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Accelerated Life Testing in Interference Models with Monte-Carlo Simulation

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Abstract

Here we have presented two accelerated Life Testing (ALT) models for interference theory of reliability. We have assumed that instead of a single stress, as faced by a system in normal operating conditions, for accelerated conditions a number of stresses are applied to the system simultaneously. In the first model we assume that the system fails if the sum of the stresses exceeds the strength of the system and assumption in the second is that the system fails if maximum of the stresses exceeds the strength. For the first model assuming that both stress and strength follow either exponential or gamma or normal distributions and in the second, generalized exponentials, we have obtained the reliability, say (R), of the system in the accelerated condition. The expressions show that when the number of stresses increases R, decreases or failure probability increases and one may get more failure data quickly, justifying the models. Using Monte-Carlo simulation we have estimated R. Another estimate of R is obtained from proportion of successes. From R we obtained estimates of reliability, say R_A, at use level. Some numerical values of R_A are tabulated for particular values of the parameters of stress-strength distributions. The numerical values also justify the use of the models.

Keywords: Stress-strength, reliability

1. INTRODUCTION

The more reliable a device is, the more difficult it is to measure its reliability. For the systems whose reliability is studied from stress-strength models i.e. interference models the situation is still more desperate. This is so because many years may be decades or century, of testing under actual operating conditions would be required to obtain numerical measures of its reliability. Even if such testing were feasible, the rate of technical advance is so great that those parts would be obsolete by the time their reliability had been measured. One approach to solving this predicament is to use accelerated life tests, in which parts are operated at higher stress levels than required for normal use, Mann et al. (1974).

Today's manufacturers face strong pressure to develop new, higher-technology products in record time, while improving productivity, product field reliability and overall quality.

Estimating the failure time distribution or long term performance of components of high reliability product is particularly difficult. Most modern products are designed to operate without failure for years, decades or longer. Thus few units will fail or degrade appreciably in a test of any practical length of time at normal use conditions. For example, the design and construction of a communications satellite may allow only eight months testing components that are expected to be in service for 10 or 15 years. For such applications Accelerated Life Testing (ALT) are used in manufacturing industries to asses, to demonstrate components and subsystems reliabilities, to certify components, to detect failure modes so that they can be correlated, to compare different manufacturers, and so on and so forth. ALTs have become increasingly important because of rapidly changing technologies, more complicated products with more components, higher customers' expectations for better reliability and the need for rapid product development, Escobar and Meeker (2006).

An ALT is one which is conducted at a stress level usually higher than that which we expect to occur in the use environment. For instance it is sometimes said that high temperature is the enemy of reliability. Increasing temperature is one of the most commonly used methods to accelerate a failure mechanism. In a biological context it could be the number of rad of γ radiation delivered to animals in radiation experiments designed to extrapolate life time test results at high stress or dose to lifetimes at low-radiation-dose environments, Barlow (1982).

But Evans (1977) made an important point that the need to make rapid reliability assessments and the fact that ALTs may be 'the only game in town' are not sufficient to justify the use of the method. Justification must be based on physical models or empirical evidence.

Given test results, the main problem is to relate lifetimes at various stress levels and to predict life at usually much lower stress level, Barlow (1982).

Some accelerated tests use more than one accelerating variables (or stresses). Such tests might be suggested when it is known that two or more potential accelerating variables contribute to degradation and failure. Using two or more variables may provide needed time acceleration without requiring levels of the individual accelerating variables to be too high. For example it is a common knowledge that a certain joint may stand a particular shock or vibration at a low temperature but it may fail even by a smaller shock at higher temperature.

Thus in ALT more severe stresses than the normal is used in the hope to get failure data quickly. In interference models stress and strength both are random variables, so more severe stress has no meaning. But if we use a sum of a number of random stresses instead of a single stress we may expect to get more failure data i.e. the combination of more than one stresses may give more failure data. The combination of stresses may be linear or non-linear. Here, we shall assume that the effect of different stresses on the system, under consideration, is linear.

Here in Sec.2 we have considered an ALT for Interference models where instead of a single stress a linear combination of m stresses work, simultaneously, on the system under study. All the stresses and the strength of the system are represented by independent random variables. For simplicity we have assumed that all the stresses are identically distributed and obtained system reliability at combined stresses and then obtained reliability for a single stress i.e. for use condition. Of course one may also use different distributions for stresses. The distributions considered here are exponential, gamma and normal. We have obtained the estimates of the parameters involved by using Monte Carlo Simulation (MCS) and there by getting estimates of system reliability. In Sec.3 we have considered another model where more than one stresses are applied simultaneously and the system works if maximum of these stresses is less than the system strength. In particular we have assumed that stress-strength are generalized exponential variates.

Most of the ALT studies in the literature are for time-to-failure (TTF) models. Here, generally, a failure time distribution of the unit under study is considered and then a relationship between the parameters of this distribution and environmental conditions (stress) is assumed. But we have not come across any ALT studies for Interference models accept an old study viz. Kakati and Sriwastav (1986).

2. AN ALT FOR INTERFERENCE MODELS

Suppose we use a sum of m (fixed) independently and identically distributed stresses $Y_1, Y_2, \dots, Y_m$, instead of a single stress Y_1 (say), on a component (or system) with strength X (say).

