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Two Warehouse Inventory Model with Different Deterioration Rates under Linear Demand and Time Varying Holding Cost

S.R. Sheikh
Department of Statistics
Prof.V.B. Shah Institute of Management
Amroli, Surat, INDIA
E-mail: shehnazrsheikh@yahoo.com

R.D. Patel
Department of Statistics
Veer Narmad South Gujarat University
Surat, INDIA
E-mail: patelramanb@yahoo.co.in

Abstract

A two warehouse inventory model with different deterioration rates under linear demand is developed. Holding cost is considered as linear function of time. Shortages are not allowed. Numerical case is given to represent the model. Affectability investigation is likewise done for parameters.

Keywords: Two warehouse, deterioration, Linear demand, Time varying holding costs.

1. INTRODUCTION:

In real life situation many time retailers decides to by goods exceeding their Own Warehouse (OW) capacity to take advantage of price discounts. Therefore an additional stock is managed in Rented Warehouse (RW) which has better storage facilities with higher inventory carrying cost and low rate of deterioration. Bhunia [1] proposed two warehouse inventory model for deteriorating items with linear demand

and shortages. Yu [11] gave two warehouse inventory model for deteriorating items with decreasing rental over time. Tyagi [10] proposed model with time dependent and variable holding cost.

Goyal [2] studied recent trends in deteriorating inventory modeling. Gupta [3] suggested inventory model for stock dependent consumption rate. Parekh and Patel [5] developed deteriorating item inventory models for two warehouses with linear demand under inflation and permissible delay in payments. Ruxian [8] gave a review on deteriorating inventory study. Sana et al. [9] proposed model on pricing decision. Jaggi et al. [4] gave replenishment policy for non- instantaneous deteriorating items in two storage facilities under inflation. Raafat [6] gave comprehensive survey of literature on continuous deteriorating inventory model. Yu [12] developed a two warehouse inventory model with quantity discounts and maintenance actions under imperfect production processes.

Generally the products are such that there is no deterioration initially. After certain time deterioration starts and again after certain time the rate of deterioration increases with time. Here we have used such a concept and developed the deteriorating items inventory models.

In this paper we have developed a two warehouse inventory model with different deterioration rates. Demand function is linear, holding cost is time varying, Shortages are not allowed. Numerical case is given to represent the model. Affectability investigation is likewise done for parameters.

2. ASSUMPTIONS AND NOTATIONS:

NOTATIONS:

The following notations are used for the development of the model:

D(t) : Demand rate is a linear function of time ($a + bt$, $a>0$, $0<b<1$)

HC(OW) : Holding cost is linear function of time $t(x_1+y_1t$, $x_1>0$, $0<y_1<1)$ in OW.

HC(RW) : Holding cost is linear function of time $t(x_2+y_2t$, $x_2>0$, $0<y_2<1)$ in RW.

A : Replenishment cost per order

c : Purchasing cost per unit

p : Selling price per unit

T : Length of inventory cycle

$I_0(t)$: Inventory level in OW at time t.

$I_r(t)$: Inventory level in RW at time t.

Q : Order quantity

t_r : time at which inventory level becomes zero in RW.

W : capacity of own warehouse

θ : Deterioration rate during $\mu_1 \leq t \leq \mu_2$, $0< \theta<1$

θt : Deterioration rate during , $\mu_2 \leq t \leq T$, $0< \theta<1$

π : Total relevant profit per unit time.

ASSUMPTIONS:

The following assumptions are considered for the development of model.

- The demand of the product is declining as a linear function of time.
- Replenishment rate is infinite and instantaneous.
- Lead time is zero.
- Shortages are not allowed.
- OW has fixed capacity W units and RW has unlimited capacity.
- The goods of OW are consumed only after consuming the goods kept in RW.
- The unit inventory cost per unit in the RW is higher than those in the OW.
- Deteriorated units neither be repaired nor replaced during the cycle time.

