3.3 Example: Consider the ring Z_6. $\tilde{h}_A: Z_6 \to [0,1]$ defined by $\tilde{h}_A(0) = 0.6$, $\tilde{h}_A(1) = 1.0$, $\tilde{h}_A(2) = 0.6$, $\tilde{h}_A(3) = 0.3$, $\tilde{h}_A(4) = 0.0$, $\tilde{h}_A(5) = 0.0$. Consider
$\tilde{h}_A(2) = \tilde{h}_A(1+1) \geq \rho\min\{0.4,1,0.4,1\}$

0.6≥0.4. Hence it is an interval-valued hesitant fuzzy subnearing. Also

$\tilde{h}_A(2) = \tilde{h}_A(1+1) \leq \rho\max\{0.4,1,0.4,1\}$

$0.6 \leq 1$

Hence it is an interval-valued anti-hesitant fuzzy subnearring.

3.4 Definition: Let X be a set, an Interval-Valued Dual Hesitant Fuzzy Set (IVDHFS) D on X is defined as:

D = $\{\langle x, h(x), g(x)\rangle / x\epsilon X\}$ where $h(x) and g(x)$are two sets of values in the interval [0,1], denoting the possible membership and non – membership degree of the element $x\epsilon X$ to the set D respectively, with the following conditions, $0 \leq \gamma, \eta \leq 1, \quad 0 \leq \gamma^+ + \eta^+ \leq 1$ where

$\gamma \in h(x), \eta \in g(x), \gamma^+ = \max_{\gamma\epsilon h(x)}\{\gamma\}, and \eta^+ = \max_{\eta\epsilon g(x)}\{\eta\}$ for all $x\epsilon X$. The pair d(x)= (h(x), g(x)) is called Dual Hesitant Fuzzy Element (DHFE) and noted by d=(h,g).

3.5 Definition: Let $\tilde{h}_A^{\sigma(\kappa)}$ and $\tilde{h}_B^{\sigma(\kappa)}$ be any two interval – valued hesitant fuzzy subsets R and H̃, respectively. The anti product of $\tilde{h}_A^{\sigma(\kappa)} \times \tilde{h}_B^{\sigma(\kappa)}$ is defined as
$\tilde{h}_A^{\sigma(\kappa)} \times \tilde{h}_B^{\sigma(\kappa)} = \{\langle (x,y), \tilde{h}_A^{\sigma(\kappa)} \times \tilde{h}_B^{\sigma(\kappa)}(x,y)\rangle / for\ all\ x\ in\ R\ and\ y\ in$ H̃ where
$\tilde{h}_A^{\sigma(\kappa)} \times \tilde{h}_B^{\sigma(\kappa)}(x,y) = \wp\max\{\tilde{h}_A^{\sigma(\kappa)}(x), \tilde{h}_B^{\sigma(\kappa)}(y)\}$.

3.6 Definition: Let $\tilde{h}_A^{\sigma(\kappa)}$ be an interval-valued hesitant fuzzy subset in a set S, the strongest interval-valued anti-hesitant fuzzy relation on S, that is an interval-valued anti-hesitant fuzzy relation $\tilde{h}_V^{\sigma(\kappa)}$ with respect to $\tilde{h}_A^{\sigma(\kappa)}$given by
$\tilde{h}_V^{\sigma(\kappa)}(x,y) = \wp\max\{\tilde{h}_A^{\sigma(\kappa)}(x), \tilde{h}_A^{\sigma(\kappa)}(y)\}$ for all x and y in S.

4. PROPERTIES OF IVAHFS

4.1 Theorem: If$\tilde{h}_A^{\sigma(\kappa)}$ is an interval – valued anti-hesitant fuzzy subnearring of a nearring $(R,+,\cdot)$ then $\tilde{h}_A^{\sigma(\kappa)}(-x) = \tilde{h}_A^{\sigma(\kappa)}(x)$ and $\tilde{h}_A^{\sigma(\kappa)}(x) \geq \tilde{h}_A^{\sigma(\kappa)}(e)$ for x in R, the identity element e in R.

Proof: Let x and the identity element e be in R.

Now $\tilde{h}_A^{\sigma(\kappa)}(x) = \tilde{h}_A^{\sigma(\kappa)}(-(-x))$

$\leq \tilde{h}_A^{\sigma(\kappa)}(-x)$

$\leq \tilde{h}_A^{\sigma(\kappa)}(x)$

Therefore $\tilde{h}_A^{\sigma(\kappa)}(-x) = \tilde{h}_A^{\sigma(\kappa)}(x)$ for all x in R.

Now $\tilde{h}_A^{\sigma(\kappa)}(e) = \tilde{h}_A^{\sigma(\kappa)}(x-x)$

$\leq \wp \max\left\{\left[\tilde{h}_A^{\sigma(\kappa)L}(x), \tilde{h}_A^{\sigma(\kappa)U}(x)\right], \left[\tilde{h}_A^{\sigma(\kappa)L}(-x), \tilde{h}_A^{\sigma(\kappa)U}(-x)\right]\right\}$

$\leq \wp max \left\{\left[\tilde{h}_A^{\sigma(\kappa)L}(x), \tilde{h}_A^{\sigma(\kappa)L}(-x)\right], \left[\tilde{h}_A^{\sigma(\kappa)U}(x), \tilde{h}_A^{\sigma(\kappa)U}(-x)\right]\right\}$

$\leq \wp max \left\{inf\left[\tilde{h}_A^{\sigma(\kappa)}(x), \tilde{h}_A^{\sigma(\kappa)}(-x)\right], sup\left[\tilde{h}_A^{\sigma(\kappa)}(x), \tilde{h}_A^{\sigma(\kappa)}(-x)\right]\right\}$

$\leq \wp max \left\{\left[\tilde{h}_A^{\sigma(\kappa)}(x), \tilde{h}_A^{\sigma(\kappa)}(-x)\right]\right\}$

$\leq \tilde{h}_A^{\sigma(\kappa)}(x)$

Therefore $\tilde{h}_A^{\sigma(\kappa)}(e) \leq \tilde{h}_A^{\sigma(\kappa)}(x)$ for all x in R.

4.2 Theorem: If $\tilde{h}_A^{\sigma(\kappa)}$ is an interval – valued anti-hesitant fuzzy subnearring of a nearring $(R, +, \cdot)$ and $\tilde{h}_A^{\sigma(\kappa)}(x-y) = \tilde{h}_A^{\sigma(\kappa)}(e)$ then $\tilde{h}_A^{\sigma(\kappa)}(x) = \tilde{h}_A^{\sigma(\kappa)}(y)$ for all x and y in R, the identity element e in R.

