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A Study of Service Surrender Secondary Queuing Model

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Abstract

Existing Queuing models are insufficient in dealing with day to day changes in the way queues are generated. Here we study queues generated when services are surrendered by customers who are awaiting service when and where such facility exists. In certain systems customers can surrender the service by a cancellation if they desire to opt out of the service giving way to a customer in a secondary queue. In this paper we consider such a model for applicants in a large educational institution.

Keywords Service Surrender rate, Secondary queue, waiting time in secondary queue

AMS Mathematics Subject Classification: 60K25

1. INTRODUCTION

When thousands of applicants apply for a few hundred seats in a large educational university with diversified courses and various criteria for qualification, fees etc. the system obviously needs to deal with large data dynamically incorporating the changes which the applicants may make from time to time .Here there may be applicants waiting to enter the system to be considered for an appropriate vacancy that may arise if someone opts out or prefers one course to another. These applicants form a secondary queue. Some of the criteria are i) Merit list ii) Reservations for some seats iii) Scholarships etc.

Literature available till date considers queuing analysis where the entire study is focused on the one queue or multiple queues which is formed in front of a service counter or multiple service counters as per Kendall notation. We know that depending on the input distribution and the output distribution the results differ in the corresponding models.

However it is observed in recent times that there are many situations where the applicants have to wait not only in one single queue but also in the second queue. The analysis of the primary queue differs from the analysis of the secondary queue. Further, it is always not necessary that the applicants have to go through two different queues. The existence of secondary queue is possible only when the system has a characteristic of service surrender facility. That is, returning back the utilized service by the applicant. In this paper we focus on the study of the secondary queue which is not carried out so far in the available queuing literature. If surrendering the service is available for applicants in the primary queue, this creates a vacancy when the service can be offered to applicants waiting in the secondary queue on a FIFO.

2. THE QUEUING MODEL

The primary queue consists of a single server and inter arrivals and service rates follows the Markovian model (inter arrival rate is Poisson and service rate is exponential) on a FIFO service rule.

(i) The server restricts its service to first N applicants who form the primary queue

(ii) From the $N+1^{th}$ applicant onwards the server allows registration to be included in a waitlist which forms the secondary queue

(iii) The applicants in the waiting or secondary queue are reordered according to merit and FIFO is applied

(iv) Applicants in the secondary queue are serviced when applicants in the primary queue surrender service

(v) The waiting time in the secondary queue is unknown and service for customers in this queue is not guaranteed.

A queue is formed in front of a counter and serviced. If there are N services available these are given on FIFO policy. When this is exhausted the next person gets to be registered as a waiting candidate. He has an option to register or leave the queue. This forms the secondary queue and the aim of this work is to find the waiting time of such an applicant who waited in the primary queue and then in the secondary queue.

3. ASSUMPTIONS FOR THE MODEL:

(i) The system starts with single server and non-empty queue with Kendall notation(M/M/1): (∞/FIFO), where N is finite

(ii) An applicant can surrender his service after N services are completed

(iii) An applicant in the primary queue who surrenders his service cannot reenter the queue.

(iv) The applicants in the waiting or secondary queue are reordered according to merit and FIFO is applied.

4. PERFORMANCE MEASURES OF THE PRIMARY QUEUE

If λ is the average arrival time and μ the mean service rate for a single server Markovian model[3], then $\frac{\lambda}{\mu} = \rho$ gives the traffic intensity.The probability that the system is idle is $P_o = 1 - \rho$.If $\rho = .60$ this means that the applicants have a 60% chance that they will have to wait in the primary queue. From above $\lambda\,\tau = \rho$, where $\tau = \frac{1}{\mu}$ then

i) Waiting time in the queue $= \frac{\rho}{1-\rho}\tau$

ii) Length of the queue $L_q = \frac{\rho^2}{1-\rho}$

iii) Waiting time in the queue $W_q = \frac{L_q}{\lambda}$

5. OTHER PERFORMANCE MEASURES

Probability that there are n customers in system $P_n = (1-\rho)\rho^n$

Probability that the applicant has to wait for more than time t = $\rho e^{-\mu(1-\rho)t}$

Probability that time-in-system is greater than t = $e^{-\mu(1-\rho)t}$

The above formulae [2] give the waiting time of the applicant in the primary queue. The waiting time in the secondary queue is a random variable

If ST is the average time the applicant keeps the service before surrendering it (discrete random variable), the probability distribution of ST=φ has to be found. Here it can be shown that the surrender time follows Geometric distribution.