Let $Y = Y_1+Y_2+ \dots + Y_m$. Then the reliability, R, of the system will be given by

$$R = P[X \geq Y]. \tag{2.1}$$

2.1 Exponential stress-strength

Let us assume that X and each Y's follow exponential distributions with parameters λ and 1 (without loss of generality), respectively. Then we know that Y will follow a gamma distribution with degrees of freedom ‘m’, so the p.d.f.’s of X and Y, viz. f(x) and g(y), are given by

$$f(x) = \frac{1}{\lambda} e^{-\frac{x}{\lambda}} \quad \text{and} \quad g(y) = \frac{y^{m-1} e^{-y}}{\Gamma(m)}. \tag{2.2}$$

Then from Eq.(2.1), the reliability of the system at accelerated stress Y is given by

$$R = \left(\frac{\lambda}{1+\lambda} \right)^m. \tag{2.3}$$

From Eq.(2.3) it is obvious that as m (the number of stresses) increases the reliability decreases and $R \to 0$ as $m \to \infty$ and so the probability of failure (=1 – R) increases and expectedly we may get more failure data quickly. The reliability, R_A, at the use level or actual level of stress is given by

$$R_A = \frac{\lambda}{1+\lambda} = (R)^{1/m}. \tag{2.4}$$

We use Monte Carlo Simulation (MCS) to estimate R_A.

To use MCS we proceed as follows:

We estimate R and R_A in two ways by MCS. (1) For a particular values of λ and m we have taken random samples of size M from exponential (λ) and gamma (m) distributions for X and Y, respectively, using MATLAB. Means of samples of X and Y give the estimates of the parameters λ and m, respectively. Then substituting these estimates in Eq.(2.3), we get the estimates $\hat{R}$ (say) of R. Then from Eq.(2.4) the estimate of R_A, say $\hat{R}_A$, is given by the mth root of $\hat{R}$. (2) Alternatively, for the same sample as in (1) each value of X is compared with corresponding value of Y and whenever an X is greater than or equal to Y, it will be called a success. Then another estimate of R, say R_P, is given as

$$R_P = \frac{\text{Number of Successes}}{n} \tag{2.5}$$

Obviously, for a given m, the corresponding estimate of R_A, say R_{AP} is given by

$$R_{AP} = (R_P)^{1/m} \tag{2.6}$$

If the above two processes are replicated k times then we get estimated reliability data sets $\hat{R}$, $\hat{R}_A$, R_P and R_{AP} of size k.

For particular values of λ and m taking M = 200, 500, 1000 and k = 100, we have obtained means and s.d.’s of $\hat{R}_A$, R_{AP}, and tabulated in the Table 1. Then, we have drawn normal probability plot (NPP) graphs for each data set of estimated reliability $\hat{R}_A$ and R_{AP} for different parameter values of λ and m as given in Table 1. All NPP graphs suggests that the distribution of $\hat{R}_A$ and R_{AP} are normal. For illustration purpose, we have given only one NPP graph for data sets $\hat{R}_A$ and R_{AP} when $\lambda = 2$ and $m = 2$ in Fig.1. We have also applied tests of significance viz. z-test (since $k > 30$) for a given λ and m, to test the significance of difference between true R_A given by Eq.(2.4) and corresponding $\hat{R}_A$. Similarly, we have used z-test to test the significance of the difference between R_A and R_{AP} for the same λ and m. All these values of z's viz. z_1 for R_A and $\hat{R}_A$, z_2 for R_A and R_{AP} are also given in Table 1.

Note: In Fig.1, we have written Estimated R_A for $\hat{R}_A$; similarly in all the graphs.

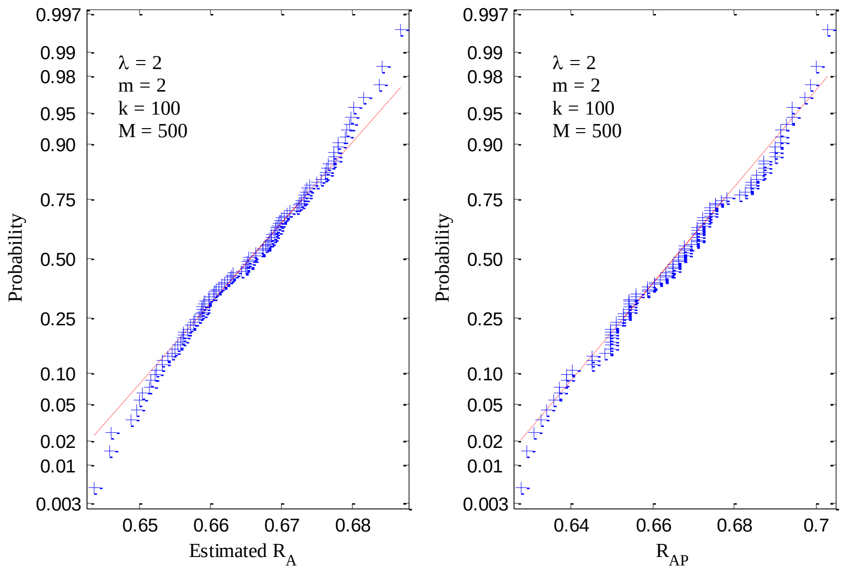

Figure 1: (Exponential S-S, Sec.2.1)