Proof: Let x and y in R, the identity element e in R. Now

$\tilde{h}_A^{\sigma(\kappa)}(x) = \tilde{h}_A^{\sigma(\kappa)}(x-y+y) \leq \wp \max\{\tilde{h}_A^{\sigma(\kappa)}(x-y), \tilde{h}_A^{\sigma(\kappa)}(y)\}$

$= \wp \max\{\tilde{h}_A^{\sigma(\kappa)}(e), \tilde{h}_A^{\sigma(\kappa)}(y)\} \quad = \tilde{h}_A^{\sigma(\kappa)}(y)$

$= \tilde{h}_A^{\sigma(\kappa)}(x-(x-y)$

$\leq \wp \max\left\{\left[\tilde{h}_A^{\sigma(\kappa)L}(x-y), \tilde{h}_A^{\sigma(\kappa)U}(x-y)\right], \left[\tilde{h}_A^{\sigma(\kappa)L}(x), \tilde{h}_A^{\sigma(\kappa)U}(x)\right]\right\}$

$\leq \wp max \left\{\left[\tilde{h}_A^{\sigma(\kappa)L}(x-y), \tilde{h}_A^{\sigma(\kappa)L}(x)\right], \left[\tilde{h}_A^{\sigma(\kappa)U}(x-y), \tilde{h}_A^{\sigma(\kappa)U}(x)\right]\right\}$

$\leq \wp max \left\{inf\left[\tilde{h}_A^{\sigma(\kappa)}(x-y), \tilde{h}_A^{\sigma(\kappa)}(x)\right], sup\left[\tilde{h}_A^{\sigma(\kappa)}(x-y), \tilde{h}_A^{\sigma(\kappa)}(x)\right]\right\}$

$\leq \wp max \left\{\left[\tilde{h}_A^{\sigma(\kappa)}(x-y), \tilde{h}_A^{\sigma(\kappa)}(x)\right]\right\}$

$$= \wp max\ \{[\tilde{h}_A^{\sigma(\kappa)}(e), \tilde{h}_A^{\sigma(\kappa)}(x)]\}$$

$$= \tilde{h}_A^{\sigma(\kappa)}(x)$$

Therefore $\tilde{h}_A^{\sigma(\kappa)}(x) = \tilde{h}_A^{\sigma(\kappa)}(y)$ for all x and y in R.

4.3 Theorem: If $\tilde{h}_A^{\sigma(\kappa)}$ is an interval–valued anti-hesitant fuzzy subnearring of a nearring $(R, +, \cdot)$ then $\tilde{\text{H}} = \left\{x/X\epsilon R: \tilde{h}_A^{\sigma(\kappa)}(x) = [0]\right\}$ is either empty or a subnearring of R.

Proof: If no, the element satisfies this condition then $\tilde{\text{H}}$ is empty.

If x and y in $\tilde{\text{H}}$ then

$$\tilde{h}_A^{\sigma(\kappa)}(x-y) \le \wp \max\left\{\left[\tilde{h}_A^{\sigma(\kappa)L}(x), \tilde{h}_A^{\sigma(\kappa)U}(x)\right], \left[\tilde{h}_A^{\sigma(\kappa)L}(y), \tilde{h}_A^{\sigma(\kappa)U}(y)\right]\right\}$$

$$\le \wp \max\left\{\left[\tilde{h}_A^{\sigma(\kappa)L}(x), \tilde{h}_A^{\sigma(\kappa)L}(y)\right], \left[\tilde{h}_A^{\sigma(\kappa)U}(x), \tilde{h}_A^{\sigma(\kappa)U}(y)\right]\right\}$$

$$\le \wp max\ \{inf\left[\tilde{h}_A^{\sigma(\kappa)}(x), \tilde{h}_A^{\sigma(\kappa)}(y)\right], sup\left[\tilde{h}_A^{\sigma(\kappa)}(x), \tilde{h}_A^{\sigma(\kappa)}(y)\right]\}$$

$$\le \wp max\ \left\{\left[\tilde{h}_A^{\sigma(\kappa)}(x), \tilde{h}_A^{\sigma(\kappa)}(y)\right]\right\}$$

$$= \wp max\ \{[[0], [0]]\}$$

$$= [0]$$

Therefore $\tilde{h}_A^{\sigma(\kappa)}(x-y) = [0]$ so get $x - y$ in $\tilde{\text{H}}$.

And $\tilde{h}_A^{\sigma(\kappa)}(xy) \le \wp \max\left\{\left[\tilde{h}_A^{\sigma(\kappa)L}(x), \tilde{h}_A^{\sigma(\kappa)U}(x)\right], \left[\tilde{h}_A^{\sigma(\kappa)L}(y), \tilde{h}_A^{\sigma(\kappa)U}(y)\right]\right\}$

$$\le \wp \max\left\{\left[\tilde{h}_A^{\sigma(\kappa)L}(x), \tilde{h}_A^{\sigma(\kappa)L}(y)\right], \left[\tilde{h}_A^{\sigma(\kappa)U}(x), \tilde{h}_A^{\sigma(\kappa)U}(y)\right]\right\}$$

$$\le \wp max\ \{inf\left[\tilde{h}_A^{\sigma(\kappa)}(x), \tilde{h}_A^{\sigma(\kappa)}(y)\right], sup\left[\tilde{h}_A^{\sigma(\kappa)}(x), \tilde{h}_A^{\sigma(\kappa)}(y)\right]\}$$

$$\le \wp max\ \left\{\left[\tilde{h}_A^{\sigma(\kappa)}(x), \tilde{h}_A^{\sigma(\kappa)}(y)\right]\right\}$$

$$= \wp max\ \{[[0], [0]]\}$$

$$= [0]$$

Therefore $\tilde{h}_A^{\sigma(\kappa)}(xy) = [0]$ so get xy in $\tilde{\text{H}}$.

Therefore $\tilde{\text{H}}$ is a subnearring of R. Hence $\tilde{\text{H}}$ is entire empty or a subnearring of R.

4.4 Theorem: If $\tilde{h}_A^{\sigma(\kappa)}$ is an interval–valued anti-hesitant fuzzy subnearring of a nearring $(R,+,\cdot)$ then $\widetilde{\mathrm{H}} = \left\{x\epsilon R: \tilde{h}_A^{\sigma(\kappa)}(x) = \tilde{h}_A^{\sigma(\kappa)}(e)\right\}$ is a subnearring of R.

Proof: Let *x* and *y* be in $\widetilde{\mathrm{H}}$.