If p denotes the probability that the applicant keeps the service for a unit time. 1-p is the probability that the applicant surrenders the service some time during a unit time.

Hence the probability that an applicant completes t units of time before he surrenders the service is P (T = t) = (1-p) p^{t-1}, t = 1, 2, 3…..which is the Geometric distribution 0 < p < 1.This p differs and is a random variable .Assume it follows a beta distribution of first kind, where p.d.f is given by

f (p) = $\frac{1}{B(\alpha,\beta)} p^{\alpha-1}(1-p)^{\beta-1}$ The unconditional distribution of T [1] is given by

f(t)= $\frac{\beta(\alpha+\beta-1)!}{(\alpha-1)!} \frac{(\alpha+t-2)!}{(\alpha+\beta+t-1)!}$. The proof for the same is given below.

$$P(T=t) = f(t) = \int_0^1 (1-p)\, p^{t-1} [f(p)]dp$$

$$= \int_0^1 (1-p)\, p^{t-1} \left[\frac{1}{B(\alpha,\beta)} p^{\alpha-1}(1-p)^{\beta-1}\right] dp$$

$$f(t) = \frac{1}{B(\alpha,\beta)} \int_0^1 (1-p)^{\beta}\, p^{\alpha+t-2} dp = \frac{1}{B(\alpha,\beta)} \left[B[(\alpha+t-1),(\beta+1)]\right]$$

$$= \frac{1}{\frac{\Gamma(\beta)\,\Gamma(\alpha)}{\Gamma(\alpha+\beta)}} \frac{\Gamma(\alpha+t-1)\,\Gamma(\beta+1)}{\Gamma(\alpha+\beta+t)} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\beta)\,\Gamma(\alpha)} \frac{\Gamma(\alpha+t-1)\,\Gamma(\beta+1)}{\Gamma(\alpha+\beta+t)}$$

$$= \frac{(\alpha+\beta-1)!}{(\beta-1)!(\alpha-1)!} \frac{(\alpha+t-2)!(\beta!)}{(\alpha+\beta+t-1)!} = \frac{(\alpha+\beta-1)!}{(\alpha-1)!} \frac{(\alpha+t-2)!(\beta!)}{(\alpha+\beta+t-1)!} \quad [1]$$

6 . PARAMETER ESTIMATION

To find the mean of T

Mean of T =E (T) = $\frac{1}{B(\alpha,\beta)}\sum_{1}^{\infty} tB(\beta+1,\alpha+t-1)$, giving values for t we get,

E(T)=$\frac{1}{B(\alpha,\beta)}$[1.B$(\beta+1,\alpha)$ + 2.B $(\beta+1,\alpha+1)$ +3.B $(\beta+1,\alpha+2)$+……

= $\frac{1}{B(\alpha,\beta)}\left[\int_0^1 p^{\beta}(1-p)^{\alpha-1}dp + 2\int_0^1 p^{\beta}(1-p)^{\alpha}dp + 3\int_0^1 p^{\beta}(1-p)^{\alpha-2}dp + \ldots\right]$

$$= \frac{1}{B(\alpha,\beta)}\int_0^1 p^{\beta}(1-p)^{\alpha-1}[1+2(1-p)+3(1-p)^2+\cdots]dp$$

$= \frac{1}{B(\alpha,\beta)}\int_0^1 p^{\beta}(1-p)^{\alpha-1}[p^{-2}]dp = \frac{1}{B(\alpha,\beta)}\int_0^1 p^{\beta-2}(1-p)^{\alpha-1}dp = \frac{B(\alpha,\beta-1)}{B(\alpha,\beta)}$

$= \frac{\Gamma(\alpha)\Gamma(\beta-1)\Gamma(\alpha+\beta)}{\Gamma(\alpha+\beta-)\,\Gamma(\alpha)\Gamma(\beta)} = \frac{\alpha+\beta-1}{\beta-1}$, which is valid for $\beta \neq 1$. If $\alpha+\beta = 1$ the mean is 0. [2]

7. PERFORMANCE MEASURE: RATE OF SERVICE SURRENDER

The above measure is obtained from reliability models and is termed hazard rate given by the conditional probability R_{ss} = P {t < T < t + dt / T > t}

Let *T* be a continuous random variable with cumulative distribution function *F*(*t*) on the interval [0,∞). Its survival function or reliability function is S (t) =P (T > t). We have to find the distribution of T. In this case T is discrete random variable and S (t) = 1- F (t) if T is discrete or continuous.