3. THE MATHEMATICAL MODEL AND ANALYSIS:

At time t=0, Q units enters into the system of which W are stored in OW and rest (Q-W) are stored in RW. At time t_r level of inventory in RW reaches to zero because of depletion due to linear demand and OW inventory remains W. during the interval (t_r,μ_1) inventory depletes in OW due to linear demand, during interval (μ_1, μ_2) inventory depletes from OW due to deterioration at rate θ and linear demand. During interval (μ_2, T) inventory depletes due to joint effect of deterioration at rate θt and demand. By time T both the warehouses are empty.

Let I(t) be the inventory at time t $(0 \le t \le T)$ as shown in figure.

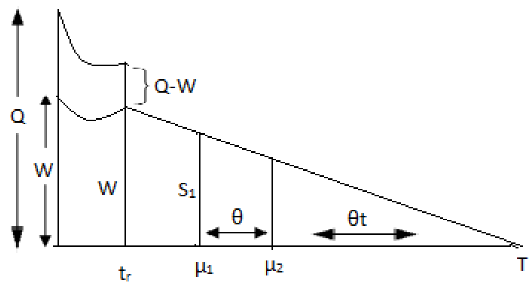


Figure 1

The differential equations which describes the instantaneous states of I(t) over the period (0, T) is given by

$$\frac{dI_r(t)}{dt} = -(a+bt) \qquad 0 \le t \le t_r \qquad (1)$$

$$\frac{dI_0(t)}{dt} = 0 \qquad 0 \le t \le t_r \qquad (2)$$

$$\frac{dI_0(t)}{dt}=-(a+bt) \qquad t_r \le t \le \mu_1 \qquad (3)$$

$$\frac{dI_0(t)}{dt}+\theta I_0(t)=-(a+bt) \qquad \mu_1 \le t \le \mu_2 \qquad (4)$$

$$\frac{dI_0(t)}{dt}+\theta t I_0(t)=-(a+bt) \qquad \mu_2 \le t \le T \qquad (5)$$

with initial conditions $I_0(0) = W$, $I(\mu_1) = S_1$, $I_0(t_r) = W$, $I_0(t_0) = 0$, $I_r(0) = Q\text{-}W$, $I_r(t_r) = 0$ and $I(T)=0$.

Solutions of these equations are given by

$$I_r(t)=Q-W-at-\frac{1}{2}bt^2 \qquad (6)$$

$$I_0(t)=W \qquad (7)$$

$$I_0(t) = S_1 + a\left(\mu_1 - t\right) + \frac{1}{2}b\left(\mu_1{}^2 - t^2\right) \qquad (8)$$

$$I_0(t) = \begin{bmatrix} S_1\left(1+\theta(\mu_1 - t)\right)+a\left(\mu_1 - t\right) + \frac{1}{2}a\theta\left(\mu_1{}^2 - t^2\right) + \frac{1}{2}b\left(\mu_1{}^2 - t^2\right) \\ + \frac{1}{3}b\theta\left(\mu_1{}^3 - t^3\right) - a\theta t\left(\mu_1 - t\right) - \frac{1}{2}b\theta t\left(\mu_1{}^2 - t^2\right) \end{bmatrix} \qquad (9)$$

$$I_0(t) = \begin{bmatrix} a\left(T-t\right)+\frac{1}{6}a\theta\left(T^3-t^3\right)+\frac{1}{2}b\left(T^2-t^2\right) \\ + \frac{1}{8}b\theta\left(T^4-t^4\right) - \frac{1}{2}a\theta t^2\left(T-t\right) - \frac{1}{4}b\theta t^2\left(T-t\right) \end{bmatrix} \qquad (10)$$

(by neglecting higher powers of θ)

Putting $\mu = t_r$ in equation (7) and (8), we get

$$I_0(t_r)=W \qquad (11)$$

$$I_0(t_r) = S_1 + a\left(\mu_1 - t_r\right) + \frac{1}{2}b\left(\mu_1{}^2 - t_r^2\right) \qquad (12)$$

Solving (11) and (12), we get

$$S_1 = W - a\left(\mu_1 - t_r\right) - \frac{1}{2}b\left(\mu_1{}^2 - t_r{}^2\right) \qquad (13)$$

