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Super Magic and Arithmetic Labelings of the Graphs $P_3 \times P_n$ AND $C_3 \times C_{2n}$

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Abstract:

Figueroa-Centeno, *et al.* [2002a, 2002a] found that the following graphs are total edge-magic: $P_4 \cup nK_2$ for n odd; $P_3 \cup nK_2$; $P_5 \cup nK_2$; nP_i for n odd and $i = 3, 4, 5$; $2P_n$; $P_1 \cup P_2 \cup \cdots \cup P_n$; $mK_{1,n}$; $C_m \odot nK_1$; $K_1 \odot nK_2$ for n even; W_{2n}; $K_2 \times K_n$, nK_3 for n odd; binary trees, generalized Petersen graphs, ladders , books, fans, and odd cycles with pendant edges attached to one vertex. Wijaya and Baskoro [2000] explained that $P_m \times C_n$ is total edge-magic for odd n at least 3. Ngurah and Baskoro [2003] stated that $P_2 \times C_n$ is not total edge-magic. They [2007b] obtained that certain subdivisions of the star $K_{1,3}$ have total edge-magic and super total edge-magic labelings. Fukuchi [2001a, 2001b] obtained edge-magic labelings for wheel, and generalized Peterson graph P[n,2]. Ngurah et.al. [2007a, 2007b] investigated super edge-magic labeling for subdivision of bipartite graph $K_{1,3}$. *In this paper, we proved (1) $C_3 \times P_n$ is super magic. (2) The graph $C_3 \times C_{2n}$ is bi-magic (3) $C_3 \times P_n$ has (k,1)-arithmetic labeling, where k is an odd positive integer. (4) $C_3 \times P_n$ has (k, 2)-arithmetic labeling for all even positive integer k ≥ 2. (5) $C_3 \times P_n$ has (k,3)-arithmetic labeling for all odd positive integer k ≥ 3.*

Section 1 - Introduction:

Kotzig and Rosa [1970] proved: $K_{m,n}$ has an total edge-magic labeling for all m and n; C_n has an total edge-magic labeling for all $n \geq 3$ and the disjoint union of n

copies of P_2 has a total edge-magic labeling if and only if n is odd. They further stated that K_n has a total edge-magic labeling if and only if $n = 1, 2, 3, 5$ or 6 and they asked whether all trees have total edge-magic labeling.

Wallis, *et al.* [2000] enumerated every total edge-magic labeling of complete graphs. They also established that the following graphs are total edge-magic: paths, crowns, complete bipartite graphs, and cycles with a single edge attached to one vertex. He [2001] also elaborated that a cycle with one pendant edge is total edge-magic. He posed a large number of research problems about total edge-magic graphs.

Slamin, *et al.* [2002] further revealed that all fans are total edge-magic. Baskar Babujee and Rao [2002] maintained that the path with n vertices has an total edge-magic labeling with magic constant $(5n + 2)/2$ when n is even and $(5n + 1)/2$ when n is odd. For stars with n vertices they provided an total edge-magic labeling with magic constant $3n$.

Yegnanarayanan [2001] confirmed that the following graphs have total edge-magic labeling: nP_3 where n is odd; $P_n + K_1$; $P_n \times C_3$ $(n \geq 2)$; the crown $C_n \odot K_1$; and $P_m \times C_3$ with n pendant vertices attached to each vertex of the outermost C_3. He further conjected that $C_n \odot K_n$ (for all n) as a n-cycle with n pendent vertices have attached at each vertex of the cycle, and nP_3 have total edge-magic labeling.

Section 2: Super magic labeling

Kotzig and Rosa [1970] introduced the concept of total edge-magic labeling. A total edge magic labeling of a graph with n vertices and m edges is a bijection f from $V \cup E$ to the integers $1, 2, \ldots, m + n$ such that there exists a constant s for any edge (u, v) in E satisfying $f(u) + f(v) + f(u, v) = s$. A graph is called a *total edge magic* graph if it has a total edge magic labeling.

Definition 2.1: (super magic) A graph G (p, q) with p vertices and q edges is called super magic if there exists a bijection $f: V \to \{1,2,\ldots p\}$ and the same map $f: E \to \{p + 1, p + 2,\ldots, p + q\}$ such that $f(u) + f(uv) + f(v)$ is a constant k for all edge uv in G.

