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Weak Forms of Fuzzy γ-Open Sets

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Abstract

By using an operation γ on topological space (X,τ), H. Ogata defined the concept of γ-open sets and investigated some properties of γ-open sets. In this paper, we introduce some generalizations of fuzzy pre γ-open sets and investigate some properties of the fuzzy sets. Moreover, we use them to obtain new separation axioms.

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1. INTRODUCTION

The usual notion of fuzzy set was introduced by Zadeh in his celebrated paper [10]. The idea of fuzzy topological spaces was introduced by Chang [4]. Different aspects of such spaces have been developed, by several investigators. This paper is also on development of the theory of fuzzy topological spaces. In 1979, S. Kasahara [6] defined the concept of an operation γ on a topological space and introduced the concept of γ–continuous mapping and γ-compact. Recently N. R. Das and B. Kalita [5] introduced the notion of fuzzy operation γ on fts. Also the authors introduced γ-open fuzzy set, γ-closed fuzzy set and γ-closure of a fuzzy set. In this paper we introduce and investigate the new notions called fuzzy $\alpha-\gamma-$open sets, fuzzy pre-$\gamma-$open sets, fuzzy $\beta-\gamma-$open sets and fuzzy $b-\gamma-$open sets which are weaker than fuzzy $\gamma-$open sets. Moreover, we use these notions to obtain new separation axioms.

2. PRELIMINARIES

For X, I^X denote the collection of all mappings from X into $I=[0,1]$. A member λ of I^X is called a fuzzy set of X. By (X,τ) or simply by X we denote a fuzzy topological space (fts) due to Chang. The interior, the closure and complement of a fuzzy set $\lambda\in I^X$ will be denoted by $\text{int}(\lambda), \text{cl}(\lambda)$ and λ^c respectively. By 0_X and 1_X we mean the constant fuzzy sets taking on the values 0 and 1 on X, respectively.

Definition 2.1.[9] A fuzzy set in X is called a fuzzy point if and only if it takes the value 0 for all $y\in X$ except one say $x\in X$. If its value at x is $\lambda(0<\lambda\le 1)$, we denote this fuzzy point p_x^λ where the point x is called its support.

Definition 2.2.[9] A fuzzy set λ is said to be contained in a fuzzy set μ if and only if $\lambda(x)\le\mu(x)$ for every $x\in X$.

Definition 2.3.[9] A fuzzy set λ is said to be quasi coincident with a fuzzy set μ, denoted by $\lambda q\mu$, if and only if there exists $x\in X$ such that $\lambda(x)\ge\mu^c(x)$, i.e., $\lambda(x)+\mu(x)>1$.

Definition 2.4.[9] A fuzzy set λ in fuzzy topological space (X,τ) is called a Q-neighborhood of p_x^λ if and only if there exists a $\mu\in\tau$ such that $p_x^\lambda q\mu\le\lambda$.

Definition 2.5.[9] A fuzzy set λ in (X,τ) is called a neighborhood of a fuzzy point p_x^λ if and only if and if there exists a $\mu\in\tau$ such that $p_x^\lambda\in\mu\subset\lambda$. A neighborhood λ is said to be fuzzy open if and only if λ is fuzzy open.

Definition 2.6.[9] The closure and interior of a fuzzy set λ of X are defined as $\overline{\lambda}$ or $cl(\lambda)=\inf\{\mu:\lambda\le\mu,\mu^c\in\tau\}$ and $\text{int}(\lambda)=\sup\{\mu:\mu\le\lambda,\mu\in\tau\}$.

Definition 2.7.[5] Let (X,τ) be a fuzzy topological space. A fuzzy operation γ on the topology τ is a mapping from τ into set I^X (family of all subsets of X) such that $V \subset \gamma(V)$ for each $V \in \tau$ where $\gamma(V)$ denotes the value of γ at *V*. The mapping defined by $\gamma(G)=G, \gamma(G)=cl(G), \gamma(G)=\text{int}(cl(G))$, etc are examples of fuzzy operations.

