Weak Forms of Fuzzy $\gamma$-Open Sets

C. Sivashanmugaraja
Department of Mathematics, Periyar Arts College,
Cuddalore-607 001, Tamil Nadu, India.

A. Vadivel
Department of Mathematics, Annamalai University,
Annamalai Nagar-608 002, Tamil Nadu, India.

Abstract

By using an operation $\gamma$ on topological space $(X, \tau)$, H. Ogata defined the concept of $\gamma$-open sets and investigated some properties of $\gamma$-open sets. In this paper, we introduce some generalizations of fuzzy pre $\gamma$-open sets and investigate some properties of the fuzzy sets. Moreover, we use them to obtain new separation axioms.

**Keywords and phrases:** Fuzzy $\gamma$-open, fuzzy $\alpha-\gamma$-open, fuzzy pre $-\gamma$-open, fuzzy $\beta-\gamma$-open, fuzzy $b-\gamma$-open.

**AMS (2000) subject classification:** 54A40

1. INTRODUCTION

The usual notion of fuzzy set was introduced by Zadeh in his celebrated paper [10]. The idea of fuzzy topological spaces was introduced by Chang [4]. Different aspects of such spaces have been developed, by several investigators. This paper is also on development of the theory of fuzzy topological spaces. In 1979, S. Kasahara [6] defined the concept of an operation $\gamma$ on a topological space and introduced the concept of $\gamma$–continuous mapping and $\gamma$-compact. Recently N. R. Das and B. Kalita [5] introduced the notion of fuzzy operation $\gamma$ on fts. Also the authors introduced $\gamma$-open fuzzy set, $\gamma$-closed fuzzy set and $\gamma$-closure of a fuzzy set. In this paper we
introduce and investigate the new notions called fuzzy \( \alpha - \gamma \)–open sets, fuzzy \( \beta - \gamma \)–open sets, fuzzy \( \alpha - \beta \)–open sets and fuzzy \( b - \gamma \)–open sets which are weaker than fuzzy \( \gamma \)–open sets. Moreover, we use these notions to obtain new separation axioms.

2. PRELIMINARIES

For \( X, \mathcal{I} \) denote the collection of all mappings from \( X \) into \( I = [0,1] \). A member \( \lambda \) of \( \mathcal{I} \) is called a fuzzy set of \( X \). By \( (X, \tau) \) or simply by \( X \) we denote a fuzzy topological space (fts) due to Chang. The interior, the closure and complement of a fuzzy set \( \lambda \in \mathcal{I} \) will be denoted by \( \text{int}(\lambda), \text{cl}(\lambda) \) and \( \lambda^c \) respectively. By \( 0_x \) and \( 1_x \) we mean the constant fuzzy sets taking on the values 0 and 1 on \( X \), respectively.

**Definition 2.1.**[9] A fuzzy set in \( X \) is called a fuzzy point if and only if it takes the value 0 for all \( y \in X \) except one say \( x \in X \). If its value at \( x \) is \( \lambda(0 < \lambda \leq 1) \), we denote this fuzzy point \( p^x_\lambda \) where the point \( x \) is called its support.

**Definition 2.2.**[9] A fuzzy set \( \lambda \) is said to be contained in a fuzzy set \( \mu \) if and only if \( \lambda(x) \leq \mu(x) \) for every \( x \in X \).

**Definition 2.3.**[9] A fuzzy set \( \lambda \) is said to be quasi coincident with a fuzzy set \( \mu \), denoted by \( \lambda q \mu \), if and only if there exists \( x \in X \) such that \( \lambda(x) \geq \mu^c(x) \), i.e., \( \lambda(x) + \mu(x) > 1 \).

**Definition 2.4.**[9] A fuzzy set \( \lambda \) in fuzzy topological space \( (X, \tau) \) is called a \( Q \)-neighborhood of \( p^x_\lambda \) if and only if there exists a \( \mu \in \tau \) such that \( p^x_\lambda q \mu \leq \lambda \).

