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Hadamard Product (Convolution) of Generalized k-Mittag-Leffler Function and A Class of Function

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Abstract

In this paper we introduce Hadamard Product (Convolution) of Generalized k-Mittag-Leffler function and A Class of Function in the Open Unit Disk $\mathbb{D}=\{z \in \mathbb{C} : |z| < 1\}$, Which are expressed in terms of the A Class of Function. Some interesting special cases of our main results are also considered.

AMS subject classification: 30C45, 33E12.
Keywords: Mittag-Leffler function, A Class of Function, Hadamard Product.

1. INTRODUCTION

In 1903, the Swedish mathematician Gosta Mittag-Leffler [11] (see also [12]) introduced and investigated the so-called Mittag-Leffler function

$$E_\alpha(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n + 1)} \qquad \dots (1.1)$$

Where $z \in \mathbb{C}, \alpha \in \mathbb{C}; Re(\alpha) > 0,\ \Gamma$ represents well known Gamma function.

Several properties of Mittag-Leffler function and generalized Mittag-Leffler function can be found e.g. in [2], [5], [6], [7], [8], [10], [13], [14], [15], [16], [23], [24] and [25].

The generalization of $E_\alpha(z)$, also known as Wiman function, is given by Wiman [25]:

$$E_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n + \beta)} \qquad \ldots (1.2)$$

where $\alpha, \beta \in \mathbb{C}, ; Re(\alpha) > 0$and $Re(\beta) > 0$.

Further, in 1971, Prabhakar [15] proposed the more general function $E_{\alpha,\beta}^{\gamma}(z)$ as:

$$E_{\alpha,\beta}^{\gamma}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_n}{n!\ \Gamma(\alpha n + \beta)}\ z^n \qquad \ldots (1.3)$$

for which $\alpha, \beta, \gamma \in \mathbb{C}, Re(\alpha) > 0, Re(\beta) > 0$ and $Re(\gamma) > 0$. The importance and great considerations of Mittag-Leffler function have led many researchers in the theory of special functions for exploring the possible generalizations and applications. Many more extensions or unifications for these functions are found in large number of papers [4], [18], [19], [20] and [22]. A useful generalization of the Mittag-Leffler function called as k-Mittag-Leffler function $E_{k,\alpha,\beta}^{\gamma}(z)$, introduced in [6], and it is given by

$$E_{k,\alpha,\beta}^{\gamma}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_{n,k}}{n!\ \Gamma_k(\alpha n + \beta)}\ z^n \qquad \ldots (1.4)$$

where $\alpha, \beta, \gamma \in \mathbb{C},\ k \in \mathbb{R}, \{Re(\alpha) > 0, Re(\beta) > 0$ and $Re(\gamma) > 0\}$ and $(\gamma)_{n,k}$ is the k-Pochhammer symbol defined as:

$$(\gamma)_{n,k} = \gamma(\gamma + k)(\gamma + 2k) \ldots (\gamma + (n-1)k) \qquad \ldots (1.5)$$

Where $\gamma \in \mathbb{C},\ k \in \mathbb{R}, n \in \mathbb{N}$.

Lately, a generalized form of k-Mittag-Leffler function was introduced and studied in [3] as:

Let $\alpha, \beta, \gamma \in \mathbb{C},\ k \in \mathbb{R}, \{Re(\alpha) > 0, Re(\beta) > 0$ and $Re(\gamma) > 0\}$ and $q \in (0,1) \cup \mathbb{N}$, then

$$E_{k,\alpha,\beta}^{\gamma,q}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_{nq,k}}{\Gamma_k(\alpha n + \beta)} \frac{z^n}{n!} \qquad \ldots (1.6)$$

where $(\gamma)_{nq,k}$ is defined as (1.5) and the generalized Pochhammer symbol is defined as (see [17]):

$$(\gamma)_{nq} = \frac{\Gamma(\gamma + nq)}{\Gamma(\gamma)} \qquad \text{... (1.7)}$$

In the integral representation, the generalized k-Gamma function is defined as:

$$\Gamma_k(n) = \int_0^\infty e^{\frac{-t^k}{k}} t^{n-1}\, dt \qquad \text{... (1.8)}$$

Let A denote the class of functions $f(z)$ normalized by

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \qquad \text{... (1.9)}$$

which are analytic in the open unit disc $\mathbb{D}= \{z \in \mathbb{C} : |z| < 1\}$.

