A Correlative Information-Theoretic Measure for Image Similarity

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Abstract
A hybrid measure is proposed for assessing the similarity among gray-scale images. The well-known Structural Similarity Index Measure (SSIM) has been designed using a statistical approach that fails under significant noise (low PSNR). The proposed measure, denoted by SjhCorr2, uses a combination of two parts: the first part is information-theoretic, while the second part is based on 2D correlation. The concept of symmetric joint histogram is used in the information-theoretic part. The new measure shows the advantages of statistical approaches and information-theoretic approaches. The proposed similarity approach is robust under noise. The new measure outperforms the classical SSIM in detecting image similarity at low PSNR under Gaussian and impulse noise, with significant difference in performance.

AMS subject classification:
Keywords: Joint histogram, image structural similarity, information-theoretic similarity, image similarity, Gaussian noise, impulse noise, 2D correlation.

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1. **Introduction**

In image processing, applications that require comparing two images according to their content, image matching is an essential component in this process. One of the most important examples is the image database retrieval systems [1]. Image similarity has become in the recent years a basic point in image processing applications like monitoring, image compression, restoration, and many other applications.

Image similarity can be defined as the difference between two images, and image similarity measure is a numerical difference between two different images under comparison. Similarity techniques can be classified according to the methods they use in deriving or defining the difference. The first kind of techniques is the statistical - based methods, and the second important type is the information - theoretical techniques [2]. An old statistical measure that has been widely used to detect image similarity is the mean squared error (MSE) [3, 4, 5].

Recently, light has been shed on a new measure that coincides with the Human Visual System (HSV): Structural Similarity Index Measure (SSIM) is proposed in 2004 by Wang and Bovik. SSIM proved to be distinguished due to its notable performance as compared to the previous metrics [1, 6].

Image recognition has become an interesting subject for researchers over the past two decades because of its potential applications in many important fields like character recognition, human computer interfaces, identity authentication and video surveillance.

Different methods for image recognition, mostly for face image recognition, have been proposed [7]. Many algorithms of face and object recognition systems have recently been designed based on image similarity measure like SSIM [1].

In this work we propose a hybrid similarity measure that combines features of 2D correlation and the information - theoretic features as represented by a new formula for joint histogram. The proposed measure is shown to outperform SSIM in detecting similarity under Gaussian and impulsive noise.

The paper is organized as follows: Section 2 deals with SSIM, which is the well-known structural similarity, and information - theoretic measures. Section 3 presents a method that applies a formula for symmetric joint histogram. In Section 4, experimental results are presented; along with performance measures. The conclusions of this study are given in Section 5.

2. **Methodology**

There have been two major approaches for image similarity: statistical approaches and information - theoretic approaches.

**Statistical methods:**
Mean-squared error (MSE) is a well-known statistical measure. However, MSE is too weak for modern applications of image processing like face recognition. The first significant structural similarity measure, called Structural Similarity Index Measure (SSIM),
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has been proposed in 2004 [1]. SSIM used statistical image parameters such as mean, variance, co-variance, and standard deviation as follows [1, 8]:

\[ \rho(x, y) = \frac{(2\mu_x \mu_y + C_1)(2\sigma_{xy} + C_2)}{\left(\mu_x^2 + \mu_y^2 + C_1\right)\left(\sigma_x^2 + \sigma_y^2 + C_2\right)} \]  

(1)

where \( \rho(x, y) \) is the SSIM metric between images \( x \) and \( y \), while \( \mu_x, \mu_y, \sigma_x^2 \) and \( \sigma_y^2 \) are the statistical means and variances of \( x \) and \( y \), respectively; \( \sigma_{xy} \) is the covariance of \( x \) and \( y \), and finally the constants \( C_1 \) and \( C_2 \) are inserted to avoid unstable results that may be reached due to division by zero, and are defined as \( C_1 = (K_1 L)^2 \) and \( C_2 = (K_2 L)^2 \), with \( K_1 \) and \( K_2 \) are small constants and \( L = 255 \) (maximum pixel value).

Information - theoretic methods:

In information - theoretic approach, the similarity between \( x \) and \( y \) (where \( x \) and \( y \) are images) can be defined as the corresponding differences between information - theoretic features of the two images. The more differences they have, the less similar they are [9].

In 2013, D. Mistry, A. Banerjee and A. Tatu proposed a new similarity measure that base on joint entropy (joint histogram) [10]. The proposed measure is based on the fact of the joint entropy is the measure of uncertainty among two images, so if the joint entropy is low then the similarity between two images are high, and vice-versa. The joint entropy was first applied on two compared images using joint histogram as in [9].

In 2014, A. F. Hassan, D. Cai-lin and Z. M. Hussain proposed a new measure called HSSIM that based on joint histogram. HSSIM outperforms statistical similarity of SSIM; it has the ability to detect similarity under significant noise (low PSNR), with an average difference of nearly 20 dB with SSIM [2].

3. A Hybrid Measure

A symmetric formula for joint histogram is utilized here to create a new measure, designed to be a hybrid measure combining statistical features (represented by 2D correlation) with information - theoretic features. The new measure is a combination of two parts as follows.