**Definition 2.5.**[9] A fuzzy set \( \lambda \) in \( (X, \tau) \) is called a neighborhood of a fuzzy point \( p^x_\lambda \) if and only if and if there exists a \( \mu \in \tau \) such that \( p^x_\lambda \subset \mu \). A neighborhood \( \lambda \) is said to be fuzzy open if and only if \( \lambda \) is fuzzy open.

**Definition 2.6.**[9] The closure and interior of a fuzzy set \( \lambda \) of \( X \) are defined as \( \overline{\lambda} \) or \( \text{cl}(\lambda) = \inf \{ \mu : \lambda \leq \mu, \mu^c \in \tau \} \) and \( \text{int}(\lambda) = \sup \{ \mu : \mu \leq \lambda, \mu \in \tau \} \).
Definition 2.7.[5] Let \((X, \tau)\) be a fuzzy topological space. A fuzzy operation \(\gamma\) on the topology \(\tau\) is a mapping from \(\tau\) into set \(I^X\) (family of all subsets of \(X\)) such that \(V \subseteq \gamma(V)\) for each \(V \in \tau\) where \(\gamma(V)\) denotes the value of \(\gamma\) at \(V\). The mapping defined by \(\gamma(G) = G, \gamma(G) = cl(G), \gamma(G) = int(cl(G))\), etc are examples of fuzzy operations.

Definition 2.8.[5] A fuzzy subset \(\lambda\) of \((X, \tau)\) will be called a fuzzy \(\gamma\)-open if for each \(p, q, \lambda\), there exists a \(V \in \tau\) and \(p, q, V\) such that \(\gamma(V) \subseteq \lambda\). \(\tau_\gamma\) denotes the set of all fuzzy \(\gamma\)-open sets. Clearly we have \(\tau_\gamma \subseteq \tau\).

Definition 2.9. [5] For a fuzzy subset \(\lambda\) of \((X, \tau)\) and \(\tau_\gamma\), we defined \(\tau_\gamma - cl(\lambda)\) as follows
\[
\tau_\gamma - cl(\lambda) = \inf \{\mu : \lambda \leq \mu, \mu \in \tau_\gamma\}.
\]

Definition 2.10. [5] For a fuzzy subset \(\lambda\) of \((X, \tau)\) and \(\tau_\gamma\), we defined \(\tau_\gamma - int(\lambda)\) as follows
\[
\tau_\gamma - int(\lambda) = \sup \{\mu : \mu \leq \lambda, \mu \in \tau_\gamma\}.
\]

Definition 2.11. A fuzzy subset \(\lambda\) in a space \(X\) is said to be

1. fuzzy \(\alpha\)-open [3] if \(\lambda \leq int(cl(int(\lambda)))\).
2. fuzzy semi-open[1] if \(\lambda \leq cl(int(\lambda))\).
3. fuzzy preopen [3] if \(\lambda \leq int(cl(\lambda))\).
4. fuzzy \(\beta\)-open [7] if \(\lambda \leq cl(int(cl(\lambda)))\).
5. fuzzy b-open [2] if \(\lambda \leq int(cl(\lambda)) \lor cl(int(\lambda))\).

3. WEAK FORMS OF FUZZY \(\gamma\)-OPEN SETS

Definition 3.1. A fuzzy subset \(\lambda\) of \((X, \tau)\) is said to be:

1. fuzzy \(\alpha-\gamma\)-open is \(\lambda \leq \tau_\gamma - int(cl(\tau_\gamma - int(\lambda)))\).
2. fuzzy pre- \(\gamma\)-open is \(\lambda \leq \tau_\gamma - int(cl(\lambda))\).
3. fuzzy \(\beta-\gamma\)-open is \(\lambda \leq cl(\tau_\gamma - int(cl(\lambda)))\).
Lemma 3.1. Let \((X, \tau)\) be a fts, then the following properties hold:

1. Every fuzzy \(\gamma\)–open set is fuzzy \(\alpha\)–open.
2. Every fuzzy \(\alpha\)–open set is fuzzy \(\alpha\)–open.
3. Every fuzzy pre-\(\gamma\)–open set is fuzzy \(\gamma\)–open.
4. Every fuzzy \(b\)–\(\gamma\)–open set is fuzzy \(\beta\)–\(\gamma\)–open.