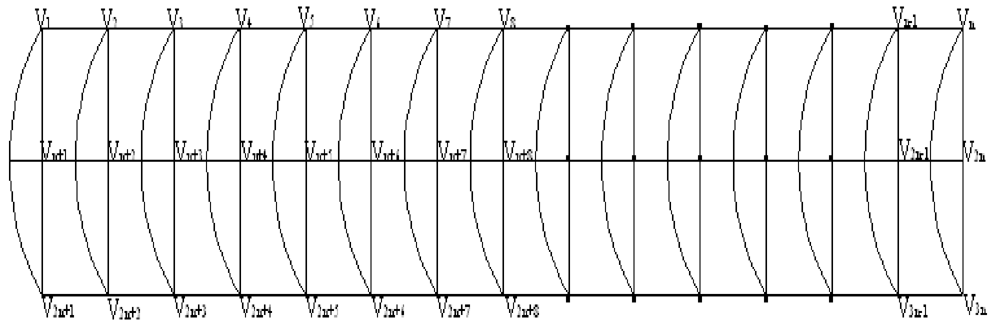
Definition 2.2: (super bi-magic) A graph G (p, q) with p vertices and q edges is called super bi-magic if there exists a bijection $f: V \to \{1,2,\ldots p\}$ and $E \to \{p + 1, p + 2,\ldots p + q\}$ such that $f(u) + f(uv) + f(v)$ is a constant k_1 or k_2 for all edge uv in G.

Definition 2.3: ((k, d)-arithmetic) A graph G (p, q) is (k, d)-arithmetic if its vertices can be assigned distinct nonnegative integers so that the values of the edges, obtained as the sums of the numbers assigned to their end vertices, can be arranged in the arithmetic progression $\{k, k + d, k + 2d ,\ldots, k + (q-1)d\}$.

The contributed results are as follows:

Theorem 2.4: The graph $C_3 \times P_n$ is super magic where n is any positive integer.

Proof: The given graph has vertex set as $\{v_1,v_2,\ldots v_{3n}\}$, and edge set is $\{v_iv_{i+1}, v_{n+i}v_{n+i+1}, v_{2n+i}v_{2n+i+1}, i = 1 \text{ to } (n-1)\} \cup \{v_iv_{n+i},v_{n+i}v_{2n+i},v_iv_{2n+i}, i =1 \text{ to } n\}$.One of the labeling for vertices is chosen as follows (figure 1):



To get super magic labeling, define f: $V(G) \rightarrow \{1, 2,\ldots, p\}$ by

$f(v_1) = 3$; $f(v_2) = 4$; $f(v_{n+1}) = 2$; $f(v_{n+2}) = 6$; $f(v_{2n+1}) = 1$; $f(v_{2n+2}) = 5$.

<u>i varies from 1 to n</u>

$f(v_i) = f(v_1)+3(i-1)$, i is odd; $f(v_i) = f(v_2)+3(i-2)$, i is even.

$f(v_{n+i}) = f(v_{n+1})+3(i-1)$, i is odd; $f(v_{n+i}) = f(v_{n+2})+3(i-2)$, i is even.

$f(v_{2n+i}) = f(v_{2n+1})+3(i-1)$, i is odd; $f(v_{2n+i}) = f(v_{2n+2})+3(i-2)$, i is even.

<u>i varies from 1 to (n-1)</u>

$f(v_i v_{i+1}) = (p+q-4)-6(i-1)$; $f(v_{n+i} v_{n+i+1}) = (p+q-5)-6(i-1)$; $f(v_{2n+i} v_{2n+i+1}) = (p+q-3)-6(i-1)$