8. THE DISTRIBUTION OF T

The distribution function of the random variable T is given by

F (T) = $\sum_{1}^{T} f(t)$ where f (t) = $\frac{1}{B(\alpha,\beta)}\int_0^1 (1-p)^{\beta}\, p^{\alpha+t-2}dp = \frac{1}{B(\alpha,\beta)}\big[B[(\alpha+t-1),(\beta+1)]\big]$

F (T) = $\sum_{1}^{T}\frac{1}{B(\alpha,\beta)}\big[B[(\alpha+t-1),(\beta+1)]\big]$ Simplifying the above it can be shown that

F (T)= 1 - $\frac{\alpha+\beta+T}{\beta} f(T+1)$

The survival function is S (T) = $\frac{(\alpha+\beta+T-1)}{\beta} f(T)$.

$\frac{f(T)}{S(T)} = \frac{\beta}{(\alpha+\beta+T-1)}$ is the R_{ss} [3]

9. ANALYSIS OF THE ABOVE QUEUING SYSTEM

The total waiting time of an applicant in the queue is waiting time in primary queue + waiting time in secondary queue $= \frac{\rho}{1-\rho}\tau + \frac{\alpha+\beta-1}{\beta-1}$ where $\frac{\lambda}{\mu} = \rho$.

a) E (T) Average waiting time of an applicant in the secondary queue.

For different values of α *and* β the waiting time in the secondary queue is studied from Table 1.

Table 1. Average waiting time of an applicant in the secondary queue

α \ β	**4**	**6**	**8**	**10**	**12**	**14**	**16**	**18**	**20**	**30**	**40**	**50**
4	2.33	1.8	1.57	1.44	1.36	1.30	1.26	1.23	1.21	1.13	1.10	1.08
6	3	2.2	1.85	1.66	1.54	1.46	1.4	1.35	1.31	1.20	1.15	1.12
8	3.66	2.6	2.14	1.88	1.72	1.61	1.53	1.47	1.42	1.27	1.20	1.16
10	4.33	3	2.42	2.11	1.90	1.76	1.66	1.58	.152	1.34	1.25	1.20
12	5	3.4	2.71	2.33	2.09	1.92	1.8	1.70	1.63	1.41	1.30	1.24
14	5.6	3.8	3	2.55	2.27	2.07	1.93	1.82	1.76	1.48	1.35	1.28
16	6.33	4.2	3.28	2.77	2.45	2.23	2.06	1.94	1.84	1.55	1.41	1.32
18	7	4.6	3.57	3	2.63	2.38	2.2	2.05	1.94	1.62	1.46	1.36
20	7.6	5	3.85	3.22	2.81	2.53	2.33	2.17	2.05	1.68	1.51	1.40
30	11	7	5.28	4.33	3.72	3.30	3	2.76	2.57	2.03	1.76	1.61
40	14.3	9	6.71	5.44	4.63	4.07	3.66	3.35	3.10	2.37	2.02	1.81
50	17.66	11	8.14	6.55	5.54	4.84	4.33	3.94	3.63	2.72	2.28	2.02

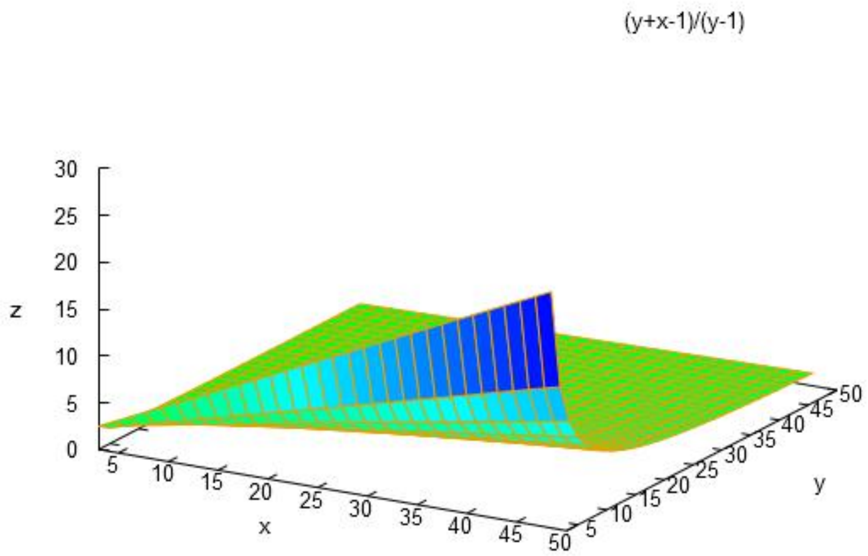

Figure 1. A 3D plot of Table 1 values

From the above table the following inferences can be made.

i) For fixed α and increasing β the expected waiting time in secondary queue decreases.

ii) For fixed β and increasing α the expected waiting time in secondary queue increases.

iii) For $\alpha = \beta$ (the diagonal elements) the expected waiting time in secondary queue converges to 2. Except for $\alpha = \beta = 2$ where E(T) =3.