Putting $t= t_r$ in equation (6), we get

$$Q = W + at_r + \frac{1}{2}bt_r^2 \qquad (14)$$

Putting $t = \mu_2$ in equations (9) and (10), we have

$$I_0(\mu_2) = \begin{bmatrix} S_1\left(1+\theta(\mu_1 - \mu_2)\right)+a\left(\mu_1 - \mu_2\right) + \frac{1}{2}a\theta\left(\mu_1^2 - \mu_2^2\right) + \frac{1}{2}b\left(\mu_1^2 - \mu_2^2\right) \\ + \frac{1}{3}b\theta\left(\mu_1^3 - \mu_2^3\right) - a\theta\mu_2\left(\mu_1 - \mu_2\right) - \frac{1}{2}b\theta\mu_2\left(\mu_1^2 - \mu_2^2\right) \end{bmatrix} \qquad (15)$$

$$I_0(\mu_2)=\begin{bmatrix} a\left(T-\mu_2\right)+\frac{1}{6}a\theta\left(T^3-\mu_2^3\right)+\frac{1}{2}b\left(T^2-\mu_2^2\right) \\ +\frac{1}{8}b\theta\left(T^4-\mu_1^4\right)-\frac{1}{2}a\theta\mu_2^2\left(T-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(T-\mu_2\right) \end{bmatrix} \tag{16}$$

From (15) and (16) solving for T and substituting S_1 from (13), we have

$$T=\frac{1}{a}\left[\left(W-a\left(\mu_1-t_r\right)-\frac{1}{2}b\left(\mu_1^{\,2}-t_r^{\,2}\right)\right)(1+\theta(\mu_1-\mu_2))+a\mu_1\right] \tag{17}$$

From (17), we see that T is a function of μ_1 and t_r, hence is not a decision variable.

Based on the assumptions and descriptions of the model, the total annual relevant profit (π), include the following elements:

(i) Ordering cost (OC) = A (18)

(ii) $HC(RW)=\int_0^{t_r} I_r(t)(x_2+y_2t)dt$

$$=\left[-\frac{1}{8}by_2t_r^{\,4}+\frac{1}{3}\left(-ay_2-\frac{1}{2}bx_2\right)t_r^{\,3}+\frac{1}{2}\left(\left(at_r+\frac{1}{2}bt_r^{\,2}\right)y_2-ax_2\right)t_r^{\,2}+\left(at_r+\frac{1}{2}bt_r^{\,2}\right)x_2t_r\right] \tag{19}$$

(iii) $HC(OW)=\int_0^{t_r}W(x_1+y_1t)dt+\int_{t_r}^{\mu_1}I_0(t)(x_1+y_1t)dt+\int_{\mu_1}^{\mu_2}I_0(t)(x_1+y_1t)dt+\int_{\mu_2}^{T}I_0(x_1+y_1t)dt$

$$=\begin{bmatrix} \frac{1}{2}Wy_1t_r^{\,2}+Wx_1t_r-\frac{1}{8}by_1\left(\mu_1^{\,4}-t_r^{\,4}\right)+\frac{1}{3}\left(-ay_1-\frac{1}{2}bx_1\right)\left(\mu_1^{\,3}-t_r^{\,3}\right) \\ +\frac{1}{2}\left(\left(W-a\left(\mu_1-t_r\right)-\frac{1}{2}b\left(\mu_1^{\,2}-t_r^{\,2}\right)+a\mu_1+\frac{1}{2}b\mu_1^{\,2}\right)y_1-ax_1\right)\left(\mu_1^{\,2}-t_r^{\,2}\right) \\ +\left(W-a\left(\mu_1-t_r\right)-\frac{1}{2}b\left(\mu_1^{\,2}-t_r^{\,2}\right)+a\mu_1+\frac{1}{2}b\mu_1^{\,2}\right)x_1\left(\mu_1-t_r\right) \end{bmatrix}$$

