Finite Difference Analysis of Boundary Layer Flow of Walters-B Viscoelastic Fluid In Porous Media with Variable Surface Heat and Mass Flux

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Abstract

Numerical study of the unsteady free convective heat and mass transfer in a Walters-B viscoelastic fluid past a vertical cone with electric conductivity and radiation is presented. The heat and mass flux at the surface of the cone is modeled as a power law, here $x$ denotes the coordinate along the slant face of the cone. The dimensionless unsteady, coupled and non-linear partial differential conservation equations for the boundary layer regime are solved by the implicit finite difference scheme. The velocity, temperature and concentration fields have been studied for the effect of electric conductivity parameter, Prandtl number (Pr), viscoelasticity parameter. The local skin-friction Nusselt number and Sherwood number are also presented and analyzed graphically. It is observed that the electric conductivity has significant effect on the viscoelastic fluid flow past vertical cone close to the cone surface. The present results are compared with available results in literature and are found to be in good agreement.

Keywords: cone; viscoelasticity; non-Darcian porous medium; finite difference method;
I. INTRODUCTION

Sivaraj and Rushi Kumar [7] analyze the effects of variable electric conductivity, non-uniform heat source/sink and higher order chemical reaction on unsteady, free convective viscoelastic fluid flow over a moving vertical cone and a flat plate saturated with porous medium.

The above studies did not consider combined viscoelastic, heat and mass transfer past a vertical cone in the presence of electric conductivity and thermal radiation. Owing to the significance of this problem in chemical technological processing the unsteady natural convection heat and mass transfer from a vertical cone with electric conductivity is considered in this paper.

II. MATHEMATICAL FORMULATION

An axi-symmetric transient natural convective heat and mass transfer in a viscoelastic fluid from an isothermal vertical cone with uniform surface temperature and concentration in a Darcy-Forchheimer fluid saturated isotropic porous medium in the presence of electric conductivity is considered. The Boussinesq’s approximation is taken into account for the buoyancy effects induced by thermal and mass diffusion. The co-ordinate system chosen (as shown in Fig. 1) is such that the \( x \)-coordinate is directed along the surface of the cone from the apex \( x = 0 \) and the \( y \)-coordinate is orientated perpendicular to this i.e. at right angles to the cone surface, outwards. Here, \( \phi \) designates the semi-vertical angle of the cone and \( r \) is the local radius of the cone.
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Under the above assumptions, implementing the shear-stress strain tensor for a Walters-B liquid, the appropriate unsteady incompressible conservation equations for the regime may be shown to take the form:

Proceeding with analysis, we implement the following non-dimensional quantities:

\[ X = \frac{x}{L}, \quad Y = \frac{y}{(Gr_L)^{\frac{1}{2}}}, \quad R = \frac{r}{L}, \quad \text{where} \ r = x \sin \phi, \]

\[ V = \frac{vL}{\nu} (Gr_L)^{-\frac{1}{2}}, \quad U = \frac{uL}{\nu} (Gr_L)^{-\frac{1}{2}}, \quad t = \frac{vt}{L} (Gr_L)^{\frac{1}{2}}, \quad Da = \frac{K}{L^2} \]

\[ T = \frac{T' - T''}{T''(L) - T''}, \quad Gr_L = \frac{g \beta (T''(L) - T'') L^3}{\nu^2}, \quad Pr = \frac{\nu}{\alpha}, \quad F = \frac{k k}{4 \sigma T''^3} \]

\[ C = \frac{C' - C''}{C''_w - C''_m}, \quad M = \frac{\sigma_r B_0^2 L}{\rho}, \quad \Gamma = \frac{k_g Gr_L^{\frac{1}{2}}}{L^2}, \quad N = \frac{\beta (C' - C')}{\beta (T'' - T'')} \]

\[ S_c = \frac{\nu}{D}, \quad F_s = \frac{b}{L} \]

Governing Equations in the non-dimensional form are

\[ \frac{\partial (UR)}{\partial X} + \frac{\partial (VR)}{\partial Y} = 0 \]

\[ \frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \frac{\partial^2 U}{\partial Y^2} - \Gamma \frac{\partial^3 U}{\partial Y^3} - MU^2 + T \cos \phi + NC \cos \phi - \frac{U}{DaGr_L} - \frac{F_s}{Da} U^2 \]

\[ \frac{\partial T}{\partial t} + U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = \frac{1}{Pr} \left[ 1 + \frac{4}{3 F} \right] \frac{\partial^2 T}{\partial Y^2} \]

\[ \frac{\partial C}{\partial t} + U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial Y} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2} \]