Proof.

1. Let \(\lambda\) be a fuzzy \(\gamma\)–open set. Then \(\lambda = \tau_\gamma - \text{int}(\lambda)\). Since \(\lambda \leq \text{cl}(\lambda)\), then \(\lambda \leq \text{cl}(\tau_\gamma - \text{int}(\lambda))\) and \(\lambda \leq \tau_\gamma - \text{int}(\text{cl}(\tau_\gamma - \text{int}(\lambda)))\). Therefore \(\lambda\) is fuzzy \(\alpha\)–\(\gamma\)–open.

2. Let \(\lambda\) be a fuzzy \(\alpha\)–\(\gamma\)–open set. Then \(\lambda \leq \tau_\gamma - \text{int}(\text{cl}(\tau_\gamma - \text{int}(\lambda))) \leq \tau_\gamma - \text{int}(\text{cl}(\lambda))\). Therefore \(\lambda\) is fuzzy pre-\(\gamma\)–open.

3. Let \(\lambda\) be a fuzzy pre-\(\gamma\)–open set. Then \(\lambda \leq \tau_\gamma - \text{int}(\text{cl}(\lambda)) \leq \tau_\gamma - \text{int}(\text{cl}(\tau_\gamma - \text{int}(\lambda))) \tau_\gamma - \text{int}(\text{cl}(\lambda))\). Therefore \(\lambda\) is fuzzy \(b\)–\(\gamma\)–open.

4. Let \(\lambda\) be a fuzzy \(b\)–\(\gamma\)–open. Then \(\lambda \leq \tau_\gamma - \text{int}(\text{cl}(\lambda)) \leq \tau_\gamma - \text{int}(\text{cl}(\text{cl}(\lambda))) \leq \tau_\gamma - \text{int}(\text{cl}(\lambda))\). Therefore \(\lambda\) is fuzzy \(\beta\)–\(\gamma\)–open.

Note that every fuzzy \(\gamma\)–open set is fuzzy open. The converses need not be true as shown by the following examples.

Examples 3.1. Let \(X = \{a, b\}\) and \(A, B, C \subseteq I^X\) defined by \(A = 0.6, B = 0.7, C = 0.9\) where \(\alpha\) denotes the constant mapping with value \(\alpha\). Let \(\tau = \{X, \emptyset, A, B, C\}\). Then
(X, τ) is a fts. Define ρ : τ → I^X by
\[ \gamma(X) = X, \gamma(\emptyset) = \emptyset, \gamma(A) = cl(A), \gamma(B) = B, \gamma(C) = cl(C). \] Then

i) The fuzzy set C is fuzzy α − ρ − open but not fuzzy ρ − open set.

ii) The fuzzy set A is fuzzy pre−ρ − open but not fuzzy α−ρ − open set.

Example 3.2. Let \( X = \{a, b\} \) and \( A, B, C, D, E \in I^X \) defined by
\[ A = 0.15, B = 0.2, C = 0.9, D = 0.6, E = 0.7 \] where \( \alpha \) denotes the constant mapping with value \( \alpha \). Let \( \tau = \{X, \emptyset, A, B, C, D, E\} \). Then \((X, \tau)\) is a fts. Define \( \gamma : \tau \rightarrow I^X \) by
\[ \gamma(X) = X, \gamma(\emptyset) = \emptyset, \gamma(A) = cl(A), \gamma(B) = B, \gamma(C) = cl(C), \gamma(D) = cl(D), \gamma(E) = E. \] Then the fuzzy set B is fuzzy b−ρ − open but not fuzzy pre−ρ − open set.