Table 1: Exponential stress-strength

M	k	λ	m	R	R_A	Mean of $\hat{R}_A$	SD of $\hat{R}_A$	Mean of $\hat{R}_{AP}$	SD of $\hat{R}_{AP}$	z_1 for $\hat{R}_A$	z_2 for $\hat{R}_{AP}$
200	100	2	2	0.4444	0.6667	0.6664	0.0174	0.6599	0.0255	0.1516	2.6300
500	100	2	2	0.4444	0.6667	0.6662	0.0092	0.6672	0.0168	0.4608	0.3292
1000	100	2	2	0.4444	0.6667	0.6676	0.0131	0.6669	0.0117	1.2100	0.1614
500	100	3	2	0.5625	0.7500	0.7505	0.0089	0.7502	0.0147	0.5641	0.1208
500	100	2	3	0.2963	0.6667	0.6679	0.0154	0.6646	0.0152	1.2617	0.3601
500	100	2	4	0.1975	0.6667	0.6659	0.0110	0.6653	0.0163	0.6006	0.8370

In Table 1, we see that for both the estimator $\hat{R}_A$ and R_{AP}, z_1 and z_2 are insignificant except for M = 200, z_2 is significant when $\lambda = 2$, m = 2, k =100. We have increased M to 500 and then z_2 for the same values of λ, m and k, becomes insignificant i.e. accuracy of estimator R_{AP} increases with n which is expected; but for M = 1000 there is not much change. So, we have decided to take M = 500 throughout. Obviously R appreciably decreases with m i.e. failure probability increases so we may get more failure data. For example, R = 0.4444 for m = 2 but R = 0.2963 for m = 3 when $\lambda = 2$, M = 500 and k = 100.

2.2 Gamma stress-strength

Suppose $X \sim \Gamma(1,\lambda)$ and $Y_i \sim \Gamma(1,\mu)$, i = 1, 2, …, m, then from the reproductive property of gamma distribution Y (=$Y_1+Y_2+ \dots +Y_m$) $\sim \Gamma(1, m\mu)$. Therefore, if λ is an integer [Pandit and Sriwastav (1975)],

$$R = \sum_{i=0}^{\lambda-1} \frac{\Gamma(\lambda + m\mu - i - 1)}{\Gamma(m\mu)(\lambda - i - 1)!\,2^{\lambda+m\mu-i-1}}\,. \tag{2.7}$$

or if λ and μ are not necessarily integers, Kapur and Lamberson (1977), then

$$R = \mathrm{Beta}(\lambda, m\mu)\mathrm{Beta}_{1/2}(\lambda, m\mu) \tag{2.8}$$

$$R_A = \mathrm{Beta}(\lambda, \mu)\mathrm{Beta}_{1/2}(\lambda, \mu) \tag{2.9}$$

From Eq.(2.7) we see that if $m\mu$ is much larger than λ then $\Gamma(m\mu) \simeq \Gamma(\lambda + m\mu - i - 1)$ and then

$$R \simeq \sum_{i=0}^{\lambda-1} \frac{1}{(\lambda - i - 1)!\,2^{\lambda+m\mu-i-1}}\,, \tag{2.10}$$

which $\rightarrow$ 0 as m $\rightarrow \infty$. So that as m increases reliability decreases or failure probability increases, thereby we may get more failure data quickly.

Further, from Eq.(2.10)

$$R_A \simeq \sum_{i=0}^{\lambda-1} \frac{1}{(\lambda - i - 1)!\,2^{\lambda+\mu-i-1}}\,, \text{ for } m = 1. \tag{2.11}$$

Then, from Eq.(2.10) and Eq.(2.11)

$$\frac{R}{R_A} \simeq \frac{\dfrac{1}{2^{\lambda+m\lambda-1}}\sum_{i=0}^{\lambda-1} 2^i}{\dfrac{1}{2^{\lambda+\mu-1}}\sum_{i=0}^{\lambda-1} 2^i} = \frac{2^{-m\mu}}{2^{-\mu}}\,, \tag{2.12}$$

which implies that in this case also

$$R_A \simeq (R)^{\frac{1}{m}}. \tag{2.13}$$

We may expect similar relation to hold approximately for R and R_A given by Eq.(2.8) and Eq.(2.9) also.

Now, as in Sec.2.1 from MCS we can estimate of λ and $m\mu$. Since the estimates of λ and $m\mu$ may not be integers, so we use Eq.(2.8) instead of Eq.(2.7). Substituting estimated values of λ and $m\mu$ in Eq.(2.8) we get the estimate, say $\hat{R}$, of reliability R at accelerated stress. The estimate of μ for use level of stress, is given by

$$\hat{\mu}_A = \frac{m\mu}{m}, \tag{2.14}$$

and the estimate, say $\hat{R}_A$, of reliability at use level of stress is given by substituting $\hat{\lambda}$ for λ and $\hat{\mu}_A$ for μ in Eq.(2.9).

Similarly as in Eq.(2.5) and Eq.(2.6), by MCS, we can get an alternative estimate of R_A by $\hat{R}_{AP}$.

For given n, k, λ, μ we have obtained means and s.d.'s of $\hat{R}_A$ and $\hat{R}_{AP}$, and tabulated in Table 2. We have drawn NPP graphs for each data set of estimated reliability $\hat{R}_A$ and R_{AP} for different values of λ, μ and m as given in Table 2. All NPP graphs suggest that the data sets of $\hat{R}_A$ and R_{AP} follow normal distribution. For illustration purpose, we have given a NPP graph for data sets $\hat{R}_A$ and R_{AP} when λ = 2, μ = 2 and m = 3 in Fig.2. We have performed z-test also, as in the last section, and the values of z's are given in the same Table2.