$$+\begin{bmatrix} \frac{1}{24}b\theta x_1\left(\mu_2^{\,4}-\mu_1^{\,4}\right)+\frac{1}{6}\left(a\theta-b\right)x_1\left(\mu_2^{\,3}-\mu_1^{\,3}\right) \\ +\frac{1}{2}\left(-\left(W-a\left(\mu_1-t_r\right)-\frac{1}{2}b\left(\mu_1^{\,2}-t_r^{\,2}\right)\right)\theta-a-a\theta\mu_1-\frac{1}{2}b\theta\mu_1^{\,2}\right)x_1\left(\mu_2^{\,2}-\mu_1^{\,2}\right) \\ +\begin{pmatrix} \left(W-a\left(\mu_1-t_r\right)-\frac{1}{2}b\left(\mu_1^{\,2}-t_r^{\,2}\right)\right)\left(1+\theta\mu_1\right) \\ +a\mu_1+\frac{1}{2}a\theta\mu_1^{\,2}+\frac{1}{2}b\mu_1^{\,2}+\frac{1}{3}b\theta\mu_1^{\,3} \end{pmatrix}x_1\left(\mu_2-\mu_1\right)\frac{1}{30}b\theta y_1\left(\mu_2^{\,5}-\mu_1^{\,5}\right) \\ +\frac{1}{8}\left(a\theta-b\right)y_1\left(\mu_2^{\,4}-\mu_1^{\,4}\right)+\frac{1}{3}\left(-\left(W-a\left(\mu_1-t_r\right)-\frac{1}{2}b\left(\mu_1^{\,2}-t_r^{\,2}\right)\right)\theta-a-a\theta\mu_1-\frac{1}{2}b\theta\mu_1^{\,2}\right)y_1\left(\mu_2^{\,3}-\mu_1^{\,3}\right) \\ +\frac{1}{2}\left(\left(W-a\left(\mu_1-t_r\right)-\frac{1}{2}b\left(\mu_1^{\,2}-t_r^{\,2}\right)\right)\left(1+\theta\mu_1\right)+a\mu_1+\frac{1}{2}a\theta\mu_1^{\,2}+\frac{1}{2}b\mu_1^{\,2}+\frac{1}{3}b\theta\mu_1^{\,3}\right)y_1\left(\mu_2^{\,2}-\mu_1^{\,2}\right) \end{bmatrix}$$

$$\left[\begin{array}{l} -\frac{1}{40}b\theta x_1\left(\frac{1}{a^5}\left(\left(W-a\left(\mu_1-t_r\right)-1/2b\left(\mu_1{}^2-t_r{}^2\right)\right)\left(1+\theta\mu_1-\theta\mu_2\right)+a\mu_1\right)^5-\mu_2{}^5\right) \\ +\frac{1}{4}\left(\frac{1}{3}a\theta+\frac{1}{4}b\theta\right)x_1\left(\frac{1}{a^4}\left(\left(W-a\left(\mu_1-t_r\right)-\frac{1}{2}b\left(\mu_1{}^2-t_r{}^2\right)\right)\left(1+\theta\mu_1-\theta\mu_2\right)+a\mu_1\right)^4-\mu_2{}^4\right) \\ +\frac{1}{3}\left(\begin{array}{l} -\frac{1}{2}b-\frac{1}{2}\theta\left(\left(W-a\left(\mu_1-t_r\right)-\frac{1}{2}b\left(\mu_1{}^2-t_r{}^2\right)\right)\left(1+\theta\mu_1-\theta\mu_2\right)+a\mu_1\right) \\ -\frac{1}{4a}b\theta\left(\left(W-a\left(\mu_1-t_r\right)-\frac{1}{2}b\left(\mu_1{}^2-t_r{}^2\right)\right)\left(\begin{array}{l}1+\theta\mu_1- \\ \theta\mu_2\end{array}\right)+a\mu_1\right) \end{array}\right)x_1 \\ \left(\frac{1}{a^3}\left(\left(W-a\left(\mu_1-t_r\right)-\frac{1}{2}b\left(\mu_1{}^2-t_r{}^2\right)\right)\left(1+\theta\mu_1-\theta\mu_2\right)+a\mu_1\right)^3-\mu_2{}^3\right) \end{array}\right]$$