Lemma 3.2. If \( \mu \) is a fuzzy open set, then \( cl(\mu \land \lambda) = cl(\mu \land cl(\lambda)) \) and hence \( \mu \land cl(\lambda) \leq cl(\mu \land \lambda) \) for any subset \( \lambda \) of a fts \( X \).

Theorem 3.1. If \( \lambda \) is a fuzzy pre−ρ − open subset in fts \((X, \tau)\) such that \( \mu \leq \lambda \leq cl(\mu) \) for a fuzzy subset \( \mu \) of \( X \), then \( \mu \) is a fuzzy pre−ρ − open set.

Proof. Since \( \lambda \leq \tau_\gamma - int(cl(\lambda)), \mu \leq \tau_\gamma - int(cl(\lambda)) \). Also, \( cl(\lambda) \leq cl(\mu) \) implies that \( \tau_\gamma - int(cl(\lambda)) \leq \tau_\gamma - int(cl(\mu)) \). Thus \( \mu \leq \tau_\gamma - int(cl(\lambda)) \leq \tau_\gamma - int(cl(\mu)) \) and hence \( \mu \) is a fuzzy pre−ρ − open set.

Theorem 3.2. A fuzzy set \( \lambda \) in a fts \((X, \tau)\) is fuzzy semi−open if \( \lambda \) is fuzzy β − ρ − open and \( \tau_\gamma - int(cl(\lambda)) \leq cl(int(\lambda)) \).

Proof. Let \( \lambda \) be a fuzzy β − ρ − open and \( \tau_\gamma - int(cl(\lambda)) \leq cl(int(\lambda)) \). Then \( \lambda \leq cl(\tau_\gamma - int(cl(\lambda))) \leq cl(cl(int(\lambda))) = cl(int(\lambda)) \). And hence \( \lambda \) is fuzzy semi-open.

Proposition 3.1. The intersection of a fuzzy pre−ρ − open set and a fuzzy open set is fuzzy pre-open.
Proof. Let $\lambda$ be a fuzzy pre-$\gamma$-open set and $\mu$ be a fuzzy open set in $X$. Then

\[ \lambda \leq \tau_\gamma - \operatorname{int}(\operatorname{cl}(\lambda)) \] and $\operatorname{int}(\mu) = \mu$, by Lemma 3.2., we have

\[ \mu \land \lambda \leq \mu \land \tau_\gamma - \operatorname{int}(\operatorname{cl}(\lambda)) \leq \operatorname{int}(\mu) \land \operatorname{int}(\operatorname{cl}(\lambda)) = \operatorname{int}(\mu \land \operatorname{cl}(\lambda)) \leq \operatorname{int}(\mu \land \lambda) \]

Therefore, $\lambda \land \mu$ is fuzzy pre-open.

Proposition 3.2. The intersection of a fuzzy $\beta-\gamma$-open set and a fuzzy open set is fuzzy $\beta$-open.

Proof. Let $\mu$ be a fuzzy open set and $\lambda$ be a fuzzy $\beta-\gamma$-open set. Since every fuzzy $\gamma$-open set is fuzzy open, by Lemma 3.2., we have

\[ \mu \land \lambda \leq \mu \land \tau_\gamma - \operatorname{int}(\operatorname{cl}(\lambda)) \]

\[ \leq \mu \land \operatorname{cl}(\mu \land \operatorname{int}(\operatorname{cl}(\lambda))) \]

\[ \leq \operatorname{cl}(\mu \land \operatorname{int}(\operatorname{cl}(\lambda))) \]

\[ = \operatorname{cl}(\mu \land \mu \land \operatorname{int}(\operatorname{cl}(\lambda))) \]

\[ = \operatorname{cl}(\mu \land \operatorname{int}(\mu \land \operatorname{cl}(\lambda))) \]

\[ \leq \operatorname{cl}(\mu \land \operatorname{int}(\mu \land \lambda)) \]

This shows that $\mu \land \lambda$ is fuzzy $\beta$-open.