Definition 2.8.[5] A fuzzy subset λ of (X,τ) will be called a fuzzy $\gamma-$open it for each $p_x^\lambda q\lambda$, there exists a $V \in \tau$ and $p_x^\lambda qV$ such that $\gamma(V) \subset \lambda$. τ_γ denotes the set of all fuzzy $\gamma-$open sets. Clearly we have $\tau_\gamma \subset \tau$.

Definition 2.9. [5] For a fuzzy subset λ of (X,τ) and τ_γ, we defined τ_γ- $cl(\lambda)$ as follows

$$\tau_\gamma - cl(\lambda) = \inf\{\mu : \lambda \le \mu, \mu^c \in \tau_\gamma\}.$$

Definition 2.10. [5] For a fuzzy subset λ of (X,τ) and τ_γ, we defined τ_γ-$\text{int}(\lambda)$ as follows

$$\tau_\gamma - \text{int}(\lambda) = \sup\{\mu : \mu \le \lambda, \mu \in \tau_\gamma\}.$$

Definition 2.11. A fuzzy subset λ in a space *X* is said to be

(1) fuzzy α-open [3] if $\lambda \le \text{int}(cl(\text{int}(\lambda)))$.

(2) fuzzy semi-open[1] if $\lambda \le cl(\text{int}(\lambda))$.

(3) fuzzy preopen [3] if $\lambda \le \text{int}(cl(\lambda))$.

(4) fuzzy β-open [7] if $\lambda \le cl(\text{int}(cl(\lambda)))$.

(5) fuzzy b-open [2] if $\lambda \le \text{int}(cl(\lambda)) \vee cl(\text{int}(\lambda))$.

3. WEAK FORMS OF FUZZY γ -OPEN SETS

Definition 3.1. A fuzzy subset λ of (X,τ) is said to be:

(1) fuzzy $\alpha-\gamma-$open is $\lambda \le \tau_\gamma - \text{int}(cl(\tau_\gamma - \text{int}(\lambda)))$.

(2) fuzzy pre– $\gamma-$open is $\lambda \le \tau_\gamma - \text{int}(cl(\lambda))$.

(3) fuzzy $\beta-\gamma-$open is $\lambda \le cl(\tau_\gamma - \text{int}(cl(\lambda)))$.

(4) fuzzy $b-\gamma-$open is $\lambda \le \tau_\gamma - \mathrm{int}(cl(\lambda)) \vee cl(\tau_\gamma - \mathrm{int}(\lambda))$.

Lemma 3.1. Let (X, τ) be a fts, then the following properties hold:

(1) Every fuzzy $\gamma-$open set is fuzzy $\alpha-\gamma-$open.
(2) Every fuzzy $\alpha-\gamma-$open set is fuzzy pre $\gamma-$open.
(3) Every fuzzy pre–$\gamma-$open set is fuzzy b–$\gamma-$open.
(4) Every fuzzy $b-\gamma-$open set is fuzzy $\beta-\gamma-$open.

Proof.

1. Let λ be a fuzzy γ-open set. Then $\lambda = \tau_\gamma - \mathrm{int}(\lambda)$. Since $\lambda \le cl(\lambda)$, then $\lambda \le cl(\tau_\gamma - \mathrm{int}(\lambda))$ and $\lambda \le \tau_\gamma - \mathrm{int}(cl(\tau_\gamma - \mathrm{int}(\lambda)))$. Therefore λ is fuzzy $\alpha-\gamma-$open.

2. Let λ be a fuzzy $\alpha-\gamma-$open set. Then $\lambda \le \tau_\gamma - \mathrm{int}(cl(\tau_\gamma - \mathrm{int}(\lambda))) \le \tau_\gamma - \mathrm{int}(cl(\lambda))$. Therefore λ is fuzzy pre– $\gamma-$ open.

3. Let λ be a fuzzy pre$-\gamma-$open set. Then $\lambda \le \tau_\gamma - \mathrm{int}(cl(\lambda)) \le \tau_\gamma - \mathrm{int}(cl(\lambda)) \vee cl(\tau_\gamma - \mathrm{int}(\lambda))$. Therefore λ is fuzzy b– $\gamma-$open.