$$\left[\begin{array}{l} -\frac{1}{48}b\theta y_1\left(\frac{1}{a^6}\left(\left(\begin{array}{l}W-a\left(\mu_1-t_r\right)- \\ \frac{1}{6}b\left(\mu_1{}^2-t_r{}^2\right)\end{array}\right)\left(1+\theta\mu_1-\theta\mu_2\right)+a\mu_1\right)^6-\mu_2{}^6\right) \\ +\frac{1}{5}\left(\frac{1}{3}a\theta+\frac{1}{4}b\theta\right)y_1\left(\frac{1}{a^5}\left(\left(\begin{array}{l}W-a\left(\mu_1-t_r\right)- \\ \frac{1}{2}b\left(\mu_1{}^2-t_r{}^2\right)\end{array}\right)\left(\begin{array}{l}1+\theta\mu \\ {}_1-\theta\mu_2\end{array}\right)+a\mu_1\right)^5-\mu_2{}^5\right) \end{array}\right]$$

$$+\left[\begin{array}{l} \frac{1}{4}\left(\begin{array}{l} -\frac{1}{2}b-\frac{1}{2}\theta\left(\left(W-a\left(\mu_1-t_r\right)-\frac{1}{2}b\left(\mu_1{}^2-t_r{}^2\right)\right)\left(1+\theta\mu_1-\theta\mu_2\right)+a\mu_1\right) \\ -\frac{b\theta}{4a}\left(W-a\left(\mu_1-t_r\right)-\frac{1}{2}b\left(\mu_1{}^2-t_r{}^2\right)\right)\left(1+\theta\mu_1-\theta\mu_2\right)+a\mu_1 \end{array}\right)y_1 \\ \left(\frac{1}{a^4}\left(\left(W-a\left(\mu_1-t_r\right)-\frac{1}{2}b\left(\mu_1{}^2-t_r{}^2\right)\right)\left(1+\theta\mu_1-\theta\mu_2\right)+a\mu_1\right)-\mu_2{}^4\right) \\ -\frac{1}{3}ay_1\left(\frac{1}{a^3}\left(\left(W-a\left(\mu_1-t_r\right)-\frac{1}{2}b\left(\mu_1{}^2-t_r{}^2\right)\right)\left(1+\theta\mu_1-\theta\mu_2\right)+a\mu_1\right)-\mu_2{}^3\right) \end{array}\right]$$

$$
+\left[\begin{array}{l}
\frac{1}{3}\left(\begin{array}{l}
-\frac{1}{2}b-\frac{1}{2}\theta\left(\left(W-a\left(\mu_1-t_r\right)-\frac{1}{2}b\left(\mu_1{}^2-t_r{}^2\right)\right)\left(1+\theta\mu_1-\theta\mu_2\right)+a\mu_1\right)\\
-\frac{b\theta}{4a}\left(\left(W-a\left(\mu_1-t_r\right)-\frac{1}{2}b\left(\mu_1{}^2-t_r{}^2\right)\right)\left(1+\theta\mu_1-\theta\mu_2\right)+a\mu_1\right)
\end{array}\right)x_1\\
\left(\frac{1}{a^3}\left(\left(W-a\left(\mu_1-t_r\right)-\frac{1}{2}b\left(\mu_1{}^2-t_r{}^2\right)\right)\left(1+\theta\mu_1-\theta\mu_2\right)+a\mu_1\right)^3-\mu_2{}^3\right)\\
\left(W-a\left(\mu_1-t_r\right)-\frac{1}{2}b\left(\mu_1{}^2-t_r{}^2\right)\right)\left(1+\theta\mu_1-\theta\mu_2\right)-a\mu_1
\end{array}\right]
$$