Now

$$\tilde{h}_A^{\sigma(\kappa)}(x-y) \leq \wp \max\left\{\left[\tilde{h}_A^{\sigma(\kappa)L}(x), \tilde{h}_A^{\sigma(\kappa)U}(x)\right], \left[\tilde{h}_A^{\sigma(\kappa)L}(y), \tilde{h}_A^{\sigma(\kappa)U}(y)\right]\right\}$$

$$\leq \wp \max\left\{\left[\tilde{h}_A^{\sigma(\kappa)L}(x), \tilde{h}_A^{\sigma(\kappa)L}(y)\right], \left[\tilde{h}_A^{\sigma(\kappa)U}(x), \tilde{h}_A^{\sigma(\kappa)U}(y)\right]\right\}$$

$$\leq \wp max\ \{inf\left[\tilde{h}_A^{\sigma(\kappa)}(x), \tilde{h}_A^{\sigma(\kappa)}(y)\right], sup\left[\tilde{h}_A^{\sigma(\kappa)}(x), \tilde{h}_A^{\sigma(\kappa)}(y)\right]\}$$

$$\leq \wp max\ \left\{\left[\tilde{h}_A^{\sigma(\kappa)}(x), \tilde{h}_A^{\sigma(\kappa)}(y)\right]\right\}$$

$$= \wp max\ \left\{\left[\tilde{h}_A^{\sigma(\kappa)}(e), \tilde{h}_A^{\sigma(\kappa)}(e)\right]\right\}$$

$$= \tilde{h}_A^{\sigma(\kappa)}(e)$$

Therefore $\tilde{h}_A^{\sigma(\kappa)}(x-y) \leq \tilde{h}_A^{\sigma(\kappa)}(e)$

Hence $\tilde{h}_A^{\sigma(\kappa)}(e) = \tilde{h}_A^{\sigma(\kappa)}(x-y)$

Therefore $(x-y)$ in $\widetilde{\mathrm{H}}$.

And $\tilde{h}_A^{\sigma(\kappa)}(xy) \leq \wp \max\left\{\left[\tilde{h}_A^{\sigma(\kappa)L}(x), \tilde{h}_A^{\sigma(\kappa)U}(x)\right], \left[\tilde{h}_A^{\sigma(\kappa)L}(y), \tilde{h}_A^{\sigma(\kappa)U}(y)\right]\right\}$

$$\leq \wp \max\left\{\left[\tilde{h}_A^{\sigma(\kappa)L}(x), \tilde{h}_A^{\sigma(\kappa)L}(y)\right], \left[\tilde{h}_A^{\sigma(\kappa)U}(x), \tilde{h}_A^{\sigma(\kappa)U}(y)\right]\right\}$$

$$\leq \wp max\ \{inf\left[\tilde{h}_A^{\sigma(\kappa)}(x), \tilde{h}_A^{\sigma(\kappa)}(y)\right], sup\left[\tilde{h}_A^{\sigma(\kappa)}(x), \tilde{h}_A^{\sigma(\kappa)}(y)\right]\}$$

$$\leq \wp max\ \left\{\left[\tilde{h}_A^{\sigma(\kappa)}(x), \tilde{h}_A^{\sigma(\kappa)}(y)\right]\right\}$$

$$= \wp max\ \left\{\left[\tilde{h}_A^{\sigma(\kappa)}(e), \tilde{h}_A^{\sigma(\kappa)}(e)\right]\right\}$$

$$= \tilde{h}_A^{\sigma(\kappa)}(e)$$

Therefore $\tilde{h}_A^{\sigma(\kappa)}(xy) \leq \tilde{h}_A^{\sigma(\kappa)}(e)$

Hence $\tilde{h}_A^{\sigma(\kappa)}(e) = \tilde{h}_A^{\sigma(\kappa)}(xy)$

Therefore (xy) in $\widetilde{\mathrm{H}}$. Hence $\widetilde{\mathrm{H}}$ is a subnearring of *R*.

4.5 Theorem: Let $\tilde{h}_A^{\sigma(\kappa)}$ is an interval–valued anti-hesitant fuzzy subnearring of a nearring $(R,+,\cdot)$. If $\tilde{h}_A^{\sigma(\kappa)}(x-y) = [0]$, then $\tilde{h}_A^{\sigma(\kappa)}(x) = \tilde{h}_A^{\sigma(\kappa)}(y)$ for all *x* and *y* in R.

Proof:

Let *x* and *y* in R.

$$\tilde{h}_A^{\sigma(\kappa)}(x) = \tilde{h}_A^{\sigma(\kappa)}(x-y+y)$$
$$\leq \wp max\left\{\left[\tilde{h}_A^{\sigma(\kappa)}(x-y), \tilde{h}_A^{\sigma(\kappa)}(y)\right]\right\}$$
$$\leq \wp max\left\{[0], \tilde{h}_A^{\sigma(\kappa)}(y)\right\}$$
$$= \tilde{h}_A^{\sigma(\kappa)}(y)$$
$$= \tilde{h}_A^{\sigma(\kappa)}(-y)$$
$$= \tilde{h}_A^{\sigma(\kappa)}(-x+x-y)$$
$$\leq \wp max\left\{\left[\tilde{h}_A^{\sigma(\kappa)}(x), \tilde{h}_A^{\sigma(\kappa)}(x-y)\right]\right\}$$
$$\leq \wp max\left\{\tilde{h}_A^{\sigma(\kappa)}(y), [0]\right\}$$
$$= \tilde{h}_A^{\sigma(\kappa)}(x)$$

Therefore $\tilde{h}_A^{\sigma(\kappa)}(x) = \tilde{h}_A^{\sigma(\kappa)}(y)$ for all *x* and *y* in *R*.

4.6 Theorem: Let $\tilde{h}_A^{\sigma(\kappa)}$ is an interval–valued anti-hesitant fuzzy subnearring of a nearring $(R,+,\cdot)$. If $\tilde{h}_A^{\sigma(\kappa)}(x-y) = [1]$ then either $\tilde{h}_A^{\sigma(\kappa)}(x) = [1]$ or $\tilde{h}_A^{\sigma(\kappa)}(y) = [1]$ for all x and y in R.

Proof: Let *x* and *y* in *R*. By the definition

$$\tilde{h}_A^{\sigma(\kappa)}(x-y) \leq \wp \max\left\{\left[\tilde{h}_A^{\sigma(\kappa)L}(x), \tilde{h}_A^{\sigma(\kappa)U}(x)\right], \left[\tilde{h}_A^{\sigma(\kappa)L}(y), \tilde{h}_A^{\sigma(\kappa)U}(y)\right]\right\}$$
$$\leq \wp \max\left\{\left[\tilde{h}_A^{\sigma(\kappa)L}(x), \tilde{h}_A^{\sigma(\kappa)L}(y)\right], \left[\tilde{h}_A^{\sigma(\kappa)U}(x), \tilde{h}_A^{\sigma(\kappa)U}(y)\right]\right\}$$
$$\leq \wp max\,\{inf\left[\tilde{h}_A^{\sigma(\kappa)}(x), \tilde{h}_A^{\sigma(\kappa)}(y)\right], sup\left[\tilde{h}_A^{\sigma(\kappa)}(x), \tilde{h}_A^{\sigma(\kappa)}(y)\right]\}$$
$$\leq \wp max\left\{\tilde{h}_A^{\sigma(\kappa)}(x), \tilde{h}_A^{\sigma(\kappa)}(y)\right\}$$

which implies that $[1] \leq \wp max\left\{\tilde{h}_A^{\sigma(\kappa)}(x), \tilde{h}_A^{\sigma(\kappa)}(y)\right\}$

Therefore either $\tilde{h}_A^{\sigma(\kappa)}(x) = [1]$ or $\tilde{h}_A^{\sigma(\kappa)}(y) = [1]$

4.7 Theorem: If $\widetilde{\mathrm{A}}$ and $\widetilde{\mathrm{B}}$ are two interval-valued anti-hesitant fuzzy subnearrings of a nearring *R*, then their union $\widetilde{\mathrm{A}} \cup \widetilde{\mathrm{B}}$ is an interval-valued anti hesitant fuzzy subnearring of *R*.