Srivastava and Tomovski [22] proved that the generalized Mittag-Leffler function is an entire function in the complex z-plane.

Using Srivastava and Tomovski [22, theorem 1, P-201], we find that, if $Re(\alpha) \geq 0$ when $Re(q) \geq 0$ with $\beta \neq 0$, Then, the power series in the defining equation (1.6) is still analytic and converges absolutely in open unit disc $\mathbb{D}$ for all $\gamma \in \mathbb{C}$.

2. MAIN THEOREM

Theorem 2.1:

If $f(z) \in A$, $(z \in \mathbb{D})$, then Convolution of Generalized k-Mittag-Leffler function $\mathfrak{I}_{k,\alpha,\beta}^{\gamma,q}(z)$ and A Class of Function $f(z)$ is

$$\aleph_{k,\alpha,\beta}^{\gamma,q}(f)(z) = z + \sum_{n=2}^{\infty} \frac{(\gamma)_{nq,k}}{\Gamma_k(\alpha n + \beta)} \frac{\Gamma\left(\frac{\gamma}{k}\right) \Gamma\left(\frac{\alpha + \beta}{k}\right)}{k^{1+q-\left(\frac{\alpha+\beta}{k}\right)}\, \Gamma\left(\frac{\gamma + kq}{k}\right)} \frac{a_n}{n!} z^n \qquad \text{... (2.1)}$$

Provided that

- $\alpha, \beta, \gamma \in \mathbb{C},\ k \in \mathbb{R}, \{Re(\alpha) > 0, Re(\beta) > 0$ and $Re(\gamma) > 0\}$ and $q \in (0,1) \cup \mathbb{N}$.
- $(\gamma)_{nq,k}$ is generalized Pochhammer symbol.
- $f(z)$ is analytic in the open unit disc $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$.
- $\beta, \gamma \in \mathbb{C};\ Re(\alpha) > max\{0, Re(q) - 1\}, Re(q) > 0$ and $Re(\alpha) = 0$ when $Re(q) = 1$ with $\beta, \gamma \neq 0$.

Proof:

By definition of k-Mittag-Leffler function [3]

$$E_{k,\alpha,\beta}^{\gamma,q}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_{nq,k}}{\Gamma_k(\alpha n + \beta)} \frac{z^n}{n!}$$

Where $\alpha, \beta, \gamma \in \mathbb{C},\ k \in \mathbb{R}, \{Re(\alpha) > 0, Re(\beta) > 0$ and $Re(\gamma) > 0\}$ and $q \in (0,1) \cup \mathbb{N}$.

$$E_{k,\alpha,\beta}^{\gamma,q}(z) = \frac{1}{k^{\frac{\beta}{k}-1}\,\Gamma\left(\frac{\beta}{k}\right)} + k^{1+q-\left(\frac{\alpha+\beta}{k}\right)} \frac{\Gamma\left(\frac{\gamma+kq}{k}\right)}{\Gamma\left(\frac{\gamma}{k}\right)\Gamma\left(\frac{\alpha+\beta}{k}\right)}\, z + \sum_{n=2}^{\infty} \frac{(\gamma)_{nq,k}}{\Gamma_k(\alpha n + \beta)} \frac{z^n}{n!} \qquad \ldots (2.2)$$