Proposition 3.3. The intersection of a fuzzy $b-\gamma$-open set and a fuzzy open set is fuzzy $b$-open.

Proof. Let $\lambda$ be fuzzy $b-\gamma$-open and $\mu$ be fuzzy open, then

\[ \lambda \leq \tau_\gamma - \operatorname{int}(\operatorname{cl}(\lambda)) \lor \operatorname{cl}(\tau_\gamma - \operatorname{int}(\lambda)) \] and $\mu = \operatorname{int}(\mu)$. Then we have

\[ \mu \land \lambda \leq \mu \land [\tau_\gamma - \operatorname{int}(\operatorname{cl}(\lambda)) \lor \operatorname{cl}(\tau_\gamma - \operatorname{int}(\lambda))] \]

\[ = [\mu \land \tau_\gamma - \operatorname{int}(\operatorname{cl}(\lambda))] \lor [\mu \land \operatorname{cl}(\tau_\gamma - \operatorname{int}(\lambda))] \]
Weak Forms of Fuzzy $\gamma$-Open Sets

\[ = \left[ \text{int}(\mu) \cap \tau_{\gamma} \cap \text{int}(\lambda) \right] \cup \left[ \mu \cap \text{cl}(\tau_{\gamma} \cap \text{int}(\lambda)) \right] \]

\[ \leq \left[ \text{int}(\mu) \cap \text{cl}(\lambda) \right] \cup \left[ \mu \cap \text{cl}(\text{int}(\lambda)) \right] \]

\[ \leq \left[ \text{int}(\mu \cap \text{cl}(\lambda)) \right] \cup \left[ \text{cl}(\mu \cap \text{int}(\lambda)) \right] \]

\[ \leq \left[ \text{int}(\text{cl}(\mu \cap \lambda)) \right] \cup \left[ \text{cl}(\text{int}(\mu \cap \lambda)) \right] \]

This shows that $\mu \cap \lambda$ is fuzzy $b$-open.

**Proposition 3.4.** The intersection of a fuzzy $\alpha-\gamma$-open set and a fuzzy open set is fuzzy $\alpha-\gamma$-open.

**Proof.** Proof is similar to the above propositions 3.2 and 3.3.

**Theorem 3.3.** If $\lambda_k : k \in \Delta$ is a collection of fuzzy $b-\gamma$-open (resp. fuzzy $\alpha-\gamma$-open, fuzzy pre-$\gamma$-open, fuzzy $\beta-\gamma$-open) sets of a fts $\{X, \tau\}$, then $\bigcup_{k \in \Delta} \lambda_k$ is fuzzy $b-\gamma$-open (resp. fuzzy $\alpha-\gamma$-open, fuzzy pre-$\gamma$-open, fuzzy $\beta-\gamma$-open).

**Proof.** We prove only the first case since the other cases are similarly shown. Since

\[ \lambda \leq \tau_{\gamma} \cap \text{int}(\text{cl}(\lambda_k)) \cup \text{cl}(\tau_{\gamma} \cap \text{int}(\lambda_k)) \]

for every $k \in \Delta$, we have

\[ \bigcup_{k \in \Delta} \lambda_k \leq \bigcup_{k \in \Delta} \left[ \tau_{\gamma} \cap \text{int}(\text{cl}(\lambda_k)) \cup \text{cl}(\tau_{\gamma} \cap \text{int}(\lambda_k)) \right] \]

\[ \leq \left[ \bigcup_{k \in \Delta} \tau_{\gamma} \cap \text{int}(\text{cl}(\lambda_k)) \right] \cup \left[ \bigcup_{k \in \Delta} \text{cl}(\tau_{\gamma} \cap \text{int}(\lambda_k)) \right] \]

\[ \leq \left[ \tau_{\gamma} \cap \text{int}\left(\bigcup_{k \in \Delta} \text{cl}(\lambda_k)\right)\right] \cup \left[ \text{cl}(\bigcup_{k \in \Delta} \tau_{\gamma} \cap \text{int}(\lambda_k)) \right] \]

\[ \leq \left[ \tau_{\gamma} \cap \text{int}\left(\bigcup_{k \in \Delta} \lambda_k\right)\right] \cup \left[ \text{cl}(\bigcup_{k \in \Delta} \tau_{\gamma} \cap \text{int}(\lambda_k)) \right] \]

Therefore, $\bigcup_{k \in \Delta} \lambda_k$ is fuzzy $b-\gamma$-open.