4. Let λ be a fuzzy b$-\gamma-$open. Then $\lambda \le \tau_\gamma - \mathrm{int}(cl(\lambda)) \vee cl(\tau_\gamma - \mathrm{int}(\lambda)) \le cl(\tau_\gamma - \mathrm{int}(cl(\lambda))) \vee cl(\tau_\gamma - \mathrm{int}(\lambda)) \le cl(\tau_\gamma - \mathrm{int}(cl(\lambda)))$ Therefore λ is fuzzy $\beta-\gamma-$open.

Note that every fuzzy $\gamma-$open set is fuzzy open. The converses need not be true as shown by the following examples.

Examples 3.1. Let $X = \{a, b\}$ and $A, B, C \in I^X$ defined by $A = \underline{0.6}$, $B = \underline{0.7}$, $C = \underline{0.9}$ where $\underline{\alpha}$ denotes the constant mapping with value α. Let $\tau = \{X, \emptyset, A, B, C\}$. Then

(X,τ) is a fts. Define $\gamma:\tau\to I^X$ by
$\gamma(X)=X,\gamma(\emptyset)=\emptyset,\gamma(A)=cl(A),\gamma(B)=B,\gamma(C)=cl(C)$. Then

i) The fuzzy set C is fuzzy $\alpha-\gamma-$open but not fuzzy $\gamma-$open set.
ii) The fuzzy set A is fuzzy pre–$\gamma-$open but not fuzzy $\alpha-\gamma-$open set.

Example 3.2. Let $X=\{a,b\}$ and $A,B,C,D,E\in I^X$ defined by $A=\underline{0.15},\ B=\underline{0.2},\ C=\underline{0.9},\ D=\underline{0.6},\ E=\underline{0.7}$ where $\underline{\alpha}$ denotes the constant mapping with value α. Let $\tau=\{X,\emptyset,A,B,C,D,E\}$. Then (X,τ) is a fts. Define $\gamma:\tau\to I^X$ by
$\gamma(X)=X,\ \gamma(\emptyset)=\emptyset,\ \gamma(A)=cl(A),\ \gamma(B)=cl(B),$
$\gamma(C)=C,\ \gamma(D)=cl(D),\ \gamma(E)=E$. Then the fuzzy set B is fuzzy b–$\gamma-$open but not fuzzy pre–$\gamma-$open set.

Lemma 3.2. If μ is a fuzzy open set, then $cl(\mu\wedge\lambda)=cl(\mu\wedge cl(\lambda))$ and hence $\mu\wedge cl(\lambda)\le cl(\mu\wedge\lambda)$ for any subset λ of a fts X.

Theorem 3.1. If λ is a fuzzy pre–$\gamma-$open subset in fts (X,τ) such that $\mu\le\lambda\le cl(\mu)$ for a fuzzy subset μ of X, then μ is a fuzzy pre–$\gamma-$open set.

Proof. Since $\lambda\le\tau_\gamma-\text{int}(cl(\lambda)),\mu\le\tau_\gamma-\text{int}(cl(\lambda))$. Also, $cl(\lambda)\le cl(\mu)$ implies that $\tau_\gamma-\text{int}(cl(\lambda))\le\tau_\gamma-\text{int}(cl(\mu))$. Thus $\mu\le\tau_\gamma-\text{int}(cl(\lambda))\le\tau_\gamma-\text{int}(cl(\mu))$ and hence μ is a fuzzy pre–$\gamma-$open set.

Theorem 3.2. A fuzzy set λ in a fts(X,τ) is fuzzy semi–open if λ is fuzzy $\beta-\gamma-$open and $\tau_\gamma-\text{int}(cl(\lambda))\le cl(\text{int}(\lambda))$

Proof. Let λ be a fuzzy $\beta-\gamma-$open and $\tau_\gamma-\text{int}(cl(\lambda))\le cl(\text{int}(\lambda))$. Then $\lambda\le cl(\tau_\gamma-\text{int}(cl(\lambda)))\le cl(cl(\text{int}(\lambda)))=cl(\text{int}(\lambda))$. And hence λ is fuzzy semi-open.