Equation (2.2) can be written as

$$\frac{\Gamma\left(\frac{\gamma}{k}\right)\Gamma\left(\frac{\alpha+\beta}{k}\right)}{k^{1+q-\left(\frac{\alpha+\beta}{k}\right)}\,\Gamma\left(\frac{\gamma+kq}{k}\right)} \left\{ E_{k,\alpha,\beta}^{\gamma,q}(z) - \frac{1}{k^{\frac{\beta}{k}-1}\,\Gamma\left(\frac{\beta}{k}\right)} \right\}$$

$$= z + \sum_{n=2}^{\infty} \frac{(\gamma)_{nq,k}}{\Gamma_k(\alpha n + \beta)} \frac{\Gamma\left(\frac{\gamma}{k}\right)\Gamma\left(\frac{\alpha+\beta}{k}\right)}{k^{1+q-\left(\frac{\alpha+\beta}{k}\right)}\,\Gamma\left(\frac{\gamma+kq}{k}\right)} \frac{z^n}{n!} \qquad \ldots (2.3)$$

Now, we define the function $\Im_{k,\alpha,\beta}^{\gamma,q}(z)$ by

$$\Im_{k,\alpha,\beta}^{\gamma,q}(z) = \frac{\Gamma\left(\frac{\gamma}{k}\right)\Gamma\left(\frac{\alpha+\beta}{k}\right)}{k^{1+q-\left(\frac{\alpha+\beta}{k}\right)}\Gamma\left(\frac{\gamma+kq}{k}\right)}\left\{E_{k,\alpha,\beta}^{\gamma,q}(z) - \frac{1}{k^{\frac{\beta}{k}-1}\Gamma\left(\frac{\beta}{k}\right)}\right\} \quad \dots (2.4)$$

Where $(z \in \mathbb{D})$, $\beta, \gamma \in \mathbb{C}$; $Re(\alpha) > max\{0, Re(q) - 1\}, Re(q) > 0$ and $Re(\alpha) = 0$ when $Re(q) = 1$ with $\beta, \gamma \neq 0$.

Then equation (2.3)

$$\Im_{k,\alpha,\beta}^{\gamma,q}(z) = z + \sum_{n=2}^{\infty} \frac{(\gamma)_{nq,k}}{\Gamma_k(\alpha n+\beta)} \frac{\Gamma\left(\frac{\gamma}{k}\right)\Gamma\left(\frac{\alpha+\beta}{k}\right)}{k^{1+q-\left(\frac{\alpha+\beta}{k}\right)}\Gamma\left(\frac{\gamma+kq}{k}\right)} \frac{z^n}{n!} \quad \dots (2.5)$$

Let $f(z) \in A$, Denote by $\aleph_{k,\alpha,\beta}^{\gamma,q}(f): A \to A$ the operator is defined by

$$\aleph_{k,\alpha,\beta}^{\gamma,q}(f)(z) = \Im_{k,\alpha,\beta}^{\gamma,q}(z) * f(z) \quad \dots (2.6)$$

Where the symbol $(*)$ denotes the convolution (Hadamard product).

Putting the value of A class of functions $f(z)$ from (1.9) and value of function $\Im_{k,\alpha,\beta}^{\gamma,q}(z)$ from (2.5) in equation (2.6) then apply Hadamard product we have desired result.

Theorem 2.2:

If $f(z) \in A$, $(z \in \mathbb{D})$, then Convolution of Generalized k-Mittag-Leffler function $\Im_{k,\alpha,\beta}^{\gamma,q}(z)$ and A Class of Function $f(z) = \left(\frac{z}{1-z}\right)$ is

$$\aleph_{k,\alpha,\beta}^{\gamma,q}\left(\frac{z}{1-z}\right) = z + \sum_{n=2}^{\infty} \frac{(\gamma)_{nq,k}}{\Gamma_k(\alpha n+\beta)} \frac{\Gamma\left(\frac{\gamma}{k}\right)\Gamma\left(\frac{\alpha+\beta}{k}\right)}{k^{1+q-\left(\frac{\alpha+\beta}{k}\right)}\Gamma\left(\frac{\gamma+kq}{k}\right)} \frac{z^n}{n!} \quad \dots (2.7)$$

Provided that

- $\alpha, \beta, \gamma \in \mathbb{C},\ k \in \mathbb{R}, \{Re(\alpha) > 0, Re(\beta) > 0$ and $Re(\gamma) > 0\}$ and $q \in (0,1) \cup \mathbb{N}$.
- $(\gamma)_{nq,k}$ is generalized Pochhammer symbol.
- $f(z)$ is analytic in the open unit disc $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$.
- $\beta, \gamma \in \mathbb{C};\ Re(\alpha) > max\{0, Re(q) - 1\}, Re(q) > 0$ and $Re(\alpha) = 0$ when $Re(q) = 1$ with $\beta, \gamma \neq 0$.