We note that the intersection of two fuzzy pre-$\gamma$-open (resp. fuzzy $b-\gamma$-open, fuzzy $\alpha-\gamma$-open, fuzzy $\beta-\gamma$-open) sets need not be fuzzy pre-$\gamma$-open (resp. fuzzy $b-\gamma$-open, fuzzy $\alpha-\gamma$-open, fuzzy $\beta-\gamma$-open) as can be seen from the following examples.
Example 3.3 Let $X = \{a, b, c\}$ and $A, B, C \in I^X$ defined by

$A(a) = 0.3, A(b) = 0.5, A(c) = 0.5;\ B(a) = 0.4, B(b) = 0.2, B(c) = 0.6;\ C(a) = 0.4, C(b) = 0.5, C(c) = 0.6.$

Let $\tau = \{X, \emptyset, A, B, C, A \land B\}.$ Then $(X, \tau)$ is a fts. Define $\gamma : \tau \rightarrow I^X$ by

$\gamma(X) = X, \gamma(\emptyset) = \emptyset, \gamma(A) = A, \gamma(B) = B, \gamma(C) = C, \gamma(A \land B) = \text{cl}(A \land B).$ Then the fuzzy set $A$ and $B$ are fuzzy $\alpha - \gamma -$open but $A \land B$ is not fuzzy $\alpha - \gamma -$open set.

Example 3.4. Let $X = \{a, b, c\}$ and $A, B, C, D, E, F \in I^X$ defined by

$A(a) = 0.3, A(b) = 0.4, A(c) = 0.5; B(a) = 0.6, B(b) = 0.5, B(c) = 0.6;\ C(a) = 0.6, C(b) = 0.5, C(c) = 0.5;\ D(a) = 0.6, D(b) = 0.5, D(c) = 0.4;\ E(a) = 0.3, E(b) = 0.4, E(c) = 0.4; F = 0.3$ where $\alpha$ denotes the constant mapping with value $\alpha.$ Let $\tau = \{X, \emptyset, A, B, C, D, E, F\}.$ Then $(X, \tau)$ is a fts. Define

$\gamma : \tau \rightarrow I^X$ by $\gamma(X) = X, \gamma(\emptyset) = \emptyset, \gamma(A) = A, \gamma(B) = B, \gamma(C) = C, \gamma(D) = D,$

$\gamma(E) = \text{cl}(E), \gamma(F) = F.$ Then the fuzzy set $A$ and $D$ are fuzzy pre-$\gamma -$open sets but $A \land D$ is not fuzzy pre-$\gamma -$open set.

Example 3.5 In Example 3.4 Define $\gamma : \tau \rightarrow I^X$ by

$\gamma(X) = X, \gamma(\emptyset) = \emptyset, \gamma(A) = A, \gamma(B) = B,$

$\gamma(C) = C, \gamma(D) = D, \gamma(E) = \text{cl}(E), \gamma(F) = \text{cl}(F).$ Then the fuzzy set $A$ and $D$ are fuzzy $\beta - \gamma -$open (resp. fuzzy $b - \gamma -$open) sets but $A \land D$ is not fuzzy $\beta - \gamma -$open (resp. fuzzy $b - \gamma -$open) set.

Proposition 3.5. Let $\lambda$ be fuzzy $b - \gamma -$open set such that $\tau_\gamma - \text{int}(\lambda) = \emptyset.$ Then $\lambda$ is fuzzy pre-$\gamma -$open.