The corresponding initial and boundary conditions are

\[ t \leq 0 : U = 0, \quad V = 0, \quad T = 0, \quad C = 0 \quad \text{for all} \ X, Y, \]

\[ t > 0 : U = 0, \quad V = 0, \quad \frac{\partial T}{\partial Y} = -X'', \quad \frac{\partial C}{\partial Y} = -X'' \quad \text{at} \ Y = 0, \]

\[ U = 0, \quad T = 0, \quad C = 0 \quad \text{at} \ X = 0, \]

\[ U \to 0, \quad T \to 0, \quad C \to 0 \quad \text{as} \ Y \to \infty. \]

The dimensionless local values of the skin friction (surface shear stress), the Nusselt number (surface heat transfer gradient) and the Sherwood number (surface concentration gradient) are given by the following expressions:
\[ \tau_x = -\left( \frac{\partial U}{\partial Y} \right)_{Y=0} \]  
(7)

\[ Nu_x = -X \left( \frac{\partial T}{\partial Y} \right)_{Y=0} \]  
(8)

\[ Sh_x = -X \left( \frac{\partial C}{\partial Y} \right)_{Y=0} \]  
(9)

III. NUMERICAL SOLUTION

The transient, non-linear equations (2)–(5) under the conditions (6) are solved by an implicit finite difference scheme of Crank-Nicolson type which is discussed in [1],[3],[4] and [6]. The dimensionless governing equations are reduced to tri-diagonal system of equations and solved by Thomas algorithm as discussed in [2]. The region of integration is considered as a rectangle with \( X_{\text{max}} = 1 \) and \( Y_{\text{max}} = 22 \) where \( Y_{\text{max}} \) corresponds to \( Y = \infty \) which lies very well outside both the momentum and thermal boundary layers. The maximum of \( Y \) was chosen as 22, after some preliminary investigation so that the last two boundary conditions are satisfied within the tolerance limit \( 10^{-5} \). The mesh sizes have been fixed as \( \Delta X = 0.05 \), \( \Delta Y = 0.05 \) with time step \( \Delta t = 0.01 \). The scheme is unconditionally stable. The local truncation error is \( O(\Delta t^2 + \Delta Y^2 + \Delta X) \) and it tends to zero as \( \Delta t, \Delta X \) and \( \Delta Y \) tend to zero. Hence, the scheme is compatible. Stability and compatibility ensure the convergence.

IV. RESULTS AND DISCUSSION

Only selective figures have been reproduced here for brevity. In order to prove the accuracy of the computations in steady state at \( X = 1.0 \), \( Pr = 0.7 \), \( \eta = 1 \) and considering \( Gr^*_i = Gr_i \cos \phi = \frac{g \beta \cos \phi (T_\infty' - T_i') L^3}{\nu^2} \) are compared with available similarity solutions in the literature. The local Nusselt number \( Nu_x \) values for different Prandtl number are compared with the results of isothermal case (\( n = 0 \)) of [5] and [6], and are found to be in good agreement.

Figures 1(a) and 1(b) show the influence of radiation parameter, \( F \), on steady state velocity (\( U \)) and temperature (\( T \)) distributions with distance into the boundary layer, transverse to the cone surface (\( Y \)). An increase in \( F \) from 0 through 0.5, 1.0, 3.0, 5.0, 10.0 to 100.0, causes a significant reduction in velocity with distance into the boundary layer i.e. retards the flow. It is observed that the steady state temperature (\( T \)) distributions to different values of the viscoelastic material parameter \( \Gamma \) from 0 to 0.001, 0.003 and to the largest value of 0.005, decrease the temperature distributions accompanies an increase in \( \Gamma \).
V. CONCLUSIONS

The unsteady natural convective heat and mass transfer in a viscoelastic fluid past a vertical cone with electric conductivity is considered. The dimensionless conservation equations for the boundary layer regime are solved by the implicit finite difference scheme of Crank-Nicolson type. The increasing viscoelasticity (\( \Gamma \)) accelerates the streamwise velocity and shear stress (local skin friction), local Nusselt number and local Sherwood number. Decreasing \( F \) accelerates the flow and increases temperatures in the boundary layer regime. An increase in cone semi-apex angle (\( \phi \)) is observed to decelerate the flow near the cone surface but to increase temperatures and concentrations in the boundary layer regime. Increasing cone angle and thermal radiation also considerably increase the time taken to attain the steady state.

![Steady state profiles](image1)

![Steady state profiles](image2)

REFERENCES