Table 2: Gamma stress-strength

M	k	λ	μ	m	R	R_A	Mean of $\hat{R}_A$	SD of $\hat{R}_A$	Mean of $\hat{R}_{AP}$	SD of $\hat{R}_{AP}$	z_1 for $\hat{R}_A$	z_2 for $\hat{R}_{AP}$
500	100	2	2	2	0.1875	0.5000	0.5039	0.0168	0.5034	0.0236	2.3297	1.4461
500	200	2	2	2	0.1875	0.5000	0.5012	0.0162	0.5018	0.0236	1.0900	1.0900
500	200	2	2	3	0.0625	0.5000	0.4997	0.0152	0.5014	0.0221	0.2808	0.8717
500	200	2	4	2	0.0195	0.1875	0.1876	0.0107	0.1871	0.0195	0.1488	0.2755
500	200	4	2	2	0.5000	0.8125	0.8118	0.0114	0.8120	0.0180	0.8591	0.3699
500	200	2	4	3	0.0017	0.1875	0.1875	0.0111	0.1886	0.0159	0.0431	0.9617
500	200	4	2	3	0.2539	0.8125	0.8120	0.0101	0.8133	0.0188	0.6745	0.6108
500	200	4	4	2	0.1133	0.5000	0.4990	0.0165	0.4983	0.0233	0.9558	1.0298
500	200	4	4	3	0.0176	0.5000	0.4989	0.0162	0.5001	0.0239	0.9503	0.0356

We have taken M = 500, as we saw in the last section that for M = 500, the estimates are good enough. In Table 6.2, we see that z_1 is significant but z_2 is insignificant for k =100 when $\lambda = 2$, $\mu = 2$ and m = 2. So, for achieving better estimator $\hat{R}_A$, we have taken k = 200 throughout. Then we observe that all z values are insignificant for all cases. For example, $z_1 = 0.2808$ and $z_2 = 0.8717$ for $\lambda = 2$, $\mu = 2$, m = 3 is smaller than the tabulated $|z| = 1.96$ at 5% level of significance. We also observe that the values of R decrease when m increases which is expected. For example, R = 0.1875 for m = 2 and R = 0.0625 for m = 3 when $\lambda = 2$, $\mu = 2$. The insignificant value of z_2 also indicate the relation Eq.(2.13) is true.

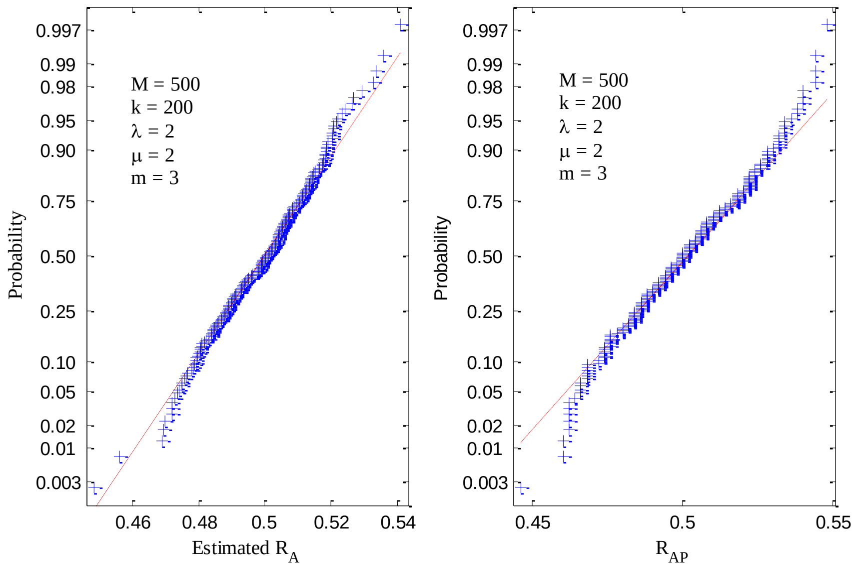

Figure 2: (Gamma S-S, Sec.2.2)

2.3 Normal stress-strength

Suppose $X \sim N(\lambda, \sigma^2)$ and $Y_i \sim N(\mu, \rho^2)$, i = 1, 2, …, m, then we know that

$Y \sim N(m\mu, m\rho^2)$ where $Y = Y_1 + Y_2 + \ldots + Y_m$. Then, Pandit and Sriwastav (1975),

$$R = \Phi\left(\frac{\lambda - m\mu}{\sqrt{\sigma^2 + m\rho^2}}\right). \tag{2.15}$$

Obviously, as in the above cases, $R \rightarrow 0$ as $m \rightarrow \infty$. So the failure probability of the system will increase with m and hopefully giving more failure data quickly.

Further, from Eq.(2.15)

$$R_A = \Phi\left(\frac{\lambda - \mu}{\sqrt{\sigma^2 + \rho^2}}\right), \text{ for m = 1.} \tag{2.16}$$

From Eq.(2.15) and Eq.(2.16) we see that here we cannot find a relation like Eq.(2.13) i.e. in this case

$$R_A \neq (R)^{\frac{1}{m}}, \tag{2.17}$$

which is reflected in numerical evaluation also [see Table 3(a)]

As above for given λ, σ^2, μ, ρ^2, m, n and k using MCS we can estimate λ, σ^2, $m\mu$ and $m\rho^2$. Then the estimates of μ and ρ^2, at the use level of stress, viz. $\hat{\mu}_A$ and $\hat{\rho}_A^2$, are given by

$$\hat{\mu}_A = \frac{(m\mu)}{m} \quad \text{and} \quad \hat{\rho}_A^2 = \frac{(m\rho)}{m}. \tag{2.18}$$

Substituting $\hat{\mu}_A$ and $\hat{\rho}_A^2$ for μ and ρ^2 in Eq.(6.2.17) we get $\hat{R}_A$, the estimate of system reliability at use level of stress.