$$
+\left[\begin{array}{l}
\frac{\theta}{6a^2}\left(\left(W-a\left(\mu_1-t_r\right)-\frac{1}{2}b\left(\mu_1{}^2-t_r{}^2\right)\right)\left(1+\theta\mu_1-\theta\mu_2\right)+a\mu_1\right)\\
+\frac{b}{2a^2}\left(\left(W-a\left(\mu_1-t_r\right)-\frac{1}{2}b\left(\mu_1{}^2-t_r{}^2\right)\right)\left(1+\theta\mu_1-\theta\mu_2\right)+a\mu_1\right)^2\\
+\frac{b\theta}{8a^4}\left(\left(W-a\left(\mu_1-t_r\right)-\frac{1}{2}b\left(\mu_1{}^2-t_r{}^2\right)\right)\left(1+\theta\mu_1-\theta\mu_2\right)+a\mu_1\right)^4\\
x_1\left(\frac{1}{a}\left(W-a\left(\mu_1-t_r\right)-\frac{1}{2}b\left(\mu_1{}^2-t_r{}^2\right)\right)\left(1+\theta\mu_1-\theta\mu_2\right)+a\mu_1-\mu_2\right)
\end{array}\right]
$$

$$
+\left[\begin{array}{l}
\frac{1}{2}\left(W-a\left(\mu_1-t_r\right)-\frac{1}{2}b\left(\mu_1{}^2-t_r{}^2\right)\right)\left(1+\theta\mu_1-\theta\mu_2\right)+a\mu_1\\
+\frac{\theta}{12a^2}\left(\left(W-a\left(\mu_1-t_r\right)-\frac{1}{2}b\left(\mu_1{}^2-t_r{}^2\right)\right)\left(1+\theta\mu_1-\theta\mu_2\right)+a\mu_1\right)+\\
\frac{b}{4a^2}\left(\left(W-a\left(\mu_1-t_r\right)-\frac{1}{2}b\left(\mu_1{}^2-t_r{}^2\right)\right)\left(1+\theta\mu_1-\theta\mu_2\right)+a\mu_1\right)^2\\
+\frac{b\theta}{16a^4}\left(\left(W-a\left(\mu_1-t_r\right)-\frac{1}{2}b\left(\mu_1{}^2-t_r{}^2\right)\right)\left(1+\theta\mu_1-\theta\mu_2\right)+a\mu_1\right)^4\\
y_1\left(\frac{1}{a^2}\left(\left(W-a\left(\mu_1-t_r\right)-\frac{1}{2}b\left(\mu_1{}^2-t_r{}^2\right)\right)\left(1+\theta\mu_1-\theta\mu_2\right)+a\mu_1\right)^2-\mu_2{}^2\right)
\end{array}\right] \qquad (20)
$$

(iv) $DC = c\theta\left[\int_{\mu_1}^{\mu_2} I_0(t)dt+\int_{\mu_2}^{T} tI_0(t)dt\right]$

$$= c\theta \left[\begin{pmatrix} -\frac{1}{48a^6} b\theta \left(\left(\begin{pmatrix} W-a(\mu_1 - t_r) \\ -1/2b(\mu_1^2 - t_r^2) \end{pmatrix} (1+\theta\mu_1 - \theta\mu_2) + a\mu_1 \right)^6 - \mu_2^6 \right) \\ +\frac{1}{5}\left(\frac{1}{3}a\theta + \frac{1}{4}b\theta\right)\left(\left(\frac{1}{a}\begin{pmatrix} W-a(\mu_1 - t_r) - \\ 1/2b(\mu_1^2 - t_r^2) \end{pmatrix} (1+\theta\mu_1 - \theta\mu_2) + a\mu_1 \right)^5 - \mu_2^5 \right) \\ +\frac{1}{4}\begin{pmatrix} -\frac{1}{2}b - \frac{1}{2}\theta\left(\left(W-a(\mu_1 - t_r) - 1/2b(\mu_1^2 - t_r^2)\right)(1+\theta\mu_1 - \theta\mu_2) + a\mu_1\right) \\ -\frac{1}{4a}b\theta\left(\left(W-a(\mu_1 - t_r) - 1/2b(\mu_1^2 - t_r^2)\right)(1+\theta\mu_1 - \theta\mu_2) + a\mu_1\right) \end{pmatrix} \\ \left(\left(\frac{1}{a^4}\left(W-a(\mu_1 - t_r) - \frac{1}{2}b(\mu_1^2 - t_r^2)\right)(1+\theta\mu_1 - \theta\mu_2) + a\mu_1 \right)^4 - \mu_2^4 \right) \end{pmatrix} \right]$$

