\theta\mu_1^3 \end{array}\right) \end{array}\right)\mu_1^2$$

$$-\frac{1}{2}\left(-xa+y\left(\begin{array}{l} \dfrac{1}{\left[1+\theta\left(\mu_1-\mu_2\right)\right]}\left(1-d\right)\left(\begin{array}{l} a\left(T-\mu_2\right)+\dfrac{1}{2}b\left(T^2-\mu_2^2\right)+\dfrac{1}{6}a\theta\left(T^3-\mu_2^3\right) \\ +\dfrac{1}{8}b\theta\left(T^4-\mu_2^4\right)-\dfrac{1}{2}a\theta\mu_2^2\left(T-\mu_2\right)-\dfrac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right) \\ -a\left(\mu_1-\mu_2\right)-\dfrac{1}{2}b\left(\mu_1^2-\mu_2^2\right)-\dfrac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right) \\ -\dfrac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2\left(\mu_1-\mu_2\right)+\dfrac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right) \end{array}\right) \\ +\left(1-d\right)\left(a\mu_1+\dfrac{1}{2}b\mu_1^2\right) \end{array}\right)\right)t_1^2$$

$$-x\left(aT+\frac{1}{2}bT^2+\frac{1}{6}a\theta T^3+\frac{1}{8}b\theta T^4\right)\mu_2-\frac{1}{48}yb\theta\mu_2^6$$

$$+\frac{1}{2}\left(-xa+y\left(\begin{array}{l}\frac{1}{\left[1+\theta\left(\mu_1-\mu_2\right)\right]}\left(\begin{array}{l}a\left(T-\mu_2\right)+\frac{1}{2}b\left(T^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(T^3-\mu_2^3\right)\\+\frac{1}{8}b\theta\left(T^4-\mu_2^4\right)-\frac{1}{2}a\theta\mu_2^2\left(T-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right)\\-a\left(\mu_1-\mu_2\right)-\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right)-\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right)\\-\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2\left(\mu_1-\mu_2\right)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right)\end{array}\right)\\+a\mu_1+\frac{1}{2}b\mu_1^2\end{array}\right)\right)t_1^2$$

$$+\frac{1}{4}\left(\frac{1}{6}xb\theta+y\left(-\frac{1}{2}b+\frac{1}{2}a\theta\right)\right)\mu_2^4$$

$$+\frac{1}{3}\left(x\left(-\frac{1}{2}b+\frac{1}{2}a\theta\right)+y\left(\begin{array}{l}-\frac{1}{\left[1+\theta\left(\mu_1-\mu_2\right)\right]}\\\left(\theta\left(\begin{array}{l}a\left(T-\mu_2\right)+\frac{1}{2}b\left(T^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(T^3-\mu_2^3\right)\\+\frac{1}{8}b\theta\left(T^4-\mu_2^4\right)-\frac{1}{2}a\theta\mu_2^2\left(T-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right)\\-a\left(\mu_1-\mu_2\right)-\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right)-\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right)\\-\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2\left(\mu_1-\mu_2\right)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right)\end{array}\right)\right)\\-a-a\theta\mu_1-\frac{1}{2}b\theta\mu_1^2\end{array}\right)\right)\mu_2^3$$

$$+x\left(\begin{array}{l}\frac{1}{\left[1+\theta\left(\mu_1-\mu_2\right)\right]}\left((1-d)\left(\begin{array}{l}a\left(T-\mu_2\right)+\frac{1}{2}b\left(T^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(T^3-\mu_2^3\right)\\+\frac{1}{8}b\theta\left(T^4-\mu_2^4\right)-\frac{1}{2}a\theta\mu_2^2\left(T-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right)\\-a\left(\mu_1-\mu_2\right)-\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right)-\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right)\\-\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2\left(\mu_1-\mu_2\right)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right)\end{array}\right)\right)\\+(1-d)\left(a\mu_1+\frac{1}{2}b\mu_1^2\right)\end{array}\right)\mu_1$$

$$-\frac{1}{8}yb\mu_1^4$$

$$-x\left(\frac{1}{\left[1+\theta(\mu_1-\mu_2)\right]}\left((1+\theta\mu_1)\left(\begin{array}{l} a(T-\mu_2)+\frac{1}{2}b(T^2-\mu_2^2)+\frac{1}{6}a\theta(T^3-\mu_2^3) \\ +\frac{1}{8}b\theta(T^4-\mu_2^4)-\frac{1}{2}a\theta\mu_2^2(T-\mu_2)-\frac{1}{4}b\theta\mu_2^2(T^2-\mu_2^2) \\ -a(\mu_1-\mu_2)-\frac{1}{2}b(\mu_1^2-\mu_2^2)-\frac{1}{2}a\theta(\mu_1^2-\mu_2^2) \\ -\frac{1}{3}b\theta(\mu_1^3-\mu_2^3)+a\theta\mu_2(\mu_1-\mu_2)+\frac{1}{2}b\theta\mu_2(\mu_1^2-\mu_2^2) \end{array}\right)\right) \\ +a\mu_1+\frac{1}{2}b\mu_1^2+\frac{1}{2}a\theta\mu_1^2+\frac{1}{3}b\theta\mu_1^3\right)\mu_1$$