Proposition 3.1. The intersection of a fuzzy pre–$\gamma-$open set and a fuzzy open set is fuzzy pre-open.

Proof. Let λ be a fuzzy pre– $\gamma-$open set and μ be a fuzzy open set in *X*. Then $\lambda \leq \tau_\gamma - \text{int}(cl(\lambda))$ and $\text{int}(\mu) = \mu,$ by Lemma 3.2., we have $\mu \wedge \lambda \leq \text{int}(\mu) \wedge \tau_\gamma - \text{int}(cl(\lambda)) \leq \text{int}(\mu) \wedge \text{int}(cl(\lambda)) = \text{int}(\mu \wedge cl(\lambda)) \leq \text{int}(cl(\mu \wedge \lambda))$. Therefore, $\lambda \wedge \mu$ is fuzzy pre-open.

Proposition 3.2. The intersection of a fuzzy $\beta-\gamma-$ open set and a fuzzy open set is fuzzy $\beta-$ open.

Proof. Let μ be a fuzzy open set and λ be a fuzzy $\beta-\gamma-$open set. Since every fuzzy $\gamma-$open set is fuzzy open, by Lemma 3.2., we have

$$\begin{aligned}
\mu \wedge \lambda &\leq \mu \wedge cl(\tau_\gamma - \text{int}(cl(\lambda))) \\
&\leq \mu \wedge cl(\text{int}(cl(\lambda))) \\
&\leq cl(\mu \wedge \text{int}(cl(\lambda))) \\
&= cl(\text{int}(\mu) \wedge \text{int}(cl(\lambda))) \\
&= cl(\text{int}(\mu \wedge cl(\lambda))) \\
&\leq cl(\text{int}(cl(\mu \wedge \lambda)))
\end{aligned}$$

This shows that $\mu \wedge \lambda$ is fuzzy $\beta-$open.

Proposition 3.3. The intersection of a fuzzy $b-\gamma-$open set and a fuzzy open set is fuzzy $b-$open.

Proof. Let λ be fuzzy $b-\gamma-$open and μ be fuzzy open, then $\lambda \leq \tau_\gamma - \text{int}(cl(\lambda)) \vee cl(\tau_\gamma - \text{int}(\lambda))$ and $\mu = \text{int}(\mu)$. Then we have

$$\begin{aligned}
\mu \wedge \lambda &\leq \mu \wedge \left[\tau_\gamma - \text{int}(cl(\lambda)) \vee cl(\tau_\gamma - \text{int}(\lambda))\right] \\
&= \left[\mu \wedge \tau_\gamma - \text{int}(cl(\lambda))\right] \vee \left[\mu \wedge cl(\tau_\gamma - \text{int}(\lambda))\right]
\end{aligned}$$

$$=\left[\operatorname{int}(\mu)\wedge\tau_{\gamma}-\operatorname{int}\left(cl(\lambda)\right)\right]\vee\left[\mu\wedge cl\left(\tau_{\gamma}-\operatorname{int}(\lambda)\right)\right]$$

$$\leq\left[\operatorname{int}(\mu)\wedge\operatorname{int}\left(cl(\lambda)\right)\right]\vee\left[\mu\wedge cl\left(\operatorname{int}(\lambda)\right)\right]$$

$$\leq\left[\operatorname{int}\left(\mu\wedge cl(\lambda)\right)\right]\vee\left[cl\left(\mu\wedge\operatorname{int}(\lambda)\right)\right]$$

$$\leq\left[\operatorname{int}\left(cl(\mu\wedge\lambda)\right)\right]\vee\left[cl\left(\operatorname{int}(\mu\wedge\lambda)\right)\right]$$

This shows that $\mu\wedge\lambda$ is fuzzy b–open.

Proposition 3.4. The intersection of a fuzzy $\alpha-\gamma-$open set and a fuzzy open set is fuzzy $\alpha-$open.

Proof. Proof is similar to the above propositions 3.2 and 3.3.