$$- c\theta \left[\begin{pmatrix} \frac{1}{3a^2}\left(\left(\begin{pmatrix} W-a(\mu_1 - t_r) - \\ \frac{1}{2}b(\mu_1^2 - t_r^2) \end{pmatrix} (1+\theta\mu_1 - \theta\mu_2) + a\mu_1 \right)^3 - \mu_2^3 \right) \\ -\frac{1}{2a^2}\begin{pmatrix} a + \frac{1}{6a^2}\theta\left(\left(W-a(\mu_1 - t_r) - \frac{1}{2}b(\mu_1^2 - t_r^2)\right)(1+\theta\mu_1 - \theta\mu_2) + a\mu_1\right)^2 + \\ \frac{1}{2a^2}b\left(\left(W-a(\mu_1 - t_r) - \frac{1}{2}b(\mu_1^2 - t_r^2)\right)(1+\theta\mu_1 - \theta\mu_2) + a\mu_1\right) + \\ \frac{1}{8a^4}b\theta\left(\left(W-a(\mu_1 - t_r) - \frac{1}{2}b(\mu_1^2 - t_r^2)\right)(1+\theta\mu_1 - \theta\mu_2) + a\mu_1\right)^3 \end{pmatrix} (T^2 - \mu_2^2) \end{pmatrix} \right]$$

$$+ c\theta \left[\begin{pmatrix} \frac{1}{24}b\theta(\mu_2^4 - \mu_1^4) + \frac{1}{3}\left(\frac{1}{2}a\theta - \frac{1}{2}b\right)(\mu_2^3 - \mu_1^3) \\ +\frac{1}{2}\left(-\left(W-a(\mu_1 - t_r) - \frac{1}{2}b(\mu_1^2 - t_r^2)\right)\theta - a - a\theta\mu_1 - \frac{1}{2}b\theta\mu_1^2 \right)(\mu_2^2 - \mu_1^2) \\ +\left(W-a(\mu_1 - t_r) - \frac{1}{2}b(\mu_1^2 - t_r^2)\right)(1+\theta\mu_1)(\mu_2 - \mu_1) + a\mu_1(\mu_2 - \mu_1) \\ +\frac{1}{2}a\theta\mu_1^2(\mu_2 - \mu_1) + \frac{1}{2}b\mu_1^2(\mu_2 - \mu_1) + \frac{1}{3}b\theta\mu_1^3(\mu_2 - \mu_1) \end{pmatrix} \right] \quad (21)$$

(v) $SR = p\int_0^T (a + bt)dt$

$$= p\left(aT + \frac{1}{2}bT^2\right) \quad (22)$$

The total profit (π) during a cycle, T consisted of the following:

$$\pi = \frac{1}{T}\left[SR - OC - HC(RW) - HC(OW) - DC\right] \quad (23)$$

Substituting values from equations (18) to (22) in equation (23), we get total profit per unit. Putting $\mu_1 = v_1T$ and $\mu_2 = v_2T$ in equation (23), value of S_1 and T from equations (13) and (17) in equation (23), we get profit in terms of t_r. Differentiating equation (23) with respect to t_r and equate it to zero, we have