$$-\frac{1}{5}\left(\frac{1}{8}xb\theta+\frac{1}{3}ya\theta\right)\mu_2^5-\frac{1}{4}\left(\frac{1}{3}xa\theta+y\left(-\frac{1}{2}b-\frac{1}{2}a\theta T-\frac{1}{4}b\theta T^2\right)\right)\mu_2^4$$

$$-\frac{1}{3}\left(x\left(-\frac{1}{2}b-\frac{1}{2}a\theta T-\frac{1}{4}b\theta T^2\right)-ya\right)\mu_2^3-\frac{1}{2}\left(-xa+y\left(aT+\frac{1}{2}bT^2+\frac{1}{6}a\theta T^3+\frac{1}{8}b\theta T^4\right)\right)\mu_2^2 \qquad (21)$$

$$+\frac{1}{3}\left(-\frac{1}{2}xb-yb\right)\mu_1^3$$

(by neglecting higher powers of θ)

(v) $$DC=c\left(\int_{\mu_1}^{\mu_2}\theta I(t)dt+\int_{\mu_2}^{T}\theta tI(t)dt\right)$$

$$=c\theta\left(\begin{array}{l} \frac{1}{1+\theta(\mu_1-\mu_2)} \\ \left(\left(\begin{array}{l} a(T-\mu_2)+\frac{1}{2}b(T^2-\mu_2^2)+\frac{1}{6}a\theta(T^3-\mu_2^3)+\frac{1}{8}b\theta(T^4-\mu_2^4)-\frac{1}{2}a\theta\mu_2^2(T-\mu_2) \\ -\frac{1}{4}b\theta\mu_2^2(T^2-\mu_2^2)-a(\mu_1-\mu_2)-\frac{1}{2}b(\mu_1^2-\mu_2^2)-\frac{1}{2}a\theta(\mu_1^2-\mu_2^2)-\frac{1}{3}b\theta(\mu_1^3-\mu_2^3) \\ +a\theta\mu_2(\mu_1-\mu_2)+\frac{1}{2}b\theta\mu_2(\mu_1^2-\mu_2^2) \end{array}\right) \\ \left(\mu_2+\theta\left(\mu_1\mu_2-\frac{1}{2}\mu_2^2\right)\right)\right) \\ +a\left(\mu_1\mu_2-\frac{1}{2}\mu_2^2\right)+\frac{1}{2}b\left(\mu_1^2\mu_2-\frac{1}{3}\mu_2^3\right)+\frac{1}{2}a\theta\left(\mu_1^2\mu_2-\frac{1}{3}\mu_2^3\right)+\frac{1}{3}b\theta\left(\mu_1^3\mu_2-\frac{1}{4}\mu_2^4\right) \\ -a\theta\left(\frac{1}{2}\mu_1\mu_2^2-\frac{1}{3}\mu_2^3\right)-\frac{1}{2}b\theta\left(\frac{1}{2}\mu_1^2\mu_2^2-\frac{1}{4}\mu_2^4\right) \end{array}\right)$$

$$- c\theta\left(\frac{1}{1+\theta(\mu_1-\mu_2)}\left(\begin{pmatrix} a\left(T-\mu_2\right)+\frac{1}{2}b\left(T^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(T^3-\mu_2^3\right)+\frac{1}{8}b\theta\left(T^4-\mu_2^4\right) \\ -\frac{1}{2}a\theta\mu_2^2\left(T-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(T^2-\mu_2^2\right)-a(\mu_1-\mu_2)-\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right) \\ -\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right)-\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2(\mu_1-\mu_2)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right)\end{pmatrix} \\ \left(\mu_1+\frac{1}{2}\theta\mu_1^2\right)\right) \\ +\frac{1}{2}a\mu_1^2+\frac{1}{3}b\mu_1^3+\frac{1}{6}a\theta\mu_1^3+\frac{1}{8}b\theta\mu_1^4\right)$$

$$+c\theta\begin{pmatrix}\frac{1}{48}b\theta T^6+\frac{1}{15}a\theta T^5+\frac{1}{4}\left(-\frac{1}{2}b-\frac{1}{2}a\theta T-\frac{1}{4}b\theta T^2\right)T^4 \\ -\frac{1}{3}aT^3+\frac{1}{2}\left(aT+\frac{1}{2}bT^2+\frac{1}{6}a\theta T^3+\frac{1}{8}b\theta T^4\right)T^2\end{pmatrix}$$

