

## **Proof of Beal's Conjecture.**

$$\text{If, } a^x + b^y = c^z$$

**Where a,b,c,x,y,z are positive integers with x,y,z >2.  
Than a,b,c have a common prime factor.**

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### **Abstract:**

Beal's conjecture is a conjecture in number theory. Billionaire banker Andrew Beal formulated this conjecture in 1993 while investigating generalizations of Fermat's last theorem. It has been claimed that the same conjecture was independently formulated by Robert Tijdeman and Don Zagier, and it has also been referred to as the Tijdeman-Zagier conjecture.

(wiki)

$$\text{If, } a^x + b^y = c^z$$

Where a,b,c,x,y,z are positive integers with x,y,z >2. Than a,b,c have a common prime factor.

With the Solving and study of the nature numbers and their power , which satisfy the Beal query the study states that Beal conjecture Is true, The whole equation runs around co prime numbers only .if abc are co prime mean if they don't have any common prime factor.

**Keywords:** Beal Conjecture, Co-Prime Numbers, Common prime factor, Divisibility of numbers .

**Proof:-**

a,b,c are the positive integer numbers mean they are natural numbers.

$$N = \{1, 2, 3, 4, \dots\}$$

a,b,c wil from above set.

In this conjecture the value of c depends on and b. hence here we will focus on a,b. We have the following basic conditions of above equation,

**(2a) if a,b,c have a common prime factor than  $x,y,z > 2$ .**

**(2b) if a,b,c have not any common prime factor than at least one value**

Among  $x,y,z = 2$  .

**(2a) If a,b,c have any common prime factor: -**

If a,b,c have any common prime factor, it means these all number a,b,c are surely divisible by any common prime number and a,b,c all are not co-prime numbers. The set of multiples of that number would contain the different powers of that number. So power may be 2 or greater than 2.

**Example:-**

$$2^3 + 2^3 = 2^4 \text{ (common prime factor 2)}$$

Here If  $X : \{ x \text{ is multiple of } 2 \}$

$$\text{And } 2^3, 2^3 \in X$$

$$3^3 + 6^3 = 3^5 \text{ (common prime factor 3)}$$

Here If  $X : \{ x \text{ is multiple of } 3 \}$

$$\text{And } 3^6, 3^5, 6^3 \in X$$

$$28^3 + 14^5 = 28^4 \text{ (common prime factor 7)}$$

Here If  $X : \{ x \text{ is multiple of } 7 \}$

$$\text{And } 14^5, 28^3, 28^5 \in X$$

$$33^5 + 66^5 = 33^5 \text{ or } 1089^3 \text{ (common prime factor 11) Here If } X : \{ x \text{ is multiple of } 11 \}$$

$$\text{And } 33^5, 66^5, 1089^3 \in X$$

Now compare above equation with Beal conjecture equation We get  $z=3, x=3, y=2$

Here this equation satisfies Beal's query therefore in different examples we can see that if there is any common prime factor, than the power would be 2 or greater than 2.

We can make find the different equations, which may satisfy the Beal conjecture, By power of any number. I.e.

If there is any number X then its power will be  $x^p$

Then let  $x^p-1=y$

Then the equation will be

$$y^p+y^{p+1}=xy^p$$

Example:-

Number (X)	Power (p)	$X^m$	$Y=x^m-1$	Equation $y^p+y^{p+1}=xy^p$
2	3	8	7	$7^3+7^4=14^3$
3	3	27	26	$26^3+26^4=78^3$
4	4	256	255	$255^4+255^5=1020^3$
5	5	3125	3124	$3124^5+3124^6=15620^5$
6	5	7776	7775	$7775^6+7775^7=38875^6$

Here with adding 3 respectively in these equations ( $y^p+y^{p+1}=xy^p$ ) we will get following equation which will also

Satisfy the Beal conjecture Example:-

$$7^3+7^4=14^3$$

$$7^6+7^7=98^3 \quad \dots: 7^{3+3}+7^{4+3}=(7*14)^3$$

$$7^9+7^{10}=686^3 \quad \dots: 7^{6+3}+7^{7+3}=(7*98)^3$$

Now compare above equation with Beal conjecture equation

We get  $z=3, x=3, y=2$

With the subtracting the any power of any two number, the resultant subtraction will also form equation which will satisfy Beal conjecture.