Proof: Let *x* and *y* belong to *R*.

$\widetilde{\mathrm{A}} = \{\langle \mathrm{x}, \tilde{h}_A^{\sigma(\kappa)}(x)\rangle / \mathrm{x} \text{ in } \mathrm{R}\}$ and $\widetilde{\mathrm{B}} = \{\langle \mathrm{x}, \tilde{h}_B^{\sigma(\kappa)}(x)\rangle / \mathrm{x} \text{ in } \mathrm{R}\}$

Let $\widetilde{\mathrm{K}} = \widetilde{\mathrm{A}} \cup \widetilde{\mathrm{B}}$ and $\widetilde{\mathrm{K}} = \left\{\langle \mathrm{x}, \tilde{h}_A^{\sigma(\kappa)}(x)\rangle / \mathrm{x} \text{ in } \mathrm{R}\right\}$

(i) $\tilde{h}_A^{\sigma(\kappa)}(x-y) = \wp \max\left\{\left[\tilde{h}_A^{\sigma(\kappa)}(x-y), \tilde{h}_B^{\sigma(\kappa)}(x-y)\right]\right\}$

$$\leq \wp max\ \{\wp max\left[\tilde{h}_A^{\sigma(\kappa)}(x), \tilde{h}_A^{\sigma(\kappa)}(y)\right], \wp max\left[\tilde{h}_B^{\sigma(\kappa)}(x), \tilde{h}_B^{\sigma(\kappa)}(y)\right]\}$$

$$= \wp max\ \left\{\wp max\left[\tilde{h}_A^{\sigma(\kappa)}(x), \tilde{h}_B^{\sigma(\kappa)}(x)\right], \wp max\left[\tilde{h}_A^{\sigma(\kappa)}(y), \tilde{h}_B^{\sigma(\kappa)}(y)\right]\right\}$$

$$= \wp max\ \left\{\tilde{h}_K^{\sigma(\kappa)}(x), \tilde{h}_K^{\sigma(\kappa)}(y)\right\}$$

Therefore $\tilde{h}_A^{\sigma(\kappa)}(x-y) \leq \wp max\ \left\{\tilde{h}_K^{\sigma(\kappa)}(x), \tilde{h}_K^{\sigma(\kappa)}(y)\right\}$

for all *x* and *y* in R.

(ii) $\tilde{h}_A^{\sigma(\kappa)}(xy) = \wp \max\left\{\left[\tilde{h}_A^{\sigma(\kappa)}(xy), \tilde{h}_B^{\sigma(\kappa)}(xy)\right]\right\}$

$$\leq \wp max\ \{\wp max\left[\tilde{h}_A^{\sigma(\kappa)}(x), \tilde{h}_A^{\sigma(\kappa)}(y)\right], \wp max\left[\tilde{h}_B^{\sigma(\kappa)}(x), \tilde{h}_B^{\sigma(\kappa)}(y)\right]\}$$

$$\leq \wp max\ \{\wp max\left[\tilde{h}_A^{\sigma(\kappa)}(x), \tilde{h}_B^{\sigma(\kappa)}(x)\right], \wp max\left[\tilde{h}_A^{\sigma(\kappa)}(y), \tilde{h}_B^{\sigma(\kappa)}(y)\right]\}$$

$$= \wp \max\left\{\left[\tilde{h}_K^{\sigma(\kappa)}(x), \tilde{h}_B^{\sigma(\kappa)}(y)\right]\right\} \text{for all } x \text{ and } y \text{ in R.}$$

Hence $\widetilde{\mathrm{A}} \cup \widetilde{\mathrm{B}}$ is an interval valued anti – hesitant fuzzy subnearring of the nearring R.

4.8 Theorem: The union of a family of interval-valued anti-hesitant fuzzy subnearrings of a nearring R is an interval-valued anti-hesitant fuzzy subnearring of R.

Proof: It is trivial**.**

4.9 Theorem: If $\tilde{h}_A^{\sigma(\kappa)}$ and $\tilde{h}_B^{\sigma(\kappa)}$ are interval-valued anti-hesitant fuzzy subnearrings of the nearrings R and H̃, respectively, then $\tilde{h}_A^{\sigma(\kappa)} \times \tilde{h}_B^{\sigma(\kappa)}$ is an interval-valued anti-hesitant fuzzy subnearring of $R \times \tilde{H}$.

Proof:

$$\tilde{h}_A^{\sigma(\kappa)} \times \tilde{h}_B^{\sigma(\kappa)}[(x_1,y_1)-(x_2,y_2)] = \tilde{h}_A^{\sigma(\kappa)} \times \tilde{h}_B^{\sigma(\kappa)}(x_1-x_2,y_1-y_2)$$

$$= \wp max\left\{\tilde{h}_A^{\sigma(\kappa)}(x_1-x_2), \tilde{h}_B^{\sigma(\kappa)}(y_1-y_2)\right\}$$

$$\leq \wp max\left\{\begin{matrix}\wp\max\left\{\left[\tilde{h}_A^{\sigma(\kappa)L}(x_1),\tilde{h}_A^{\sigma(\kappa)U}(x_1)\right],\left[\tilde{h}_A^{\sigma(\kappa)L}(x_2),\tilde{h}_A^{\sigma(\kappa)U}(x_2)\right]\right\}, \wp\max\{\left[\tilde{h}_B^{\sigma(\kappa)L}(y_1),\tilde{h}_B^{\sigma(\kappa)U}(y_1)\right], \\ \left[\tilde{h}_B^{\sigma(\kappa)L}(y_2),\tilde{h}_B^{\sigma(\kappa)U}(y_2)\right]\}\end{matrix}\right\}$$

$$\leq \wp\max\{\wp\max\left\{inf\left[\tilde{h}_A^{\sigma(\kappa)}(x_1),\tilde{h}_A^{\sigma(\kappa)}(x_2)\right], sup\left[\tilde{h}_A^{\sigma(\kappa)}(x_1),\tilde{h}_A^{\sigma(\kappa)}(x_2)\right]\right\},$$
$$\wp\max\left\{inf\left[\tilde{h}_B^{\sigma(\kappa)}(y_1),\tilde{h}_B^{\sigma(\kappa)}(y_2)\right], sup\left[\tilde{h}_B^{\sigma(\kappa)}(y_1),\tilde{h}_B^{\sigma(\kappa)}(y_2)\right]\right\}\}$$

$$\leq \wp max\ \{\wp max\left[\tilde{h}_A^{\sigma(\kappa)}(x_1),\tilde{h}_A^{\sigma(\kappa)}(x_2)\right], \wp max\left[\tilde{h}_B^{\sigma(\kappa)}(y_1),\tilde{h}_B^{\sigma(\kappa)}(y_2)\right]\}$$
$$= \wp max\ \{\wp max\left[\tilde{h}_A^{\sigma(\kappa)}(x_1),\tilde{h}_B^{\sigma(\kappa)}(y_1)\right], \wp max\left[\tilde{h}_A^{\sigma(\kappa)}(x_2),\tilde{h}_B^{\sigma(\kappa)}(y_2)\right]\}$$

$$= \wp\max\{\tilde{h}_A^{\sigma(\kappa)} \times \tilde{h}_B^{\sigma(\kappa)}(x_1,y_1),\ \tilde{h}_A^{\sigma(\kappa)} \times \tilde{h}_B^{\sigma(\kappa)}(x_2,y_2)\}$$