Theorem 3.3. If $\{\lambda_k : k\in\Delta\}$ is a collection of fuzzy $b-\gamma-$open (resp. fuzzy $\alpha-\gamma-$open, fuzzy pre– $\gamma-$open, fuzzy $\beta-\gamma-$open) sets of a fts(X,τ), then $\bigcup_{k\in\Delta}\lambda_k$ is fuzzy $b-\gamma-$open (resp.fuzzy $\alpha-\gamma-$open, fuzzy pre– $\gamma-$open, fuzzy $\beta-\gamma-$open).

Proof. We prove only the first case since the other cases are similarly shown. Since $\lambda\leq\tau_{\gamma}-\operatorname{int}\left(cl(\lambda_k)\right)\vee cl\left(\tau_{\gamma}-\operatorname{int}(\lambda_k)\right)$ for every $k\in\Delta$, we have

$$\bigcup_{k\in\Delta}\lambda_k\leq\bigcup_{k\in\Delta}\left[\tau_{\gamma}-\operatorname{int}\left(cl(\lambda_k)\right)\vee cl\left(\tau_{\gamma}-\operatorname{int}(\lambda_k)\right)\right]$$

$$\leq\left[\bigcup_{k\in\Delta}\tau_{\gamma}-\operatorname{int}\left(cl(\lambda_k)\right)\right]\vee\left[\bigcup_{k\in\Delta}cl\left(\tau_{\gamma}-\operatorname{int}(\lambda_k)\right)\right]$$

$$\leq\left[\tau_{\gamma}-\operatorname{int}\left(\bigcup_{k\in\Delta}cl(\lambda_k)\right)\right]\vee\left[cl\left(\bigcup_{k\in\Delta}\tau_{\gamma}-\operatorname{int}(\lambda_k)\right)\right]$$

$$\leq\left[\tau_{\gamma}-\operatorname{int}\left(cl\left(\bigcup_{k\in\Delta}\lambda_k\right)\right)\right]\vee\left[cl\left(\tau_{\gamma}-\operatorname{int}\left(\bigcup_{k\in\Delta}\lambda_k\right)\right)\right]$$

Therefore, $\bigcup_{k\in\Delta}\lambda_k$ is fuzzy $b-\gamma-$open.

We note that the intersection of two fuzzy pre– $\gamma-$open (resp. fuzzy $b-\gamma-$open, fuzzy $\alpha-\gamma-$open, fuzzy $\beta-\gamma-$open) sets need not be fuzzy pre– $\gamma-$open (resp. fuzzy $b-\gamma-$open, fuzzy $\alpha-\gamma-$open, fuzzy $\beta-\gamma-$open) as can be seen from the following examples.

Example 3.3 Let $X=\{a,b,c\}$ and $A,B,C\in I^X$ defined by

$A(a)=0.3, A(b)=0.5, A(c)=0.5;\; B(a)=0.4, B(b)=0.2, B(c)=0.6$;

$C(a)=0.4, C(b)=0.5, C(c)=0.6$.

Let $\tau=\{X,\emptyset,A,B,C,A\wedge B\}$. Then (X,τ) is a fts. Define $\gamma:\tau\to I^X$ by

$\gamma(X)=X, \gamma(\emptyset)=\emptyset,\; \gamma(A)=A, \gamma(B)=B, \gamma(C)=C, \gamma(A\wedge B)=Cl(A\wedge B)$. Then the fuzzy set *A* and *B* are fuzzy $\alpha-\gamma-$open but $A\wedge B$ is not fuzzy $\alpha-\gamma-$open set.

Example 3.4. Let $X=\{a,b,c\}$ and $A,B,C,D,E,F\in I^X$ defined by

$A(a)=0.3, A(b)=0.4,\; A(c)=0.5; B(a)=0.6, B(b)=0.5, B(c)=0.6;$

$C(a)=0.6, C(b)=0.5, C(c)=0.5;\; D(a)=0.6, D(b)=0.5, D(c)=0.4;$

$E(a)=0.3, E(b)=0.4, E(c)=0.4; F=\underline{0.3}$ where $\underline{\alpha}$ denotes the constant mapping with value α. Let $\tau=\{X,\emptyset,A,B,C,D,E,F\}$. Then (X,τ) is a fts. Define

$\gamma:\tau\to I^X$ by $\gamma(X)=X, \gamma(\emptyset)=\emptyset, \gamma(A)=A, \gamma(B)=B, \gamma(C)=C, \gamma(D)=D,$

$\gamma(E)=cl(E), \gamma(F)=F$. Then the fuzzy set *A* and *D* are fuzzy pre– $\gamma-$open sets but $A\wedge D$ is not fuzzy pre– $\gamma-$open set.