**Example:-**

$$2^3 - 1^3 = 7,$$

$$\text{Therefore } 7^3 + 7^4 = 14^3$$

As well as

$$3^3 - 2^3 = 19,$$

$$\text{Therefore } 19^4 + 34^3 = 57^3$$

As well as

$$2^4 - 1^4 = 15,$$

$$\text{Therefore } 15^5 + 15^4 = 30^4$$

We can see that if there is any common prime factor, than the power would be 2 or greater than 2.

**(2a) If a,b,c has not any common prime factor: -**

Since if there is any number is n,

If we will observe the numbers which fallow the Beal conjecture we will get that, the Beal equation may fallow this pattern

$$\text{If n is odd than } n^2/2 = q \text{ (take quaint without decimal) } n^x + q^2 = (q+1)^2$$

$$\text{If n is even than } n^2/4 = q$$

$$n^x + (q-1)^2 = (q+1)^2$$

The value of q will come alternate even and odd hence these numbers will not have any common prime factor. There for here n,q,p have not any common prime factor as well as they must have a square power.

So again we get conclusion that in Beal Conjecture if a,b,c have not any common prime factor than among the power x,y,z one's value must will equal to 2.

Let a,b are two numbers and the difference between a-b= d. With the subtracting the square and cube or any powers of a,b or let in Beal conjecture equation a-b=d.

Let here d=1,

It mean a-b=1

$$a^2 - b^2 = c$$

The every solution for  $a^2 - b^2 = c$  always will obey  $2b+1$   $a^3 - b^3 = c$

The every solution for  $a^3 - b^3 = c$  always will obey  $3b^2 + 3b + 1$

If we will compare both of equation we will see for every solution of both equations in equation one where powers of a,b is 2, the value of c may have of any power of any number

But in equation where powers of a,b is 2 or more than 2, the value of c may only have square of any power of any number

As well as

Let here  $d=2$ ,

It mean  $a-b=2$

$$a^2-b^2=c$$

The every solution for  $a^2-b^2=c$  always will come  $4b+4 a^3-b^3=c$

The every solution for  $a^3-b^3=c$  always will come  $4^2+12b+8$

If we will compare both of equation we will see for every solution of both equations in equation one where powers of a,b is 2, the value of c may have of any power of any number

But in equation where powers of a,b is 2 or more than 2, the value of c may only have square of any power of any number.

The difference D states that if  $d=1$ , means the number a,b are continuous integer number.

And for  $d<1$ , there will have two conditions that one series will have alternate even-odd and second would have even –even or odd-odd series. Even-odd numbers series would not Common prime factor and rest of two would have common prime factor.

If we will compare these equations with Beal's conjectures equation we will get that

The conclusion is the if we will find  $a^3-b^3=c$ ,

$\sqrt{r}$  either will have a perfect square root or no any perfect root.

Similarly whenever we will find

$$p^i-q^i=c \quad (i<2)$$

the  $\sqrt{c}$  either will have a perfect square root or no any perfect root.

So finally we get conclusion that in Beal Conjecture if  $a, b, c$  have not any common prime factor than among the power  $x, y, z$  one's value must will equal to 2.

The conclusion is the if we will find  $p^3 - q^3 = r$ ,

$\sqrt{r}$  either will have a perfect square root or no any perfect root.

Similarly whenever we will find

$$p^i - q^i = r \quad (i < 2)$$

the  $\sqrt{r}$  either will have a perfect square root or no any perfect root.

### **Symbol And Abbreviations:-**

N- Natural numbers,  $\in$ -Belongs to