Definition 3.2 A fts $(X, \tau)$ is called fuzzy door space if every fuzzy subset of $X$ is fuzzy open or fuzzy closed.

Proposition 3.6. If $(X, \tau)$ is a fuzzy door space and fuzzy $\gamma -$regular, then every fuzzy pre-$\gamma -$open set is fuzzy $\gamma -$open.
Weak Forms of Fuzzy $\gamma$-Open Sets

**Proof.** Let $\lambda$ be a fuzzy pre-$\gamma$-open set. If $\lambda$ is fuzzy open, then $\lambda$ is fuzzy $\gamma$-open. Otherwise, $\lambda$ is fuzzy closed and hence $\lambda \leq \tau_{\gamma} - \text{int}(cl(\lambda)) = \tau_{\gamma} - \text{int}(\lambda) \leq \lambda$. Therefore, $\lambda = \tau_{\gamma} - \text{int}(\lambda)$ and thus $\lambda$ is a fuzzy $\gamma$-open set.

4. NEW FUZZY SEPARATION AXIOMS

**Definition 4.1.** Let $(X, \tau)$ be a fts and $A \in I^{X}$. Then $A$ is called a pre-$Q$-neighborhood of $p_{x}^{k} \in S(X)$ if and only if there exists a fuzzy pre-$\gamma$-open set $U$ such that $p_{x}^{k} \in U$ and $U \subseteq A$. The class of all open $Q$-neighborhoods of $p_{x}^{k}$ is denoted by $N_{\text{pre-}}^{Q}(p_{x}^{k})$.

**Definition 4.2.** A fts $(X, \tau)$ is called:

(1) fuzzy pre-$\gamma-T_{1}$ (resp. fuzzy $\alpha-\gamma-T_{1}$, fuzzy $b-\gamma-T_{1}$, fuzzy $\beta-\gamma-T_{1}$) iff for any $p_{x}^{k}, p_{y}^{k} \in S(X)$ and $p_{x}^{k} \neq p_{y}^{k}$, there exists a fuzzy pre-$\gamma$-open (resp. fuzzy $\alpha-\gamma$-open, fuzzy $b-\gamma$-open, fuzzy $\beta-\gamma$-open) $Q$-neighborhoods $U$ and $V$ of $p_{x}^{k}$ and $p_{y}^{k}$ respectively such that $\gamma(U) \bar{\gamma}(V)$.

(2) fuzzy pre-$\gamma-T_{2}$ (resp. fuzzy $\alpha-\gamma-T_{2}$, fuzzy $b-\gamma-T_{2}$, fuzzy $\beta-\gamma-T_{2}$) iff for any $p_{x}^{k}, p_{y}^{k} \in S(X)$ and $p_{x}^{k} \neq p_{y}^{k}$, there exists a fuzzy pre-$\gamma$-open (resp. fuzzy $\alpha-\gamma$-open, fuzzy $b-\gamma$-open, fuzzy $\beta-\gamma$-open) $Q$-neighborhoods $U$ and $V$ of $p_{x}^{k}$ and $p_{y}^{k}$ respectively such that $\gamma(U) \bar{\gamma}(V)$.

**Theorem 4.1.** If a space $(X, \tau)$ is fuzzy pre-$\gamma-T_{2}$ (resp. $\alpha-\gamma-T_{2}, b-\gamma-T_{2}, \beta-\gamma-T_{2}$), then it is fuzzy pre-$\gamma-T_{1}$ (resp. $\alpha-\gamma-T_{1}, b-\gamma-T_{1}, \beta-\gamma-T_{1}$).