Example 3.5 In Example 3.4 Define $\gamma:\tau\to I^X$ by

$\gamma(X)=X, \gamma(\emptyset)=\emptyset, \gamma(A)=A, \gamma(B)=B,$

$\gamma(C)=C, \gamma(D)=D, \gamma(E)=cl(E), \gamma(F)=cl(F)$. Then the fuzzy set *A* and *D* are fuzzy $\beta-\gamma-$open (resp. fuzzy $b-\gamma-$open) sets but $A\wedge D$ is not fuzzy $\beta-\gamma-$open (resp. fuzzy $b-\gamma-$open) set.

Proposition 3.5. Let λ be fuzzy $b-\gamma-$open set such that $\tau_\gamma-\text{int}(\lambda)=\emptyset$. Then λ is fuzzy pre– $\gamma-$open.

Definition 3.2 A fts (X,τ) is called fuzzy door space if every fuzzy subset of *X* is fuzzy open or fuzzy closed.

Proposition 3.6. If (X,τ) is a fuzzy door space and fuzzy $\gamma-$regular, then every fuzzy pre– $\gamma-$open set is fuzzy $\gamma-$open.

Proof. Let λ be a fuzzy pre– $\gamma-$open set. If λ is fuzzy open, then λ is fuzzy $\gamma-$open. Otherwise, λ is fuzzy closed and hence $\lambda \le \tau_\gamma - \text{int}(cl(\lambda)) = \tau_\gamma - \text{int}(\lambda) \le \lambda.$ Therefore , $\lambda = \tau_\gamma - \text{int}(\lambda)$ and thus λ is a fuzzy γ–open set.

4. NEW FUZZY SEPARATION AXIOMS

Definition 4.1. Let (X,τ) be a fts and $A \in I^X$. Then *A* is called a pre–*Q*–neighborhood of $p_x^\lambda \in S(X)$ if and only if there exists a fuzzy pre– $\gamma-$open set *U* such that $p_x^\lambda qU$ and $U \subseteq A$. The class of all open *Q*-neighborhoods of p_x^λ is denoted by $N^{pre-Q}(p_x^\lambda)$.

Definition 4.2. A fts (X,τ) is called:

(1) fuzzy pre– $\gamma - T_1$ (resp. fuzzy $\alpha-\gamma-T_1$, fuzzy $b-\gamma-T_1$, fuzzy $\beta-\gamma-T_1$) iff for any $p_x^\lambda, p_y^k \in S(X)$ and $p_x^\lambda \ne p_y^k$, there exists a fuzzy pre– $\gamma-$open (resp. fuzzy $\alpha-\gamma-$open, fuzzy $b-\gamma-$open, fuzzy $\beta-\gamma-$open) *Q*–neighborhoods *U* and *V* of p_x^λ and p_y^k respectively such that $p_y^k \bar{q} \gamma(U)$ and $p_x^\lambda \bar{q} \gamma(V)$.

(2) fuzzy pre– $\gamma - T_2$ (resp. fuzzy $\alpha-\gamma-T_2$, fuzzy $b-\gamma-T_2$, fuzzy $\beta-\gamma-T_2$) iff for any $p_x^\lambda, p_y^k \in S(X)$ and $p_x^\lambda \ne p_y^k$, there exists a fuzzy pre– $\gamma-$open (resp. fuzzy $\alpha-\gamma-$open, fuzzy $b-\gamma-$open, fuzzy $\beta-\gamma-$open) *Q*-neighborhoods *U* and *V* of p_x^λ and p_y^k respectively such that $\gamma(U)\bar{q}\gamma(V)$.