Proof:

By definition of k-Mittag-Leffler function [3]

$$E_{k,\alpha,\beta}^{\gamma,q}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_{nq,k}}{\Gamma_k(\alpha n + \beta)} \frac{z^n}{n!}$$

Where $\alpha, \beta, \gamma \in \mathbb{C},\ k \in \mathbb{R}, \{Re(\alpha) > 0, Re(\beta) > 0$ and $Re(\gamma) > 0\}$ and $q \in (0,1) \cup \mathbb{N}$.

$$E_{k,\alpha,\beta}^{\gamma,q}(z) = \frac{1}{k^{\frac{\beta}{k}-1}\,\Gamma\left(\frac{\beta}{k}\right)} + k^{1+q-\left(\frac{\alpha+\beta}{k}\right)} \frac{\Gamma\left(\frac{\gamma + kq}{k}\right)}{\Gamma\left(\frac{\gamma}{k}\right)\,\Gamma\left(\frac{\alpha+\beta}{k}\right)}\ z + \sum_{n=2}^{\infty} \frac{(\gamma)_{nq,k}}{\Gamma_k(\alpha n + \beta)} \frac{z^n}{n!} \qquad \text{... (2.8)}$$

Equation (2.8) can be written as

$$\frac{\Gamma\left(\frac{\gamma}{k}\right)\,\Gamma\left(\frac{\alpha+\beta}{k}\right)}{k^{1+q-\left(\frac{\alpha+\beta}{k}\right)}\,\Gamma\left(\frac{\gamma+kq}{k}\right)} \left\{E_{k,\alpha,\beta}^{\gamma,q}(z) - \frac{1}{k^{\frac{\beta}{k}-1}\,\Gamma\left(\frac{\beta}{k}\right)}\right\} = z + \sum_{n=2}^{\infty} \frac{(\gamma)_{nq,k}}{\Gamma_k(\alpha n + \beta)} \frac{\Gamma\left(\frac{\gamma}{k}\right)\,\Gamma\left(\frac{\alpha+\beta}{k}\right)}{k^{1+q-\left(\frac{\alpha+\beta}{k}\right)}\,\Gamma\left(\frac{\gamma+kq}{k}\right)} \frac{z^n}{n!} \qquad \text{... (2.9)}$$

Now, we define the function $\mathfrak{I}_{k,\alpha,\beta}^{\gamma,q}(z)$ by

$$\mathfrak{I}_{k,\alpha,\beta}^{\gamma,q}(z) = \frac{\Gamma\left(\frac{\gamma}{k}\right)\Gamma\left(\frac{\alpha+\beta}{k}\right)}{k^{1+q-\left(\frac{\alpha+\beta}{k}\right)}\Gamma\left(\frac{\gamma+kq}{k}\right)}\left\{E_{k,\alpha,\beta}^{\gamma,q}(z) - \frac{1}{k^{\frac{\beta}{k}-1}\Gamma\left(\frac{\beta}{k}\right)}\right\} \quad \ldots(2.10)$$

Where $(z \in \mathbb{D})$, $\beta,\gamma \in \mathbb{C}$; $Re(\alpha) > max\{0, Re(q)-1\}, Re(q) > 0$ and $Re(\alpha) = 0$ when $Re(q) = 1$ with $\beta,\gamma \neq 0$.