Therefore

$$\tilde{h}_A^{\sigma(\kappa)} \times \tilde{h}_B^{\sigma(\kappa)}[(x_1,y_1)(x_2,y_2)] \leq \wp max\{\tilde{h}_A^{\sigma(\kappa)} \times \tilde{h}_B^{\sigma(\kappa)}(x_1,y_1),\ \tilde{h}_A^{\sigma(\kappa)} \times \tilde{h}_B^{\sigma(\kappa)}(x_2,y_2)\}$$

Hence $\tilde{h}_A^{\sigma(\kappa)} \times \tilde{h}_B^{\sigma(\kappa)}$ is an interval-valued anti-hesitant fuzzy subnearring of $R \times \tilde{H}$.

4.10 Theorem: Let $\tilde{h}_A^{\sigma(\kappa)}$ *and* $\tilde{h}_B^{\sigma(\kappa)}$ be interval-valued fuzzy subset of the nearrings R and H̃ respectively. Suppose that e and e^1 are the identity elements of R and H̃ respectively. If $\tilde{h}_A^{\sigma(\kappa)} \times \tilde{h}_B^{\sigma(\kappa)}$ is an interval-valued anti-hesitant fuzzy subnearring of $R \times \tilde{H}$, then at least one of the following two statements must hold.

(i) $\tilde{h}_B^{\sigma(\kappa)}(e') \leq \tilde{h}_A^{\sigma(\kappa)}(x)$for all x in R.

(ii) $\tilde{h}_A^{\sigma(\kappa)}(e) \leq \tilde{h}_B^{\sigma(\kappa)}(y)$for all y in H̃.

Proof: Let $\tilde{h}_A^{\sigma(\kappa)} \times \tilde{h}_B^{\sigma(\kappa)}$ be an interval-valued anti-hesitant fuzzy subnearring of $R \times \tilde{\mathrm{H}}$. By contra positive, suppose that none of the statements (i) and (ii) holds. Then to find a in R and b in $\tilde{\mathrm{H}}$ such that

$\tilde{h}_A^{\sigma(\kappa)}(a) < \tilde{h}_B^{\sigma(\kappa)}\ (e')$ and

$\tilde{h}_B^{\sigma(\kappa)}(b) < \tilde{h}_A^{\sigma(\kappa)} < (e)$.

Then it has,

$$\tilde{h}_A^{\sigma(\kappa)} \times \tilde{h}_B^{\sigma(\kappa)}(a,b) \leq \wp max\left\{\left[\tilde{h}_A^{\sigma(\kappa)L}(a), \tilde{h}_A^{\sigma(\kappa)U}(a)\right], \left[\tilde{h}_B^{\sigma(\kappa)L}(b), \tilde{h}_B^{\sigma(\kappa)U}(b)\right]\right\}$$

$$\leq \wp max\left\{\left[\tilde{h}_A^{\sigma(\kappa)L}(a), \tilde{h}_B^{\sigma(\kappa)L}(b)\right], \left[\tilde{h}_A^{\sigma(\kappa)U}(a), \tilde{h}_B^{\sigma(\kappa)U}(b)\right]\right\}$$

$$\leq \wp max\left\{inf\left[\tilde{h}_A^{\sigma(\kappa)}(a), \tilde{h}_B^{\sigma(\kappa)}(b)\right], sup\left[\tilde{h}_A^{\sigma(\kappa)}(a), \tilde{h}_B^{\sigma(\kappa)}(b)\right]\right\}$$

$$\leq \wp max\ \{\ \tilde{h}_A^{\sigma(\kappa)}(a), \tilde{h}_B^{\sigma(\kappa)}(b)\ \}$$

$$\leq \wp max\ \{\ \tilde{h}_A^{\sigma(\kappa)}(e), \tilde{h}_B^{\sigma(\kappa)}(e')\ \}$$

$$= \tilde{h}_A^{\sigma(\kappa)} \times \tilde{h}_B^{\sigma(\kappa)}\ (e,e')$$

Thus $\tilde{h}_A^{\sigma(\kappa)} \times \tilde{h}_B^{\sigma(\kappa)}$ is not an interval-valued anti-hesitant fuzzy subnearring of $R \times \tilde{\mathrm{H}}$.

Hence either $\tilde{h}_B^{\sigma(\kappa)}(e') \leq \tilde{h}_A^{\sigma(\kappa)}(x)$ for all x in R.

$\tilde{h}_A^{\sigma(\kappa)}(e) \leq \tilde{h}_B^{\sigma(\kappa)}(y)$ for all y in $\tilde{\mathrm{H}}$.

4.11 Theorem: Let $\tilde{h}_A^{\sigma(\kappa)}$ and $\tilde{h}_B^{\sigma(\kappa)}$ be interval-valued hesitant fuzzy subsets if the nearrings R and $\tilde{\mathrm{H}}$ respectively and $\tilde{h}_A^{\sigma(\kappa)} \times \tilde{h}_B^{\sigma(\kappa)}$ is an interval-valued anti-hesitant fuzzy subsets of $R \times \tilde{\mathrm{H}}$. Then the following are true.

(i) If $\tilde{h}_A^{\sigma(\kappa)}(x) \geq \tilde{h}_B^{\sigma(\kappa)}(e')$ then $\tilde{h}_A^{\sigma(\kappa)}$ is an interval-valued anti-hesitant fuzzy subnearring of R.

(ii) If $\tilde{h}_B^{\sigma(\kappa)}(x) \leq \tilde{h}_A^{\sigma(\kappa)}(e')$ then $\tilde{h}_B^{\sigma(\kappa)}$ is an interval-valued anti-hesitant fuzzy subnearring of $\tilde{\mathrm{H}}$.

(iii) Either $\tilde{h}_A^{\sigma(\kappa)}$ is an interval-valued anti-hesitant fuzzy subnearring of R or $\tilde{h}_B^{\sigma(\kappa)}$ is an interval-valued anti-hesitant fuzzy subnearring of $\tilde{\mathrm{H}}$.

Proof: Let $\widetilde{h}_A^{\sigma(\kappa)} \times \widetilde{h}_B^{\sigma(\kappa)}$ is an interval-valued anti-hesitant fuzzy subsets of $R \times \widetilde{\mathrm{H}}$ and (x, e') and (y, e') be in $R \times \widetilde{\mathrm{H}}$.