<u>i varies from 1 to n</u>

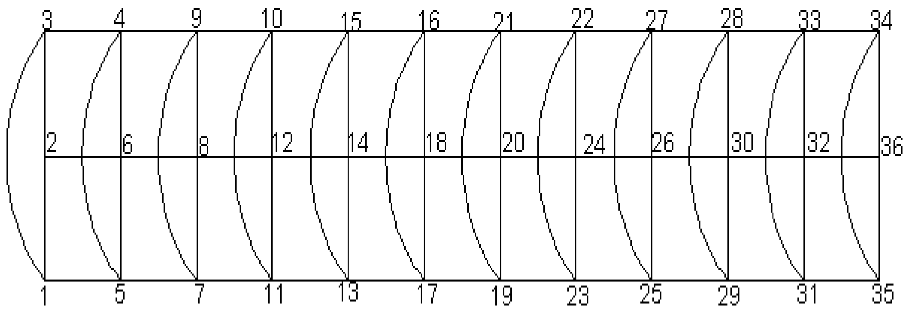
$f(v_i v_{n+i}) = (p+q-2)-6(i-1)$, i is odd; $f(v_i v_{n+i}) = (p+q-7)-6(i-2)$,i is even.

$f(v_{n+i} v_{2n+i}) = (p+q)-6(i-1)$, i is odd; $f(v_{n+i} v_{2n+i}) = (p+q-8)-6(i-2)$,i is even.

f $(v_i\, v_{2n+i})$ = (p+q-1)-6(i-1), i is odd; f $(v_i\, v_{2n+i})$ = (p+q-6)-6(i-2),i is even.

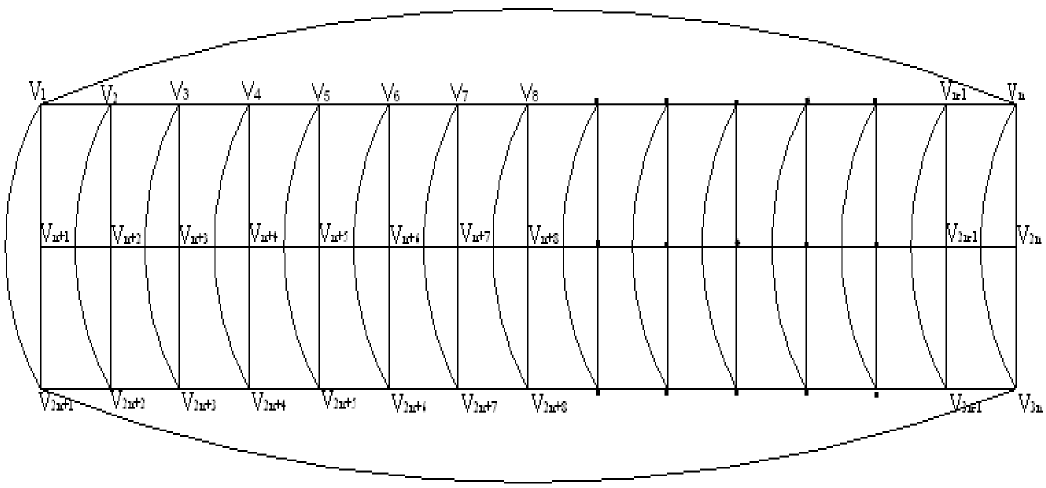
So f satisfies the conditions in the definition (2.1). Magic number of the graph is k_1 = p + q + 3.

Example 2.5: The graph $C_3\times P_{12}$ has super magic labeling with magic number 108 (figure 2).



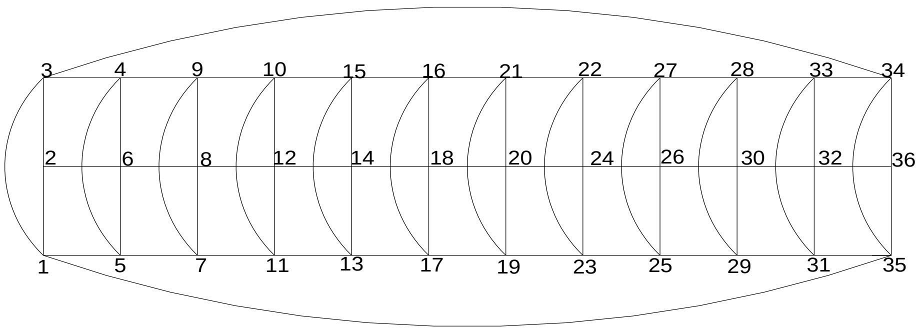
Theorem 2.6: The graph $C_3\times C_{2n}$ is bi-magic where n is positive integer.