R_P obtained as in Eq.(2.5) gives another estimate of R. But estimate of R_A from R_P cannot be obtained from Eq.(2.6) because of Eq.(2.17). Here, we have obtained R_{AP} from R_P using Eq.(2.6) and note that the difference between R_A and R_{AP} is highly significant [see Table 3(a)]. Even increasing the values of M does not improve the estimator R_{AP}. Of course, corresponding z_1 is insignificant. Thus data also show that Eq.(2.17) is true.

Table 3 (a): Normal stress-strength

n	k	λ	μ	σ	ρ	m	R	R_A	Mean of $\hat{R}_A$	SD of $\hat{R}_A$	Mean of R_{AP}	SD R_{AP}	z_1 for $\hat{R}_A$	z_2 for R_{AP}
500	100	2	2	1	1	2	0.1241	0.5000	0.5004	0.0154	0.3540	0.0216	0.2326	67.4800
200	100								0.4962	0.0240	0.3529	0.0349	1.3132	42.0624
1000	100										0.3528	0.0159	0.7846	92.6826

Further, we have obtained means and s.d.'s of $\hat{R}_A$ and R_P which are tabulated in the Table 3(b). We have drawn NPP graphs for each data set of estimated reliability $\hat{R}_A$ and R_P for different values of λ, μ, σ and m as given in Table 3(b). The NPP graphs suggest that the distribution of $\hat{R}_A$ and R_P are normal. For illustration, we have given only one such graph in Fig.3 corresponding to $\lambda = 2$, $\mu = 2$, $\sigma = 1$, $\rho = 1$, m = 2. As earlier, we have used z-test for each case; z values are also tabulated in Table 3(b).

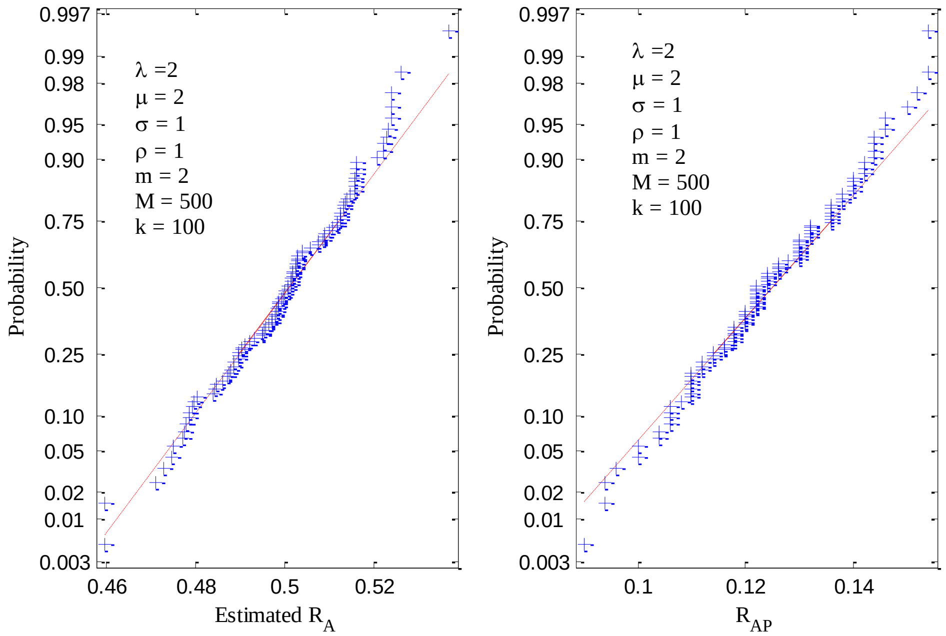

Figure 3: (Normal S-S, Sec.2.3)

Table 3 (b): Normal stress-strength

M	k	λ	μ	σ	ρ	m	R	R_A	Mean of $\hat{R}_A$	SD of $\hat{R}_A$	Mean of R_P	SD R_P	z_1 for $\hat{R}_A$	z_2 for R_P
500	100	2	2	1	1	2	0.1241	0.5000	0.4987	0.0162	0.1228	0.0179	0.7846	0.1903
500	100	3	2	1	1	2	0.2819	0.7602	0.7598	0.0132	0.2818	0.0223	0.3553	0.3328
500	100	2	3	1	1	2	0.0105	0.2397	0.2393	0.0135	0.0103	0.0043	0.3212	1.4600
500	100	2	2	1	1	3	0.0227	0.5000	0.4978	0.0134	0.0230	0.0070	1.6100	0.9192
500	100	2	2	2	2	2	0.2819	0.5000	0.5010	0.0154	0.2812	0.0226	0.6365	0.7104
500	100	2	2	2	2	3	0.1587	0.5000	0.5003	0.0144	0.1548	0.0162	0.2197	0.3514

In Table 3(b), we see that all z-values for both the estimator $\hat{R}_A$ for R_A and R_P for R are insignificant in all cases. For example, $z_1 = 0.3212$ and $z_2 = 1.4600$ for $\lambda = 2$, $\mu = 3$, $\sigma = 1$, $\rho = 1$ and m = 2 are smaller than tabulated $|z| = 1.96$, so, z_1 and z_2 are insignificant. We also see that R value decreases with increasing m which is expected. For example, R = 0.1241 for m = 2 but R = 0.0227 for m = 3 when $\lambda = 2$, $\mu = 2$, $\sigma = 1$ and $\rho = 1$.