Proof: If $\alpha = \beta$, E (T) = $\frac{\alpha+\beta-1}{\beta-1} = \frac{2\beta-1}{\beta-1} = \frac{2-\frac{1}{\beta}}{1-\frac{1}{\beta}}$, converges to 2 for large values of β.

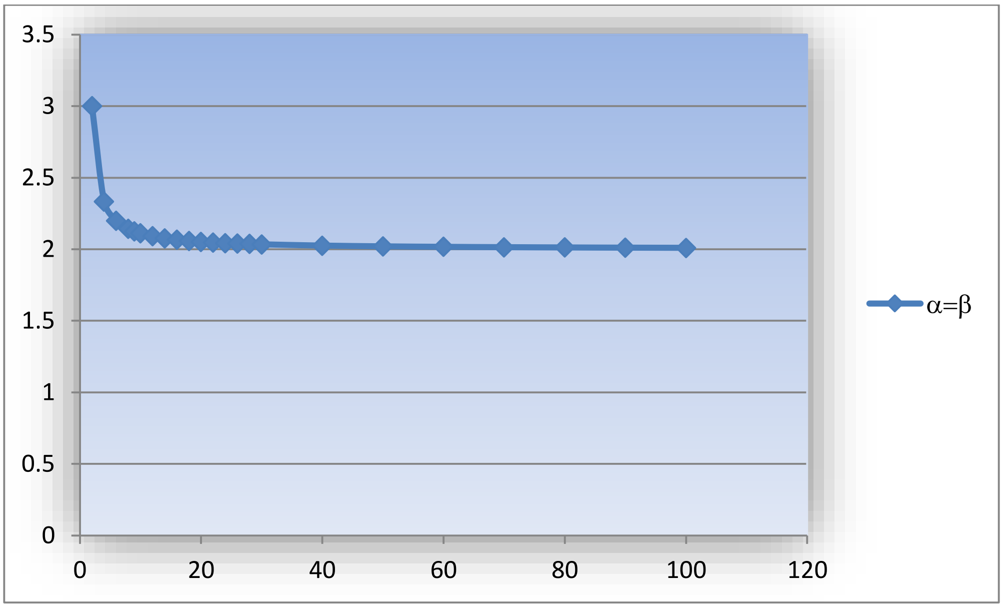

Figure 2. Plot for $\alpha = \beta$

b) R_{ss} = P {t < T < t + dt / T > t} = $\frac{\beta}{(\alpha+\beta+T-1)}$ For a faster service surrender rate, the waiting time in the secondary queue is reduced. The rate of service surrender is given in Table 2 and Table 3. Here two types are studied.

a) If α is fixed and β and t are varied.

b) If β is fixed and α and t are varied.

Table 2. Service surrender rate when α is fixed as 5 and β and t are varied

T \ β	10	20	30	40	50
4	0.5555	0.714286	0.78947368	0.833333	0.862069
6	0.5	0.666667	0.75	0.740741	0.833333
8	0.45454	0.625	0.71428571	0.8	0.806452
10	0.41666	0.588235	0.68181818	0.769231	0.78125
20	0.29411	0.454545	0.55555555	0.625	0.67567
30	0.22727	0.37037	0.46875	0.540541	0.595238
40	0.18518	0.3125	0.40540540	0.47619	0.531915
50	0.15625	0.27027	0.35714285	0.425532	0.480769
60	0.13513	0.238095	0.31914893	0.384615	0.438596
70	0.1190	0.212766	0.28846153	0.350877	0.403226
80	0.10638	0.19230	0.26315789	0.322581	0.373134
90	0.09615	0.17543	0.24193543	0.298507	0.347222
100	0.08771	0.16129	0.223880597	0.277778	0.324675