Proof: The given graph has vertex set as {v1,v2, …v3n}, and edge set is $\{v_iv_{i+1}, v_{n+i}v_{n+i+1}, v_{2n+i}v_{2n+i+1},$ i = 1 to (n-1)} ∪ $\{v_iv_{n+i}, v_{n+i}v_{2n+i}, v_iv_{2n+i}$, i =1 to n} ∪ $\{v1v_n, v_{2n+1}v_{3n}\}$..One of the labeling for vertices is chosen as follows (figure 3):



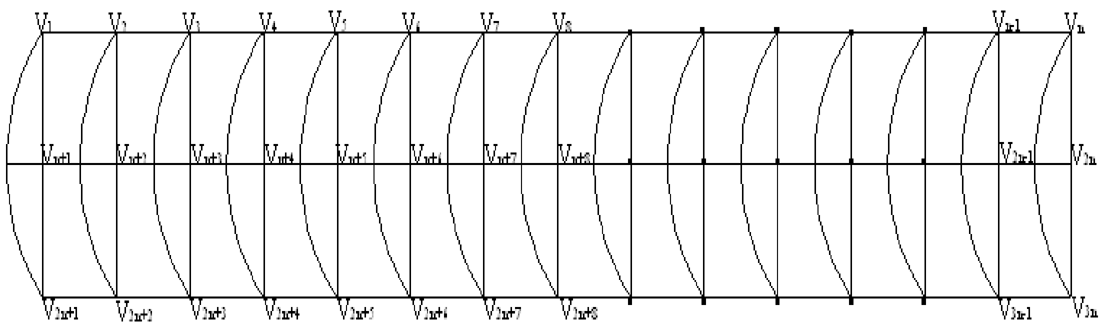
Define f (v_1v_n) = p + 1 = 3n+1 by f $(v_{2n+1}v_{3n})$ = p + 2 = (3n + 2). The remaining labelings for edges and vertices are mentioned as in the theorem (2.4). Magic number of the given graph is k_1 = p + q + 3 = 3n + (6n - 1) + 3 = 9n + 2, and k_2 = (p - 2) + 3 + p + 1 = (2p + 2) = (6n + 2).

Example 2.7: The graph $C_3{\times}P_{12}$ has bi-magic labeling with two magic numbers 74 & 110 (figure 4).



Theorem 2.8: The graph $C_3{\times}P_n$ has (k, 1)-arithmetic labeling, where n is positive integer, and k is an odd positive integer.

Proof: The given graph $C_3{\times}P_n$ has the vertex set vertex set $\{v_1, v_2,\ldots, v_{3n}\}$and edge set $\{v_iv_{i+1}, v_{n+i}v_{n+i+1}, v_{2n+i}v_{2n+i+1}$, i = 1 to (n-1)$\} \cup \{ v_iv_{n+i},v_{n+i}v_{2n+i},v_iv_{2n+i}$, i =1 to n$\}$. One of the arbitrary labelings for vertex set is as follows (figure 5):



Define f: V(G) → N in the following manner:

<u>i varies from 1 to n</u>

$f(v_i) = 6+3(i-1)$,i is odd ; $f(v_i) = 7 + 2(i-2)$,i is even

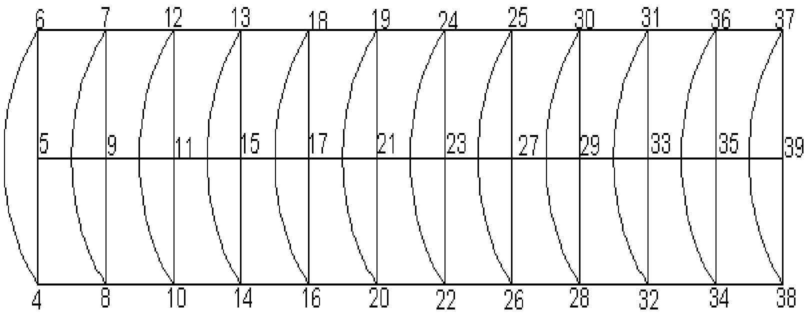
<u>i varies from (n+1) to 2n</u>

$f(v_{n+i}) = 5 + 3(i - 1)$, i is odd; $f(v_{n+i}) = 9 + 3(i-2)$,i is even i varies from (2n+1) to 3n