3. ANOTHER ALT MODEL FOR INTERFERENCE THEORY

Kakaty and Sriwastav (1986) used an ALT for interference models in a different way. They assumed that a number of random stresses are applied, simultaneously, to a system under test. If the maximum of these stresses is more than the strength of the system then the system fails otherwise it survives. They assumed that all the stresses are i.i.d. exponential variates and strength is another exponential variate. Here we have considered the same model for generalized exponential distributions, Gupta and Kundu (1997).

3.1 Mathematical formulation

Let $Y_1, Y_2, \ldots, Y_m$ be m i.i.d. random variables representing m stresses simultaneously applied on a system whose strength is represented by another r.v. X. Let G(y) be be the c.d.f. of Y's and f(x) be the p.d.f. of X. Then as per assumption of the model the system survives if

$$\text{Max}\,(Y_1, Y_2, \ldots, Y_m) \le X.$$

Then reliability R of the system at accelerated stress is given by

$$R = P\,[\text{Max}\,(Y_1, Y_2, \ldots, Y_m) \le X]$$

$$= \int_{-\infty}^{\infty} [G(x)]^m f(x)dx\,. \qquad (3.1)$$

We note that Eq.(3.1) is a decreasing function of 'm' i.e. as m, the number of stresses applied simultaneously, increases the survival probability of the system decreases and $R \to 0$ as $m \to \infty$. In other words failure probability (=1– R) increases with m and hopefully we shall get more failure data quickly. This justifies the use of more than one stresses simultaneously.

3.2 Stress - strength are generalized exponential variates

Let us assume that both X and Y_i, i = 1, 2, …,m follows generalized exponential distributions (GED) with same scale parameters but different shape parameters with p.d.f.'s f(x) and g(y), respectively, given by

$$f(x,\ \lambda,\ \alpha) = \alpha\,\lambda\,e^{-\lambda x}\left(1-e^{-\lambda x}\right)^{\alpha-1}; \qquad x \ge 0, \lambda > 0,\ \alpha > 0 \qquad (3.2)$$

where, λ is the scale parameter and α is the shape parameter.

$$g_i(y_i,\ \lambda,\ \beta) = \beta\,\lambda\,e^{-\lambda y_i}\left(1-e^{-\lambda y_i}\right)^{\beta-1}; \qquad y \ge 0, \lambda > 0,\ \beta > 0 \qquad (3.3)$$

where, λ is the scale parameter and β is the shape parameter.

Obviously, the c.d.f. of Y_i is

$$G_i(y_i,\ \lambda,\ \beta) = \left(1 - e^{-\lambda y_i}\right)^{\beta}. \quad (3.4)$$

Let $\qquad Y = \text{Max}(Y_1, Y_2, \ldots, Y_m) \quad (3.5)$

Then, the c.d.f. of Y, G(y), is given by

$$G(y) = \prod_{i=1}^{m} [G_i(y_i)] \quad (3.6)$$

Suppose $\qquad G_1(y) = G_2(y) = \ldots = G_m(y_m) = G_1(y)$, say

Then $\qquad G(y) = [G_1(y)]^m = (1 - e^{-\lambda y})^{m\beta}, \quad (3.7)$

which is the c.d.f. of a generalized exponential variate with scale parameter λ and shape parameter $m\beta$

Then, from Eq.(3.1) the reliability of the system at accelerated stress is

$$R = \int_0^{\infty} \left[\left(1 - e^{-\lambda x}\right)^{\beta}\right]^m \alpha\,\lambda\, e^{-\lambda x}\left(1 - e^{-\lambda x}\right)^{\alpha - 1} dx. \quad (3.8)$$

Substituting $z = 1 - e^{-\lambda x}$ in Eq.(4.3.8) we get

$$R = \alpha \int_0^1 z^{m\beta + \alpha - 1}\, dz$$

$$= \frac{\alpha}{m\beta + \alpha}. \quad (3.9)$$

Obviously, $R \to 0$ as $m \to \infty$, so as number of stresses (m) increases reliability decreases i.e. failure probability increases hopefully giving more failure data quickly.

Now from Eq.(3.9) the reliability R_A of the system by the application of a single stress i.e. reliability of the system at use level of stress, R_A (say), is given by

$$R_A = \frac{\alpha}{\alpha + \beta}, \text{ when } m = 1 \quad (3.10)$$

If $\hat{\alpha}$ and $\hat{\beta}$ are estimates of α and β then the estimate of R in Eq.(3.9) is given by

$$\hat{R} = \frac{\hat{\alpha}}{\hat{\alpha} + m\beta} \quad (3.11)$$

and that of $\hat{R}_A$ in Eq.(4.3.10) by

$$\hat{R}_A = \frac{\hat{\alpha}}{\hat{\alpha} + \hat{\beta}}. \quad (3.12)$$

If we have samples of sizes M from populations given by Eq.(3.2) and Eq.(3.3), respectively, then the MLE's of α and β are given by Kundu and Gupta (2005)

$$\hat{\alpha} = \frac{M}{\sum_{i=1}^{M} \log(1-e^{-\lambda x_i})} \quad \text{and} \quad \hat{\beta} = \frac{M}{\sum_{i=1}^{M} \log(1-e^{-\lambda y_i})}, \text{ respectively.} \tag{3.13}$$

Since R_A is a one-to-one function of α, β hence as per the properties of MLE's $\hat{R}_A$ is a MLE of R_A.