**Proof.** Let $(X, \tau)$ be a fuzzy pre-$\gamma-T_{2}$ (resp. $\alpha-\gamma-T_{2}, b-\gamma-T_{2}, \beta-\gamma-T_{2}$). Let $p_{x}^{k}, p_{y}^{k} \in S(X)$ and $p_{x}^{k} \neq p_{y}^{k}$, then there exists a fuzzy pre-$\gamma$-open (resp. $\alpha-\gamma$-open, $b-\gamma$-open, $\beta-\gamma$-open) $Q$-neighborhoods $U$ and $V$ of $p_{x}^{k}$ and $p_{y}^{k}$ respectively such that $\gamma(U) \bar{\gamma}(V)$. Since $p_{x}^{k} \in U$ and $p_{y}^{k} \in V$, therefore,
\[ p_i^\gamma q^\gamma(U) \text{ and } p_i^\gamma q^\gamma(V). \] Hence \((X, \tau)\) is fuzzy pre-\(\gamma-T_1\)(resp. \(\alpha-\gamma-T_1, b-\gamma-T_1, \beta-\gamma-T_1\)).

**Theorem 4.2.** A space \((X, \tau)\) be a fuzzy pre-\(\gamma-T_1\) if and only if any fuzzy singleton in \(X\) is a fuzzy pre-\(\gamma\)-closed set.

**Proof.** Obvious.

**Theorem 4.3.** Suppose \(\gamma: \tau \rightarrow I^X\) is regular operation. If \((X, \tau_\gamma)\) is a fuzzy \(T_2\)-space then \((X, \tau)\) is a fuzzy pre-\(\gamma-T_2\).

**Proof.** Let \(p_i^\gamma, p_i^\gamma \in S(X)\) and \(p_i^\gamma \neq p_i^\gamma\). Since \((X, \tau_\gamma)\) is a fuzzy \(T_2\)-space, then there exists fuzzy pre-\(\gamma\)-open \(Q\)-neighborhoods \(U, V(\in \tau_\gamma \subset \tau)\) of \(p_i^\gamma\) and \(p_i^\gamma\) respectively such that \(\gamma(U)q^\gamma(V)\). Thus, \((X, \tau)\) is a fuzzy pre-\(\gamma-T_2\).

**Definition 4.3.** Let \((X, \tau)\) be a fts and \(\gamma\) an operation on \(\tau\). A fuzzy set \(\lambda \in I^X\) is called fuzzy pre-\(\gamma\)-generalized closed (fuzzy pre-\(\gamma\)-g-closed, for short) if \(\text{cl}_\gamma(\lambda) \leq U\) whenever \(\lambda \leq U\) and \(U\) is fuzzy pre-\(\gamma\)-open in \((X, \tau)\).

**Theorem 4.4.** Every fuzzy pre-\(\gamma\)-closed set is fuzzy pre-\(\gamma\)-g-closed.

**Proof.** Obvious.

The converse is not true as shown by the following example.

**Example 4.1.** In Example 3.4 The fuzzy set \(B\) is fuzzy pre-\(\gamma\)-g-closed set but not pre-\(\gamma\)-closed.

**Definition 4.4.** A space \((X, \tau)\) is called a fuzzy pre-\(\gamma-T_{1/2}\) space if every fuzzy pre-\(\gamma\)-g-closed set of \((X, \tau)\) is fuzzy pre-\(\tau\)-closed.

**Theorem 4.5.** For each \(p_i^\gamma \in S(X)\) and \(p_i^\gamma\) is fuzzy pre-\(\gamma\)-closed or \(\left(p_i^\gamma\right)^c\) is fuzzy pre-\(\gamma\)-g-closed set in \((X, \tau)\).
Proof. Suppose \( p^\gamma_x \) is not fuzzy pre-\( \gamma \)-closed. Then \( \left( p^\gamma_x \right)^c \) is fuzzy pre-\( \gamma \)-open. Let \( U \) be any fuzzy pre-\( \gamma \)-open set such that \( \left( p^\gamma_x \right)^c \leq U \). Since \( U = X \) is the only fuzzy pre-\( \gamma \)-open, \( c\ell_{\gamma} \left( \left( p^\gamma_x \right)^c \right) \leq U \). Therefore, \( \left( p^\gamma_x \right)^c \) is the fuzzy pre-\( \gamma \)-\( \gamma \)-closed set.

REFERENCES