Then equation (2.9)

$$\mathfrak{I}_{k,\alpha,\beta}^{\gamma,q}(z) = z + \sum_{n=2}^{\infty} \frac{(\gamma)_{nq,k}}{\Gamma_k(\alpha n+\beta)} \frac{\Gamma\left(\frac{\gamma}{k}\right)\Gamma\left(\frac{\alpha+\beta}{k}\right)}{k^{1+q-\left(\frac{\alpha+\beta}{k}\right)}\Gamma\left(\frac{\gamma+kq}{k}\right)} \frac{z^n}{n!} \quad \ldots(2.11)$$

Let $f(z) = \left(\frac{z}{1-z}\right) \in A$, Denote by $\aleph_{k,\alpha,\beta}^{\gamma,q}\left(\frac{z}{1-z}\right): A \to A$ the operator is defined by

$$\aleph_{k,\alpha,\beta}^{\gamma,q}\left(\frac{z}{1-z}\right) = \mathfrak{I}_{k,\alpha,\beta}^{\gamma,q}(z) * \left(\frac{z}{1-z}\right) \quad \ldots(2.12)$$

Where the symbol $(*)$ denotes the convolution (Hadamard product).

Putting the value of A class of functions $f(z)$ from (1.9) and value of function $\mathfrak{I}_{k,\alpha,\beta}^{\gamma,q}(z)$ from (2.11) in equation (2.12) then apply Hadamard product we have desired result.

3. SPECIAL CASE

Corollary 3.1 If we take $\beta = 1, \gamma = 1, q = 1, k = 1$ and $\alpha = 0$ in equation (2.1) and let the condition of theorem 2.1 be satisfied, we arrive at the following result

$$\aleph_{1,0,1}^{1,1}(f)(z) = z + \sum_{n=2}^{\infty} a_n z^n$$

$$= f(z)$$

Corollary 3.2 Let the condition of theorem 2.1 be satisfied and set $\alpha = 0, \beta = 1, \gamma = 0, q = 1, k = 1$, the equation (2.1) reduces to

$$\aleph_{1,0,1}^{0,1}(f)(z) = z + \sum_{n=2}^{\infty} \frac{1}{n} a_n \, z^n$$

$$= \int_0^z \frac{f(z)}{z} dz$$

Corollary 3.3 If we take $\alpha = 0, \beta = 1, \gamma = 2, q = 1, k = 1$ in the theorem 2.1 and let the condition of theorem 2.1 be satisfied, we arrive at the following result

$$\aleph_{1,0,1}^{2,1}(f)(z) = z + \sum_{n=2}^{\infty} \frac{(n+1)}{2} a_n \, z^n$$

$$= \frac{1}{2}\{f(z) + zf'(z)\}$$

Corollary 3.4 Let the condition of theorem 2.2 be satisfied and set $\alpha = 1, \ \beta = 1, \gamma = 0, q = 1, k = 1$, the equation (2.7) reduces to

$$\aleph_{1,1,1}^{1,1}\left(\frac{z}{1-z}\right) = z + \sum_{n=2}^{\infty} \frac{z^n}{n!}$$

$$= e^z - 1$$

Corollary 3.5 If we take $\alpha = 0, \beta = 1, \gamma = 2, q = 1, k = 1$ in the equation (2.7) and let the condition of theorem 2.1 be satisfied, we arrive at the following result

$$\aleph_{1,1,0}^{1,1}\left(\frac{z}{1-z}\right) = z + \sum_{n=2}^{\infty} \frac{z^n}{(n-1)!}$$

$$= ze^z$$

4. CONCLUSION

In this paper we have presented Hadamard Product (Convolution) of Generalized k-Mittag-Leffler function and A Class of Function in the Open Unit Disk $\mathbb{D}=\{z \in \mathbb{C} : |z| < 1\}$. The result so established may be found useful in several interesting situation appearing in the literature on mathematical analysis. Further many known and unknown results have been established in terms of the A Class of Function. The results presented in this paper are easily converted in terms of the generalized Mittag-leffler function. We are also trying to find certain possible applications of those results presented here to some other research areas.

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