f (v_{2n+i}) =4+3(i-1) ,i is odd; f (v_{2n+i}) = 8 + 3(i-2) ,i is even

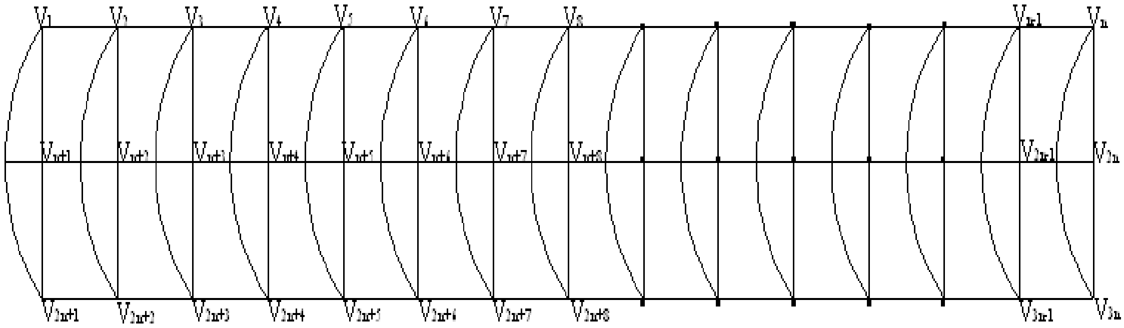
Define f (uv) = f (u) + f (v) for all uv$\in$E(G). Thus the values of the edges, obtained as the sums of the numbers assigned to their end vertices, can be arranged in the arithmetic progression {k, k + d, k + 2d ,…, k + (q-1)d}. Therefore the given graph is (k, 1)-arithmetic labeling where k is odd integer.

Example 2.9: The graph $C_3\times P_{12}$ has (9, 1)-arithmetic labeling (figure 6).



Theorem 2.10: The graph $C_3\times P_n$ has (k, 2)-arithmetic labeling for all even positive integer k $\geq$ 2.

Proof: The given one has the vertex set$\{v_1,v_2,\dots v_{3n}\}$ and edge set $\{v_iv_{i+1},v_{n+i}v_{n+i+1},v_{2n+i}v_{2n+i+1}$, i =1 to (n-1)} $\cup$ { $v_iv_{n+i},v_{n+i}v_{2n+i},v_iv_{2n+i}$, i = 1 to n}(figure 7). Let n be any positive integer.



Define f: V(G) → N in the following manner:

i varies from 1 to (n-1)

f (v_i) =8+6(i-1) ,i is odd; f (v_i) = 10+6(i-2) ,i is even.

i varies from (2n+1) to 2n

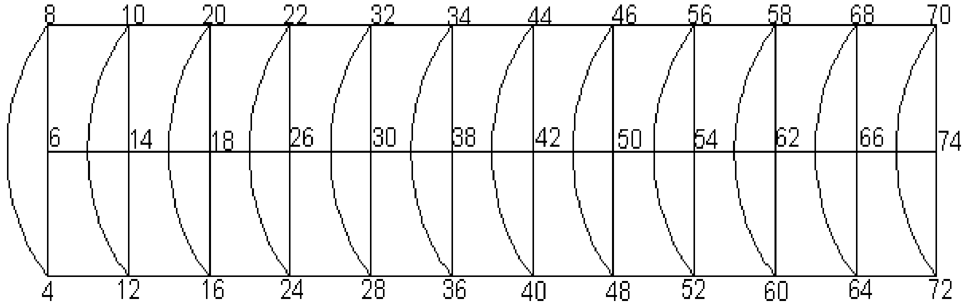
f (v_{n+i}) =6+6(i-1) ,i is odd; f (v_{n+i}) = 14+6(i-2) , i is even.

i varies from (2n+1) to 3n

f (v_{2n+i}) =4+6(i-1) ,i is odd; f (v_{2n+i}) = 12+6(i-2), i is even.