Now using the property $\widetilde{h}_A^{\sigma(\kappa)}(x) \geq \widetilde{h}_B^{\sigma(\kappa)}(e')$ for all x in R.

Then it is $\widetilde{h}_A^{\sigma(\kappa)}(x-y) = \wp max\{\widetilde{h}_A^{\sigma(\kappa)}(x-y), \widetilde{h}_B^{\sigma(\kappa)}(e'e')\}$

$$= \widetilde{h}_A^{\sigma(\kappa)} \times \widetilde{h}_B^{\sigma(\kappa)} [(x-y),(e'e')]$$

$$= \widetilde{h}_A^{\sigma(\kappa)} \times \widetilde{h}_B^{\sigma(\kappa)} [(x,e') + (-y,e')]$$

$$\leq \wp\max\{\widetilde{h}_A^{\sigma(\kappa)} \times \widetilde{h}_B^{\sigma(\kappa)}(x,e'), + \widetilde{h}_A^{\sigma(\kappa)} \times \widetilde{h}_B^{\sigma(\kappa)}(-y,e')\}$$

$$\leq \wp\max\{\wp\max\{\widetilde{h}_A^{\sigma(\kappa)}(x) \times \widetilde{h}_B^{\sigma(\kappa)}(e')\}, \wp max\{\widetilde{h}_A^{\sigma(\kappa)}(-y) \times \widetilde{h}_B^{\sigma(\kappa)}(e')\}\}$$

$$= \wp\max\{\widetilde{h}_A^{\sigma(\kappa)}(x), \widetilde{h}_A^{\sigma(\kappa)}(-y)\}$$

$$\leq \wp\max\{\widetilde{h}_A^{\sigma(\kappa)}(x), \widetilde{h}_A^{\sigma(\kappa)}(y)\}$$

Therefore $\widetilde{h}_A^{\sigma(\kappa)}(x-y) \leq \wp\max\{\widetilde{h}_A^{\sigma(\kappa)}(x), \widetilde{h}_A^{\sigma(\kappa)}(y)\}$ for all x and y in R. And

$$\widetilde{h}_A^{\sigma(\kappa)}(xy) = \wp\max\{\widetilde{h}_A^{\sigma(\kappa)}(xy), \widetilde{h}_B^{\sigma(\kappa)}(e'e')\}$$

$$= \widetilde{h}_A^{\sigma(\kappa)} \times \widetilde{h}_B^{\sigma(\kappa)} [(xy),(e'e')]$$

$$= \widetilde{h}_A^{\sigma(\kappa)} \times \widetilde{h}_B^{\sigma(\kappa)} [(x,e'),(y,e')]$$

$$\leq \wp\max\{\widetilde{h}_A^{\sigma(\kappa)} \times \widetilde{h}_B^{\sigma(\kappa)}(x,e'), \widetilde{h}_A^{\sigma(\kappa)} \times \widetilde{h}_B^{\sigma(\kappa)}(y,e')\}$$

$$= \wp\max\{\wp\max\{\widetilde{h}_A^{\sigma(\kappa)}(x), \widetilde{h}_B^{\sigma(\kappa)}(e')\}, \wp\max\{\widetilde{h}_A^{\sigma(\kappa)}(y), \widetilde{h}_B^{\sigma(\kappa)}(e')\}$$

$$= \wp\max\{\widetilde{h}_A^{\sigma(\kappa)}(x), \widetilde{h}_A^{\sigma(\kappa)}(y)\}$$

$$\leq \wp\max\{\widetilde{h}_A^{\sigma(\kappa)}(x), \widetilde{h}_A^{\sigma(\kappa)}(y)\}$$

Therefore $\widetilde{h}_A^{\sigma(\kappa)}(xy) \leq \wp\max\{\widetilde{h}_A^{\sigma(\kappa)}(x), \widetilde{h}_A^{\sigma(\kappa)}(y)\}$ for all x, y in R.

Hence $\widetilde{h}_A^{\sigma(\kappa)}$ is an internal-valued anti-hesitant fuzzy subnearring of R.

Thus (i) is proved.

Now using the property

$\widetilde{h}_B^{\sigma(\kappa)}(x) \geq \widetilde{h}_A^{\sigma(\kappa)}(e)$ for all x in $\widetilde{\mathrm{H}}$. Then it is

$$\widetilde{h}_B^{\sigma(\kappa)}(x-y) = \wp \max\left\{\widetilde{h}_B^{\sigma(\kappa)}(x-y), \widetilde{h}_A^{\sigma(\kappa)}(ee)\right\}$$

$$= \widetilde{h}_A^{\sigma(\kappa)} \times \widetilde{h}_B^{\sigma(\kappa)} [(ee),(x-y)]$$

$= \tilde{h}_A^{\sigma(\kappa)} \times \tilde{h}_B^{\sigma(\kappa)} [(e,x),(e,-y)]$

$\leq \wp \max\{ \tilde{h}_A^{\sigma(\kappa)} \times \tilde{h}_B^{\sigma(\kappa)}(e,x), \tilde{h}_A^{\sigma(\kappa)} \times \tilde{h}_B^{\sigma(\kappa)} (e,-y)$

$= \wp\max\{ \wp\max\{\tilde{h}_B^{\sigma(\kappa)}(x), \tilde{h}_A^{\sigma(\kappa)} (e)\}, \wp\max\{\tilde{h}_B^{\sigma(\kappa)}(-y), \tilde{h}_B^{\sigma(\kappa)}(e)\}$

$= \wp \max\left\{\tilde{h}_B^{\sigma(\kappa)}(x), \tilde{h}_B^{\sigma(\kappa)}(-y)\right\}$

$\leq \wp \max\left\{\tilde{h}_B^{\sigma(\kappa)}(x), \tilde{h}_B^{\sigma(\kappa)}(y)\right\}$

Therefore $\tilde{h}_B^{\sigma(\kappa)}(x-y) \leq \wp \max\left\{\tilde{h}_B^{\sigma(\kappa)}(x), \tilde{h}_B^{\sigma(\kappa)}(y)\right\}$ for all x and y in $\tilde{\text{H}}$.