It seems difficult to obtain the mean and the variance of $\hat{R}_A$, analytically. We have used Monte-Carlo Simulation to obtain $\hat{R}_A$ and its mean and variance.

To use MCS we proceed as follows:

The c.d.f. of X from Eq.(3.2) is

$$F(x) = \left(1-e^{-\lambda x}\right)^{\alpha} = Z \text{ (say)}, \tag{3.14}$$

where Z is uniform in (0, 1).

$$\text{Now,} \quad z^{\frac{1}{\alpha}} = 1-e^{-\lambda x} \quad \text{or} \quad e^{-\lambda x} = 1-z^{\frac{1}{\alpha}} \quad \text{or} \; -\lambda x = \log(1-z^{\frac{1}{\alpha}})$$
$$\Rightarrow x = -\frac{1}{\lambda}\log(1-z^{\frac{1}{\alpha}}) \tag{3.15}$$

Thus by taking M uniformly distributed random values of Z, from Eq.(3.15) we can obtain a random sample of size M of the r.v. X following generalized exponential distribution.

Now we apply m stresses, which are identically distributed as generalized exponential variates with parameters (λ, β), simultaneously, and we assume that if the maximum of these stresses exceeds the strength of the system the system fails. From Eq.(3.5) Y represents this maximum and its distribution is given by Eq.(3.7) with parameters (λ ,$m\beta$). So, first we take a set of m uniformly distributed values of Z and obtain m y_{ij}'s from the formula [as in Eq.(3.15)].

$$y_{ij} = -\frac{1}{\lambda}\log(1-z^{\frac{1}{\beta}}), \; j = 1,2, \dots ,m. \tag{3.16}$$

Then the maximum of first set of m y_{ij}'s given by Eq.(3.16) is, say y_1. We repeat this process M times and get y_1, y_2, … , y_M. As noted above, the distribution of these y_i's is generalized exponential with parameters (λ, $m\beta$). Then the estimate of $m\beta$ is given by [see Eq.(3.13)]

$$m\beta = \frac{M}{\sum_{i=1}^{M} \log(1 - e^{-\lambda y_i})}, \tag{3.17}$$

where y_i's are the maximums of y_{ij}'s given by Eq.(3.16)

Then the estimate of β is given by

$$\hat{\beta} = \frac{m\beta}{m} \tag{3.18}$$

The estimates $\hat{\alpha}$ of α is given by Eq.(3.13) where x_i's are given by Eq.(3.15). Consequently estimates of reliability with m (known) simultaneous stresses and with a single stress is given respectively by Eq.(3.11) and Eq.(3.12), respectively.

Alternatively, we can also estimate R_A by R_P where

$$R_P = \text{Number of cases out of M where } [X \geq Y] / M. \tag{3.19}$$

As earlier the whole process is repeated k times. We have obtained reliabilities estimates given by Eq.(3.11), Eq.(3.12) and Eq.(3.19) for different values of λ, α, β, m, M and k and obtained their mean and standard deviation and tabulated them in the Table 4. We have plotted NPP graphs in each case and found that the distributions of $\hat{R}_A$ and R_P are normal. For illustration purpose we have given one such NPP graph in Fig.4 corresponding to $\lambda = 5$, m = 2, $\alpha = 2$, $\beta = 2$, M = 500, k = 200. As earlier we have used z-test to test the significance of difference between R_A, $\hat{R}_A$ and R_A, R_P. The z-values are also given in the same Table 4.

Table 4: Generalized exponential stress-strength

M	k	λ	m	α	β	R	R_A	Mean of $\hat{R}_A$	SD of $\hat{R}_A$	Mean of R_P	SD of R_P	z_1 for $\hat{R}_A$	z_2 for R_P
500	100	5	2	2	2	0.3333	0.5000	0.4984	0.0157	0.3293	0.0186	0.9873	2.1677
500	200	5	2	2	2	0.3333	0.5000	0.4988	0.0144	0.3311	0.0212	1.1489	1.4843
500	200	5	2	3	2	0.4286	0.6000	0.6005	0.0158	0.4288	0.0226	0.4612	0.1181
500	200	5	2	2	2	0.2500	0.5000	0.5015	0.0150	0.2512	0.0202	1.4096	0.8271
500	200	5	2	2	3	0.2500	0.4000	0.4004	0.0149	0.2495	0.0116	0.3625	0.3300

In Table 4, we note that z_1 is insignificant but z_2 is significant when M = 500, k = 100, $\lambda = 5$, m = 2, $\alpha = 2$ and $\beta = 2$. So for achieving better estimator R_P we have taken k =200 and found that all z-values are insignificant. For example, z_1 = 0.4612, z_2 =

0.1181 are smaller than $|z| < 1.96$ when k = 200, m = 2, $\lambda = 5$, $\alpha = 3$ and $\beta = 2$. So we have taken M = 500 and k = 200 throughout. In Table 4, we see that system reliability at accelerated stress R increases when α increases. For example, R = 0.3333 for $\alpha = 2$ but R = 0.4286 for $\alpha = 3$ when $\lambda = 5$ and $\beta = 2$. Similarly, R decreases when β decreases. For example, R = 0.3333 for $\beta = 2$ but R = 0.2500 for $\beta = 3$, $\lambda = 5$ and $\alpha = 2$.