$$\text{i.e. } \frac{d\pi}{dt_r} = 0 \quad (24)$$

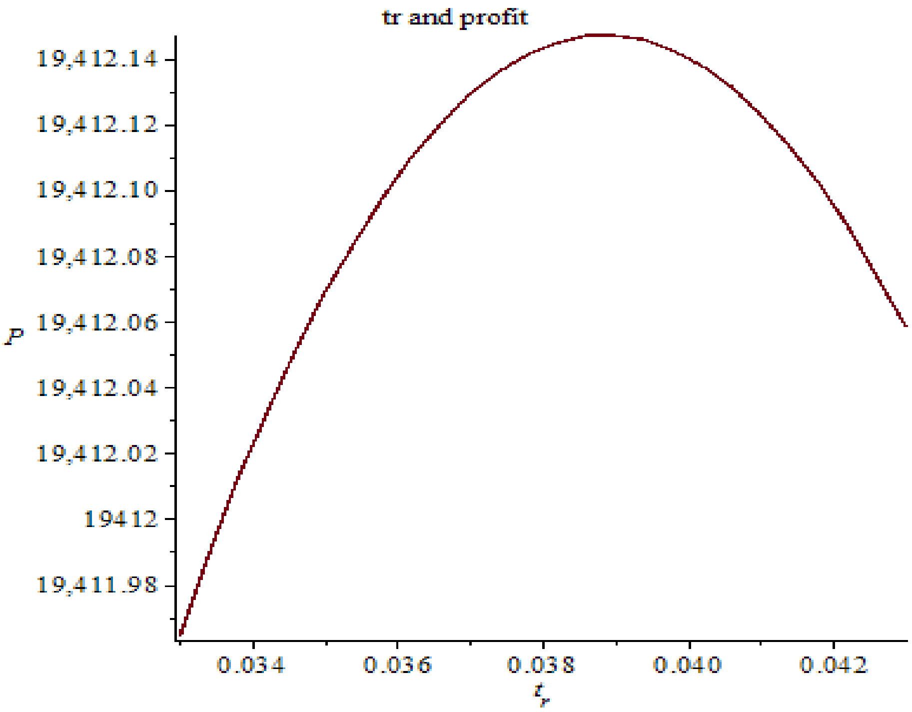
provided it satisfies the condition

$$\frac{d^2\pi}{dt_r^2} < 0 \quad (25)$$

4. NUMERICAL EXAMPLE:

Considering A= Rs.100, a = 500, W=136, b=0.05, c=Rs. 25, p = 40, θ=0.05, x_1 = Rs. 3, y_1=0.05, x_2 = Rs. 6, y_2=0.06, v_1=0.30, v_2=0.50, in appropriate units. The optimal value of t_r^* =0.0388, Profit* = 19412.1471 and Q*= Rs. 155.4000.

The second order conditions given in equation (25) are also satisfied. The graphical representation of the concavity of the profit function is also given.



5. SENSITIVITY ANALYSIS:

On the basis of the data given in example above we have studied the sensitivity analysis by changing the following parameters one at a time and keeping the rest fixed.

Table 1
Sensitivity Analysis

Parameter	%	t_r	T	Profit	Q
a	+20%	0.0488	0.2747	23350.7549	165.2800
	+10%	0.0468	0.2932	21376.6323	161.7400
	-10%	0.0316	0.3394	17444.3192	150.2200
	-20%	0.0269	0.3655	15470.6067	146.7600
θ	+20%	0.0368	0.3076	19406.3168	154.4000
	+10%	0.0378	0.3087	19409.2062	154.9000
	-10%	0.0398	0.3109	19415.0208	155.9000
	-20%	0.0409	0.3121	19418.0228	156.4500
x_1	+20%	0.0276	0.2987	19366.9121	149.8000
	+10%	0.0333	0.3043	19389.3638	152.6500
	-10%	0.0442	0.3152	19435.2442	158.1000
	-20%	0.0495	0.3204	19458.6390	160.7500
x_2	+20%	0.0331	0.3041	19410.8899	152.5500
	+10%	0.0357	0.3067	19411.4713	153.8500
	-10%	0.0425	0.3135	19412.9424	157.2500
	-20%	0.0469	0.3178	19413.8921	156.6500
A	+20%	0.0579	0.3288	19349.5301	164.9500
	+10%	0.0485	0.3194	19380.3727	160.2500
	-10%	0.0288	0.2998	19444.9447	150.4000
	-20%	0.0184	0.2895	19478.8731	145.2000

From the table we observe that as parameter a increases/ decreases average total profit increases/ decreases.

Also, we observe that with increase and decrease in the value of θ, x_1, and x_2 there is corresponding decrease/ increase in total profit.

From the table we observe that as parameter A increases/ decreases average total profit decreases/ increases.

CONCLUSION:

In this paper, we have developed a two warehouse inventory model for deteriorating items with different deterioration rates, linear demand and time varying holding cost. Sensitivity with respect to parameters has been carried out. The results show that with the increase/ decrease in the parameter values there is corresponding increase/ decrease in the value of profit.

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