And $\tilde{h}_B^{\sigma(\kappa)}(xy) = \wp \max\left\{\tilde{h}_B^{\sigma(\kappa)}(xy), \tilde{h}_A^{\sigma(\kappa)}(ee)\right\}$

$= \tilde{h}_A^{\sigma(\kappa)} \times \tilde{h}_B^{\sigma(\kappa)} [(ee),(xy)]$

$= \tilde{h}_A^{\sigma(\kappa)} \times \tilde{h}_B^{\sigma(\kappa)} [(e,x),(e,y)]$

$\leq \wp \max\{ \tilde{h}_A^{\sigma(\kappa)} \times \tilde{h}_B^{\sigma(\kappa)}(e,x), \tilde{h}_A^{\sigma(\kappa)} \times \tilde{h}_B^{\sigma(\kappa)} (e,y)$

$= \wp\max\{ \wp\max\{\tilde{h}_B^{\sigma(\kappa)}(x), \tilde{h}_A^{\sigma(\kappa)} (e)\}, \wp\max\{\tilde{h}_B^{\sigma(\kappa)}(y), \tilde{h}_B^{\sigma(\kappa)}(e)\}$

$= \wp \max\left\{\tilde{h}_B^{\sigma(\kappa)}(x), \tilde{h}_B^{\sigma(\kappa)}(y)\right\}$

$\leq \wp \max\left\{\tilde{h}_B^{\sigma(\kappa)}(x), \tilde{h}_B^{\sigma(\kappa)}(y)\right\}$

Therefore $\tilde{h}_B^{\sigma(\kappa)}(xy) \leq \wp \max\left\{\tilde{h}_B^{\sigma(\kappa)}(x), \tilde{h}_B^{\sigma(\kappa)}(y)\right\}$ for all x and y in $\tilde{\text{H}}$.

Hence $\tilde{h}_B^{\sigma(\kappa)}$ is an interval–valued anti-hesitant fuzzy subnearring of $\tilde{\text{H}}$.

Thus (ii) is proved. (iii) is clear.

4.12 Theorem: Let $\tilde{h}_A^{\sigma(\kappa)}$ be an interval-valued hesitant fuzzy subset of a nearing R and $\tilde{h}_V^{\sigma(\kappa)}$

be the strongest interval-valued anti-hesitant fuzzy relation of R with respect to $\tilde{h}_A^{\sigma(\kappa)}$. Then $\tilde{h}_A^{\sigma(\kappa)}$ is an interval-valued anti-hesitant fuzzy subnearring of R if and only if $\tilde{h}_V^{\sigma(\kappa)}$ is an interval-valued anti-hesitant fuzzy subnearring of R x R.

Proof: Suppose that $\tilde{h}_A^{\sigma(\kappa)}$ is an interval – valued anti – hesitant fuzzy subnearring of R. Then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in R x R. Then it has

$\tilde{h}_V^{\sigma(\kappa)}(x-y) = \tilde{h}_V^{\sigma(\kappa)}[(x_1,x_2) - (y_1,y_2)]$

$$= \tilde{h}_V^{\sigma(\kappa)}(x_1 - y_1, x_2 - y_2)$$

$$= \wp \max\left\{\tilde{h}_A^{\sigma(\kappa)}(x_1 - y_1), \tilde{h}_A^{\sigma(\kappa)}(x_2 - y_2)\right\}$$

$$\leq \wp\max\{ \wp\max\{\tilde{h}_A^{\sigma(\kappa)}(x_1), \tilde{h}_A^{\sigma(\kappa)}(-y_1)\}, \wp\max\{\tilde{h}_A^{\sigma(\kappa)}(x_2), \tilde{h}_A^{\sigma(\kappa)}(-y_2)\}$$

$$= \wp\max\{ \wp\max\{\tilde{h}_A^{\sigma(\kappa)}(x_1), \tilde{h}_A^{\sigma(\kappa)}(x_2)\}, \wp\max\{\tilde{h}_A^{\sigma(\kappa)}(-y_1), \tilde{h}_A^{\sigma(\kappa)}(-y_2)\}$$

$$= \wp\max\{ \wp\max\{\tilde{h}_A^{\sigma(\kappa)}(x_1), \tilde{h}_A^{\sigma(\kappa)}(x_2)\}, \wp\max\{\tilde{h}_A^{\sigma(\kappa)}(y_1), \tilde{h}_A^{\sigma(\kappa)}(y_2)\}$$

$$= \wp \max\left\{\tilde{h}_V^{\sigma(\kappa)}(x_1, x_2), \tilde{h}_V^{\sigma(\kappa)}(y_1, y_2)\right\}$$

$$= \wp \max\left\{\tilde{h}_V^{\sigma(\kappa)}(x), \tilde{h}_V^{\sigma(\kappa)}(y)\right\}$$

Therefore $\tilde{h}_V^{\sigma(\kappa)}(x - y) \leq \wp \max\left\{\tilde{h}_V^{\sigma(\kappa)}(x), \tilde{h}_V^{\sigma(\kappa)}(y)\right\}$ for all x and y in R× R.

And then it has

$$\tilde{h}_V^{\sigma(\kappa)}(xy) = \tilde{h}_V^{\sigma(\kappa)}[(x_1, x_2)(y_1, y_2)]$$

$$= \tilde{h}_V^{\sigma(\kappa)}(x_1 y_1, x_2 y_2)$$

$$= \wp \max\left\{\tilde{h}_A^{\sigma(\kappa)}(x_1 y_1), \tilde{h}_A^{\sigma(\kappa)}(x_2 y_2)\right\}$$

$$\leq \wp\max\{ \wp\max\{\tilde{h}_A^{\sigma(\kappa)}(x_1), \tilde{h}_A^{\sigma(\kappa)}(y_1)\}, \wp\max\{\tilde{h}_A^{\sigma(\kappa)}(x_2), \tilde{h}_A^{\sigma(\kappa)}(y_2)\}$$

$$= \wp\max\{ \wp\max\{\tilde{h}_A^{\sigma(\kappa)}(x_1), \tilde{h}_A^{\sigma(\kappa)}(x_2)\}, \wp\max\{\tilde{h}_A^{\sigma(\kappa)}(y_1), \tilde{h}_A^{\sigma(\kappa)}(y_2)\}$$

$$= \wp \max\left\{\tilde{h}_V^{\sigma(\kappa)}(x_1, x_2), \tilde{h}_V^{\sigma(\kappa)}(y_1, y_2)\right\}$$

$$= \wp \max\left\{\tilde{h}_V^{\sigma(\kappa)}(x), \tilde{h}_V^{\sigma(\kappa)}(y)\right\}$$

Therefore $\tilde{h}_V^{\sigma(\kappa)}(xy) \leq \wp \max\left\{\tilde{h}_V^{\sigma(\kappa)}(x), \tilde{h}_V^{\sigma(\kappa)}(y)\right\}$ for all x and y in R× R. This proves that

$\tilde{h}_V^{\sigma(\kappa)}$ is an interval-valued anti-hesitant fuzzy subnearring of R× R.