Table 3. Service surrender rate when β is fixed as 5 and α and t are varied

T \ α	**10**	**20**	**30**	**40**	**50**
4	0.3125	0.178571	0.131579	0.104167	0.086207
6	0.277778	0.166667	0.125	0.1	0.083333
8	0.25	0.15625	0.119048	0.096154	0.080645
10	0.227273	0.147059	0.113636	0.092593	0.078125
20	0.208333	0.113636	0.092593	0.078125	0.067568
30	0.147059	0.092593	0.078125	0.067568	0.059524
40	0.113636	0.078125	0.067568	0.059524	0.053191
50	0.092593	0.067568	0.059524	0.053191	0.048077
60	0.078125	0.059524	0.053191	0.048077	0.04386
70	0.067568	0.053191	0.048077	0.04386	0.040323
80	0.059524	0.048077	0.04386	0.040323	0.037313
90	0.053191	0.04386	0.040323	0.037313	0.034722
100	0.048077	0.040323	0.037313	0.034722	0.032468

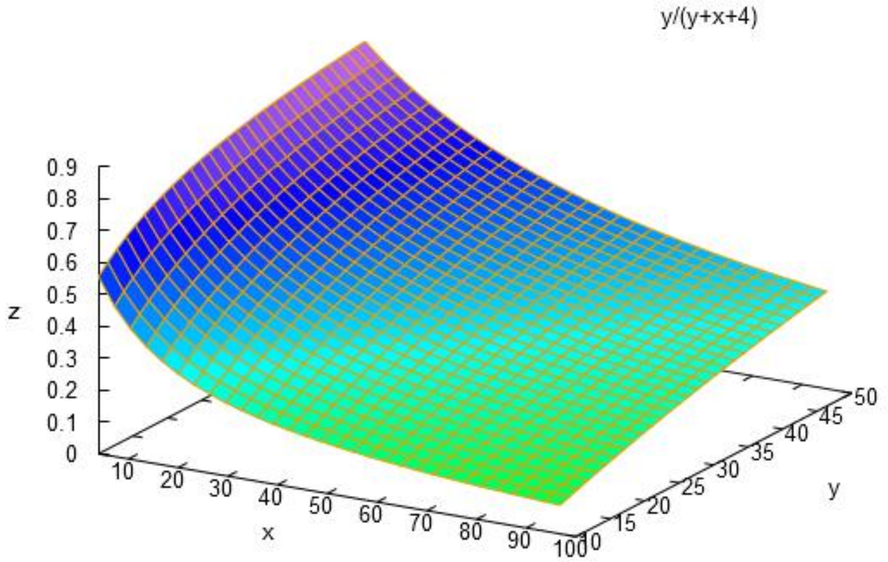

Figure 3. A 3D plot of Table 2

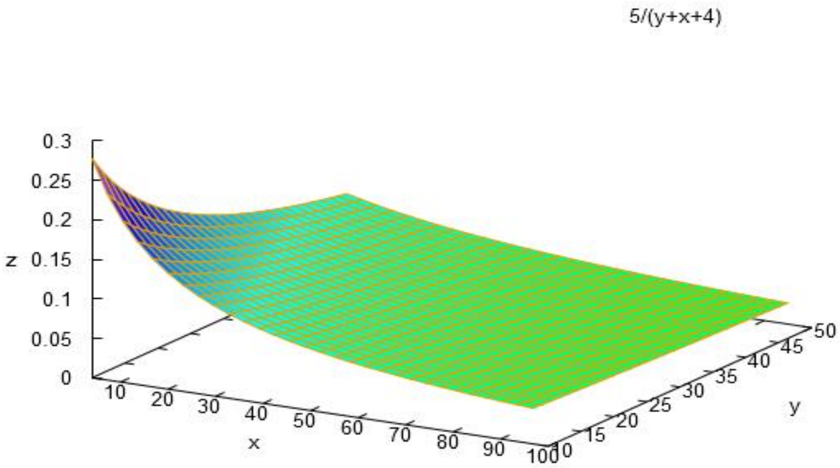

Figure 4. A 3D plot of Table 3

From the above table values it can be inferred that i) When α is fixed and β and t are varied the Service surrender rate increases as β increases ii) When β is fixed and α and t are varied the Service surrender rate decreases as α increases. This shows that in the first case service surrender rate is faster and length of the secondary queue declines at a faster rate.

CONCLUSION

This paper analyses the secondary queues with surrender facility, waiting time and service surrender rate which are the major performance measures. When this queue is modeled as a Beta- distribution of first kind with parameters α and β, for fixed α and increasing β the expected waiting time in secondary queue decreases. Also when α is fixed and β and t are varied the service surrender rate increases as β increases and length of the secondary queue declines at a faster rate. Hence in this case service surrender rate is inversely proportional to secondary queue length.

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