Theorem 4.1. If a space (X,τ) is fuzzy pre– $\gamma - T_2$ (resp. $\alpha-\gamma-T_2, b-\gamma-T_2, \beta-\gamma-T_2$), then it is fuzzy pre– $\gamma - T_1$ (resp. $\alpha-\gamma-T_1, b-\gamma-T_1, \beta-\gamma-T_1$).

Proof. Let (X,τ) be a fuzzy pre– $\gamma - T_2$ (resp. $\alpha-\gamma-T_2, b-\gamma-T_2, \beta-\gamma-T_2$). Let $p_x^\lambda, p_y^k \in S(X)$ and $p_x^\lambda \ne p_y^k$, then there exists a fuzzy pre– $\gamma-$open (resp. $\alpha-\gamma-$open, $b-\gamma-$open, $\beta-\gamma-$open) *Q*-neighborhoods *U* and *V* of p_x^λ and p_y^k respectively such that $\gamma(U)\bar{q}\gamma(V)$. Since $p_x^\lambda q\gamma(U)$ and $p_y^k q\gamma(V)$, therefore,

$p_x^\lambda \overline{q} \gamma(V)$ and $p_y^k \overline{q} \gamma(U)$. Hence (X,τ) is fuzzy pre– $\gamma - T_1$(resp. $\alpha - \gamma - T_1, b - \gamma - T_1, \beta - \gamma - T_1$).

Theorem 4.2. A space (X,τ) be a fuzzy pre– $\gamma - T_1$ if and only if any fuzzy singleton in *X* is a fuzzy pre– $\gamma -$closed set.

Proof. Obvious.

Theorem 4.3. Suppose $\gamma : \tau \to I^X$ is regular operation. If (X,τ_γ) is a fuzzy T_2-space then (X,τ) is a fuzzy pre– $\gamma - T_2$.

Proof. Let $p_x^\lambda, p_y^k \in S(X)$ and $p_x^\lambda \neq p_y^k$. Since (X,τ_γ) is a fuzzy T_2-space, then there exists fuzzy pre– $\gamma -$open *Q*-neightborhoods $U,V(\in \tau_\gamma \subset \tau)$ of p_x^λ and p_y^k respectively such that $\gamma(U)\overline{q}\gamma(V)$. Thus, (X,τ) is a fuzzy pre–$\gamma - T_2$.

Definition 4.3. Let (X,τ) be a fts and γ an operation on τ. A fuzzy set $\lambda \in I^X$ is called fuzzy pre–γ–generalied closed (fuzzy pre–γ–*g*–closed, for short) if $cl_\gamma(\lambda) \leq U$ whenever $\lambda \leq U$ and *U* is fuzzy pre–$\gamma -$open in (X,τ).

Theorem 4.4. Every fuzzy pre–$\gamma -$closed set is fuzzy pre–$\gamma - g$–closed.

Proof. Obvious.

The converse is not true as shown by the following example.

Example 4.1. In Example 3.4 The fuzzy set *B* is fuzzy pre–$\gamma - g -$closed set but not pre–$\gamma -$closed.

Definition 4.4. A space (X,τ) is called a fuzzy pre–$\gamma - T_{1/2}$ space if every fuzzy pre–$\gamma - g -$closed set of (X,τ) is fuzzy pre–$\tau -$closed.

Theorem 4.5. For each $p_x^\lambda \in S(X)$ and p_x^λ is fuzzy pre–$\gamma -$closed or $(p_x^\lambda)^c$ is fuzzy pre–$\gamma - g -$closed set in (X,τ).

Proof. Suppose p_x^λ is not fuzzy pre–$\gamma-$closed. Then $\left(p_x^\lambda\right)^c$ is fuzzy pre–$\gamma-$open. Let U be any fuzzy pre–$\gamma-$open set such that $\left(p_x^\lambda\right)^c \leq U$. Since $U = X$ is the only fuzzy pre–$\gamma-$open, $cl_\gamma\left(\left(p_x^\lambda\right)^c\right) \leq U$. Therefore, $\left(p_x^\lambda\right)^c$ is the fuzzy pre–$\gamma - g -$closed set.

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