Conversely, assume that $\tilde{h}_V^{\sigma(\kappa)}$ is an interval-valued anti-hesitant fuzzy subnearring of R× R,

then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$. If put $x_2 = y_2 = e$, where e is the identity element of R then get

$\tilde{h}_A^{\sigma(\kappa)}(x_1 - y_1) \leq \wp \max\left\{\tilde{h}_A^{\sigma(\kappa)}(x_1), \tilde{h}_A^{\sigma(\kappa)}(y_1)\right\}$ for all x_1 and y_1 in R. And

$\wp \max\left\{\tilde{h}_A^{\sigma(\kappa)}(x_1 y_1), \tilde{h}_A^{\sigma(\kappa)}(x_2 y_2)\right\} = \tilde{h}_V^{\sigma(\kappa)}(x_1 y_1, x_2 y_2)$

$$= \tilde{h}_V^{\sigma(\kappa)}[(x_1,x_2)(y_1,y_2)]$$

$$= \tilde{h}_V^{\sigma(\kappa)}(xy)$$

$$\leq \wp \max\left\{\tilde{h}_V^{\sigma(\kappa)}(x), \tilde{h}_V^{\sigma(\kappa)}(y)\right\}$$

$$= \wp \max\left\{\tilde{h}_V^{\sigma(\kappa)}(x_1,x_2), \tilde{h}_V^{\sigma(\kappa)}(y_1,y_2)\right\}$$

$$= \wp\max\{ \wp\max\{\tilde{h}_A^{\sigma(\kappa)}(x_1), \tilde{h}_A^{\sigma(\kappa)}(x_2)\}, \wp\max\{\tilde{h}_A^{\sigma(\kappa)}(y_1), \tilde{h}_A^{\sigma(\kappa)}(y_2)\}$$

Suppose that $x_2 = y_2 = e$, where e is the identity element of R which implies

$\tilde{h}_A^{\sigma(\kappa)}(x_1y_1) \leq \wp \max\left\{\tilde{h}_A^{\sigma(\kappa)}(x_1), \tilde{h}_A^{\sigma(\kappa)}(y_1)\right\}$ for all x_1 and y_1 in R.

Hence $\tilde{h}_A^{\sigma(\kappa)}$ is an interval-valued anti-hesitant fuzzy subnearring of R.

5. CONCLUSION

This paper is concluded that the concept of internal-valued hesitant and anti-hesitant fuzzy subnearrings are defined with proper example. Based on the definition of interval-valued anti-hesitant fuzzy subnearring, the properties of interval-valued anti-hesitant fuzzy subnearring are proved. In forth coming paper, the discussion will be made on interval-valued Q hesitant fuzzy subnearring.

REFERENCES

[1] AsokKumer Ray, On product of fuzzy subgroups, Fuzzy sets and systems,105,181-183(1999).

[2] K.T.Atanassov. Intuitionistic fuzzy sets. Fuzzy Sets and Systems,20:87-96,1986.

[3] Azriel Rosenfeld, Fuzzy Groups, Journal of Mathematical Analysis and Applications,35,512-517(1971).

[4] Bedregal, R. Reister, H. Bustince , C. Lopez – Molina, and V .Torra. Aggregating functions for typical hesitant fuzzy elements and the of automorphisms. Information Sciences, 256(1): 82-97,2014.

[5] Biswas.R,Fuzzy Subgroups and Anti-fuzzy subgroups,Fuzzy sets and systems,35,121- 124(1990).

[6] H.Bustince and P. Burillo. Correlation of Interval-Valued intuitionistic fuzzy sets. Fuzzy Sets and Systems, 74(2):237-244, 1995.

[7] H.Bustince, J.Fernadez, A.Kolesaroba and R.Mesiar. Generation of linear orders for intervals by means of aggregation functions. Fuzzy Sets and Systems, 220:69-77, 2013.

[8] H.Bustince, F. Herrera, and J. Montero, editors. Fuzzy Sets and Their Extensions: Representation, Aggregation and Models, Volume 220 of Studies in Fuzziness and Soft Computing. Springer, 2008.

[9] N .Chen, Z.S. Xu, and M.M. Xia. Correlation Coefficient s of hesitant fuzzy sets and their applications to clustering analysis. Applied Mathematical Modelling, 37(4):2197-2211, 2013.

[10] N .Chen, Z.S. Xu, and M.M. Xia. Interval-Valued hesitant preference relations and their applications to group decision making. Knowledge – Based Systems, 37(1):528-540, 2013.

[11] D. Dubois and H. Prade. Fundamentals of Fuzzy Sets, volume 7 of The Handbooks of Fuzzy Sets. Springer, 2000.

[12] B. Farhadinia. Information measures for hesitant fuzzy sets and interval-valued hesitant fuzzy sets. Information Sciences, 240:129-144, 2013.

[13] W.L. Hung and M.S. Yang. Similarity measures between type -2 fuzzy sets. International Journal of Uncertainty, Fuzziness and Knowledge – Based Systems, 12(6):827-841, 2004.

[14] G.J. Klir and B. Yuan. Fuzzy sets and fuzzy logic: Theory and Applications. Prentice-Hall PTR, 1995.

[15] H. Liu and R.M. Rodriguez. A fuzzy envelope for hesitant fuzzy linguistic term set and its application to multicriteria decision making. Information Sciences, 258: 266-276, 2014.

[16] D.H. Peng, C.Y. Gao, and Z.F. Gao. Generalized hesitant fuzzy synergetic weighted distance measures and their application to multiplecriteria decision-making. Applied Mathematical Modelling, 37(8):5837-5850, 2013.

[17] G. Qian , H. Wang, and X. Feng. Generalized hesitant fuzzy sets and their application in decision support system. Knowledge – Based Systems, 37(1):357-365,2013.

[18] R.M. Rodriguez, L.Martnez and F.Herrera. Hesitant fuzzy linguistic term sets for decision making. IEEE Transactions on Fuzzy Systems, 20(1):109-119, 2012.

[19] R.M. Rodriguez, L. Martinez, and F.Herrera. A group decision making model dealing with comparative linguistic expressions based on hesitant fuzzy linguistic term sets. Information Sciences, 241(1):28-42,2013.

[20] N. Sahu and G.S. Thakur. Hesitant distance similarity measures for document clustering. In Worl congress on Information and Communication Tecnologies (WICT), pages 430-438, Mumbai, India,2011.

[21] Sharma.P.K,Homomorphism of Intuitionistic fuzzy groups,International Mathematics forum,Vol.6,2011,no.64,3139-3178.

[22] V.Torra. Hesitant Fuzzy sets. International journal of Intelligent systems, 25(6):529-539,2010.

[23] I.B. Turksen. Interval-valued fuzzy sets based on normal forms. Fuzzy sets and systems, 20:191-210, 1986.

[24] M.M. Xia and Z.S. Xu. Hesitant fuzzy information aggregation in decision making. International Journal Approximate Reasoning, 52:395-407, 2011.

[25] R.R.Yager. On the theory of bags. International journal Generation system, 13:23-37, 1986.

[26] D. Yu. Triangular hesitant fuzzy set and its application to teaching quality evaluation. Journal of Information and Computational Science, 10(7):1925-1934, 2013.

[27] D.Yu, W.Zhang, and Y.Xu. Group decision making under hesitant fuzzy environment with application to personnel evaluation. Knowledge Based Systems, 52:1-10, 2013.

[28] L.Zadeh. Fuzzy sets. Information and Control, 8:338-353, 1965.

[29] Zadeh.L.A, The concept of a linguistic variable and its application to approximation reasoning-1, Inform.Sci., 8(1975),199-249.

[30] B. Zhu, Z.S. Xu and M.M. Xia. Hesitant fuzzy geometric Bonferroni means. Information Sciences.