$$- c\theta\begin{pmatrix}\frac{1}{48}b\theta\mu_2^6+\frac{1}{15}a\theta\mu_2^5+\frac{1}{4}\left(-\frac{1}{2}b-\frac{1}{2}a\theta T-\frac{1}{4}b\theta T^2\right)\mu_2^4 \\ -\frac{1}{3}a\mu_2^3+\frac{1}{2}\left(aT+\frac{1}{2}bT^2+\frac{1}{6}a\theta T^3+\frac{1}{8}b\theta T^4\right)\mu_2^2\end{pmatrix} \tag{22}$$

(vi) $$SR = \left(p\int_0^T (a+bt)dt + p_d dQ\right) = p\left(aT+\frac{1}{2}bT^2\right)+p_d dQ \tag{23}$$

The total profit (π) during a cycle consisted of the following:

$$\pi = \frac{1}{T}\left[SR - OC - PC - SrC - HC - DC\right] \tag{24}$$

Substituting values from equations (18) to (23) in equation (24), we get total profit per unit. Putting $\mu_1 = v_1T$ and $\mu_2 = v_2T$ and value of t_1 and Q in equation (24), we get profit in terms of T. Differentiating equation (24) with respect to T and equate it to zero, we have

$$\text{i.e. } \frac{d\pi}{dT} = 0 \tag{25}$$

provided it satisfies the condition

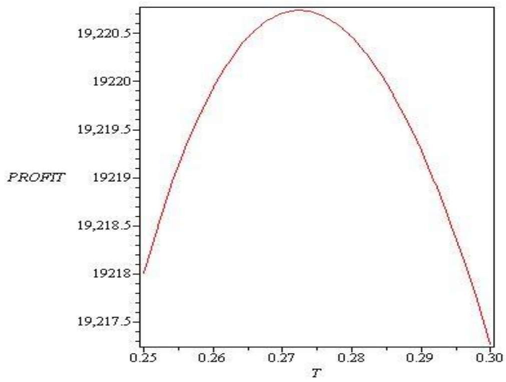
$$\frac{d^2\pi}{dT^2} < 0. \tag{26}$$

4. NUMERICAL EXAMPLE:

Considering A= Rs.100, a = 500, b=0.05, c=Rs. 25, p= Rs. 40, p_d = 15, d= 0.02, z = 0.40, λ= 10000, θ=0.05, x = Rs. 5, y=0.05, v_1=0.30, v_2 = 0.50, in appropriate units. The optimal value of T* =0.2724, Profit*= Rs. 19220.7443 and optimum order quantity Q* = 136.4673.

The second order conditions given in equation (26) are also satisfied. The graphical representation of the concavity of the profit function is also given.

T and Profit



Graph 1

5. SENSITIVITY ANALYSIS:

On the basis of the data given in example above we have studied the sensitivity analysis by changing the following parameters one at a time and keeping the rest fixed.

Table 1

Sensitivity Analysis

Parameter	%	T	Profit	Q
a	+20%	0.2489	23141.4159	149.6038
	+10%	0.2598	21180.2957	143.1553
	-10%	0.2869	17263.0045	129.3743
	-20%	0.3043	15307.3889	121.9119
x	+20%	0.2505	19156.0362	125.4731
	+10%	0.2608	19187.7131	130.6433
	-10%	0.2857	19255.3175	143.1463
	-20%	0.3011	19291.6680	150.8821
θ	+20%	0.2702	19215.7560	135.4150
	+10%	0.2713	19218.2447	135.9413
	-10%	0.2735	19223.2550	136.9928
	-20%	0.2746	19225.7770	137.5178
A	+20%	0.2980	19150.6195	149.3247
	+10%	0.2855	19184.8953	143.0459
	-10%	0.2586	19258.4097	129.5389
	-20%	0.2440	19298.2026	122.2109
λ	+20%	0.2724	19220.8583	136.4673
	+10%	0.2724	19220.8064	136.4673
	-10%	0.2723	19220.6683	136.4171
	-20%	0.2723	19220.5734	136.4171

From the table we observe that as parameter a increases/ decreases average total profit and optimum order quantity also increases/ decreases.
Also, we observe that with increase and decrease in the value of θ and x, there is corresponding decrease/ increase in total profit and optimum order quantity.
From the table we observe that as parameter A increases/ decreases average total profit decreases/ increases and optimum order quantity increases/ decreases.

6. CONCLUSION:
In this paper, we have developed an inventory model for deteriorating items with linear demand with different deterioration rates. Sensitivity with respect to parameters have been carried out. The results show that with the increase/ decrease in the parameter values there is corresponding increase/ decrease in the value of profit.

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