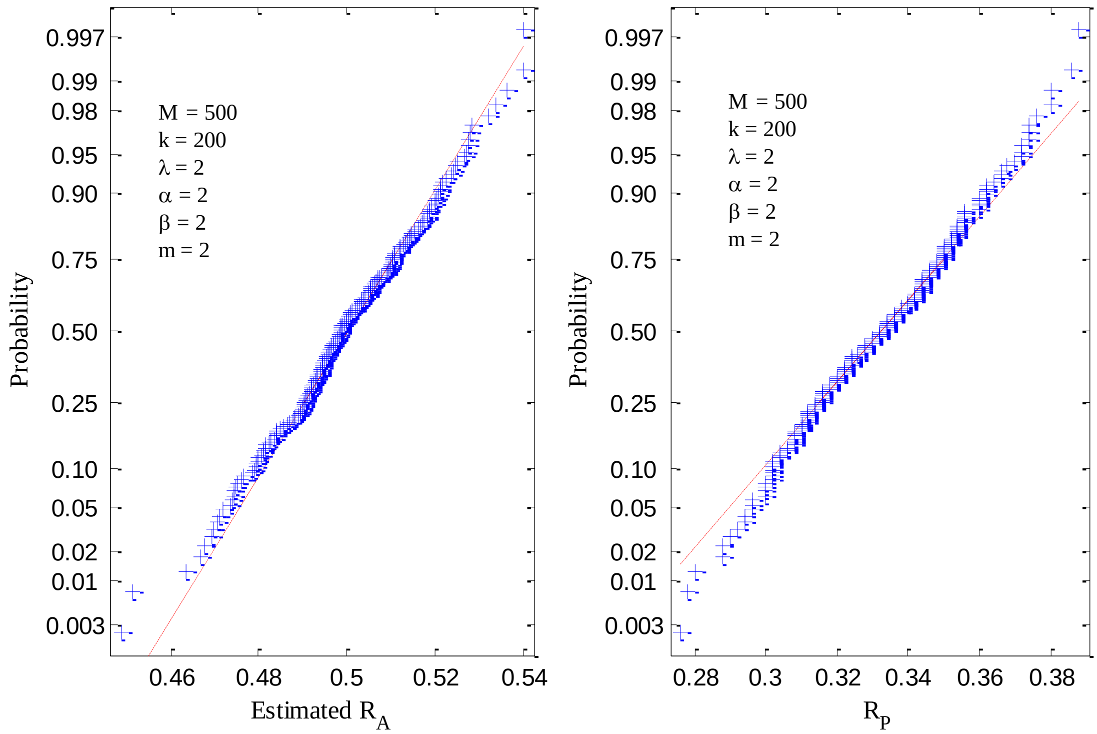

Figure 4: (Generalized exponential S-S, Sec.3.2)

4. CONCLUSION

We have studied here two ALT models and the numerical values of reliability, in each case, justify them as such. Here instead of a single stress a number of stresses are applied to get the accelerated effect. In the 1st model the effect of stresses is additive. In the second model we have assumed that maximum of the stresses cause the system to fail. We have considered exponential, gamma and normal distributions for the first model and generalized exponential for the second model as stress-strength distributions. Any other distribution may also be considered for both the models. For some systems even a minimum level of stress is required for the system to work, so minimum of the stress is applied may also be considered.

REFERENCES

[1] Barlow, R.E. (1982): ‘Accelerated Life Tests and Information’, Radiation Research, Vol. 90, pp. 90-97.

[2] Escober, L.A. and Meeker, W.Q. (2006): ‘A Review of Accelerated Test Models’, Statistical Science, Vol. 21, No. 4, pp. 552-577.

[3] Evans, R. A. (1977): ‘Accelerated Testing: The only Game in Town’, IEEE Transaction on Reliability, Vol.26, pp.241.

[4] Gupta, R.D. and Kundu, D. (1999): ‘Generalized Exponential Distributions’, Australian and New Zealand Journal of Statistics, Vol.41, pp.173-188.

[5] Kakaty, M. C. and Sriwastav, G. L., (1986): ‘Accelerated Life Testing under Stress-strength Model’, Indian Association of Productivity, Quality and Reliability Transaction, Vol.11, No.1-2, pp.51-56.

[6] Kapur, K. C. and Lamberson, L. R., (1977): ‘Reliability in Engineering Design’, John Wiley and Sons, New York.

[7] Kundu, D. and Gupta, R. D., (2005): ‘Estimation for Generalized Exponential Distribution’, METRIKA, Vol.61, pp.298-308.

[8] Mann, N. R., Schafer, R. E. and Singpurwalla, N. D., (1974): ‘Methods for Statistical Analysis of Reliability and Life Data’, John Wiley and Sons, New York.

[9] Pandit, S. N. N. and Sriwastav, G. L., (1975): ‘Studies in Cascade Reliability-I’, IEEE Transaction on Reliability, Vol.R-24, No.1, pp.53-56.