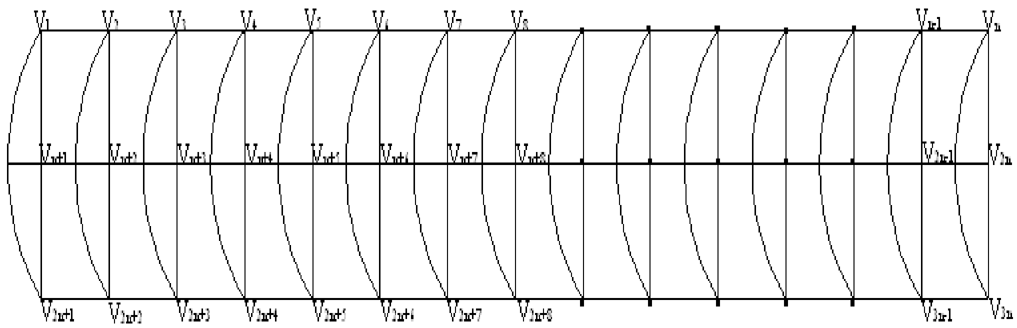
Define f (uv) = f (u) + f (v) for all uv $\in$ E(G). Thus the values of the edges, obtained as the sums of the numbers assigned to their end vertices, can be arranged in the arithmetic progression {k, k + d, k + 2d ,…, k + (q-1)d}. Therefore the given graph is (k, 2)-arithmetic labeling where k is even integer.

Example 2.11: The graph $C_3 \times P_n$ has (10, 2)-arithmetic labeling (figure 8).



Theorem 2.12: The graph $C_3 \times P_n$ has (k, 3)-arithmetic labeling for all odd positive integer $k \geq 3$.

Proof: The given one has the vertex set $\{v_1, v_2, \ldots, v_{3n}\}$ and edge set $\{v_iv_{i+1}, v_{n+i}v_{n+i+1}, v_{2n+i}v_{2n+i+1},\ i = 1 \text{ to } (n-1)\} \cup \{v_iv_{n+i}, v_{n+i}v_{2n+i}, v_iv_{2n+i},\ i = 1 \text{ to } n\}$ (figure 9).



Define f: V(G) → N by the following rule:

<u>i varies from 1 to n</u>

$f(v_i) = 9 + 9(i-1)$, i is odd ; $f(v_i) = 12 + 9(i-2)$, i is even

<u>i varies from n+1 to 2n</u>

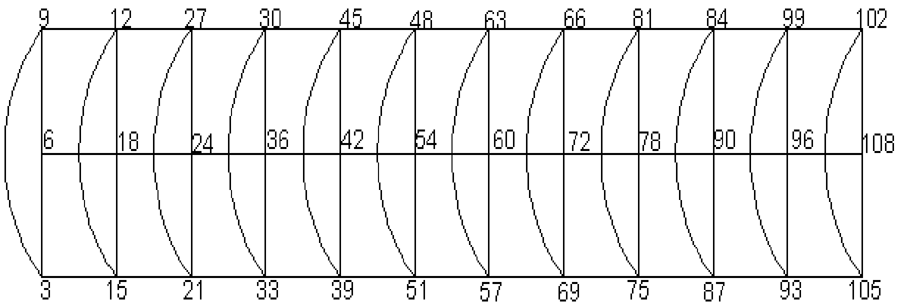
$f(v_{n+i}) = 6 + 9(i-1)$, i is odd ; $f(v_{n+i}) = 18 + 9(i-2)$, i is even

<u>i varies from (2n+1) to 3n</u>

$f(v_{2n+i}) = 3 + 9(i-1)$, i is odd ; $f(v_{2n+i}) = 15 + 9(i-2)$, i is even

Define f (uv) = f (u) + f (v) for all $uv \in E(G)$. Thus the values of the edges, obtained as the sums of the numbers assigned to their end vertices, can be arranged in the arithmetic progression {k, k + d, k + 2d ,…, k + (q-1)d}. Therefore the given graph is (k, 2)-arithmetic labeling where k is odd integer.

Example 2.13: The graph $C_3 \times P_{12}$ has (9, 3)-arithmetic labeling (figure 10).



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