The first part of the proposed measure is the information - theoretic part that uses the concept of symmetric joint histogram, defined as follows:

\[ Q(x, y) = \frac{\sum_i \sum_j \left[ (T_{ij} - \tau_{ij}) \right] \frac{1}{h_i - C} \right)^2}{2L^2} \]  

(2)

where \( T_{ij} \) is the symmetric joint histogram of image \( x \) and image \( y \), \( \tau_{ij} = T(x, x) \) is the self- symmetric joint histogram of the first (reference) image \( x \), \( h_i \) is the original histogram of \( x \) and \( C \) is a very small positive constant to avoid division by zero. Note that:

\[ Q(x, y) \geq 0 \]
The above value can be normalized by using maximal error estimated value $Q_\infty(x, y)$ in significant noise (very low PSNR) as follows:

$$v(x, y) = \frac{Q(x, y)}{Q_\infty(x, y)} \quad (3)$$

The process of normalization will ensure that:

$$0 \leq v(x, y) \leq 1$$

The final version of the first part can be stated as follows:

$$q(x, y) = 1 - v(x, y)$$

The second part of the measure is represented by 2D correlation between reference image $x$ and noisy image $y$ as follows:

$$p(x, y) = \text{corr2}(x, y) \quad (4)$$

where corr2 is given as follows [11]:

$$\text{corr2}(x, y) = \frac{\sum_i \sum_j [x(i, j) - \bar{x}] \cdot [y(i, j) - \bar{y}]}{\sqrt{[\sum_i \sum_j (x(i, j) - \bar{x})^2] \cdot [\sum_i \sum_j (y(i, j) - \bar{y})^2]}} \quad (5)$$

where $\bar{x}$ and $\bar{y}$ are the mean values of $x$ and $y$ respectively.

The effect of information-theoretic features could be incorporated with the effect of correlative features as follows:

$$S_{jh\text{Corr2}}(x, y) = (k_1 + k_3) \cdot q + k_2 \cdot p \quad (6)$$

where $k_1, k_2, k_3$ are constants.

Note that $0 \leq S_{jh\text{Corr2}} \leq 1$; giving 1 for completely similar images and 0 for completely different images.

The above hybrid measure can be calculated according to the following algorithm.

**Algorithm:**

Input:
Images $x$ and $y$, which are the reference image and the noisy version, $C$ is small constant and $L = 255$ which represents the maximum pixel value.

Output:
Similarity, a number ranging between 0 and 1.

Step 1: Convert image values into double type.
Step 2: Set $p = \text{corr2}(x, y)$
Step 3: Set $\tau_{ij} = \text{self-symmetric joint histogram of } x$.
Step 4: Set $T_{ij} = \text{symmetric joint histogram of } x$ and $y$. 
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Step 5: Set $Q$:

$$Q(x, y) = \frac{\sum_i \sum_j [(T_{ij} - \tau_{ij}) \frac{1}{h_i - \tau_j}]^2}{2L^2}$$

Step 6: Set $Q_{\infty} = Q(x, y)$ when noise is maximum.

Step 7: Set $q$ after normalization: $v = Q/Q_{\infty}$; $q = 1 - v$.

Step 8: Compute $S_{jhCorr2}(x, y) = \frac{(k_1 + k_3) \cdot q + k_2 \cdot p}{k_2 \cdot p + k_3 \cdot q + k_1}$; where $k_1, k_2, k_3$ are constants.

End of Algorithm.

4. Test Environment

Two types of noise have been considered in simulation and testing: Gaussian noise, which is one of the most popular noise types that are encountered in signal processing systems; and impulsive noise, which is common in image processing.

To test the performance of the proposed measure, different categories of images have been considered: a human face (from AT&T database, [12]), a geometric shape, and a landscape. Parameters are $k_1 = 3$, $k_2 = 2$, $k_3 = 4$.

5. Results and Discussion

The proposed measure has been tested and simulated using MATLAB.

A. Performance under Gaussian Noise:

The proposed measure has been tested under Gaussian noise, which is the most popular noise that attacks images and systems. Results are shown in Figures 1-3. Table 1 shows a comparison between the SSIM and $S_{jhCorr2}$ for different types of images. Comparison has also been made with joint histogram similarity, $S_{jh}$. The proposed measure gives larger similarity than SSIM under Gaussian noise.

Table 1: $S_{jhCORR2}$ vs. SSIM under Gaussian noise (PSNR = 28 dB).

<table>
<thead>
<tr>
<th>Image</th>
<th>Measure</th>
<th>SSIM</th>
<th>$S_{jhCORR2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Landscape</td>
<td></td>
<td>0.4181</td>
<td>0.9906</td>
</tr>
<tr>
<td>Human Face</td>
<td></td>
<td>0.7239</td>
<td>0.9940</td>
</tr>
<tr>
<td>Geometric Shape</td>
<td></td>
<td>0.2556</td>
<td>0.9797</td>
</tr>
</tbody>
</table>
Figure 1: Performance comparison of SSIM and Sjh-Corr2 using similar images (landscape image) under Gaussian noise.

B. Performance under Impulsive Noise:
Second test was performed under impulsive noise, salt & pepper model from Matlab has been used. Results are shown in Table 2 and Figures 4-6.

It can be seen that the proposed measure gives better results (larger similarity between the image and its noisy version) under impulsive noise for various kinds of images (geometric, human face, and landscape images).
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Figure 2: Performance comparison of SSIM and SjhCorr2 using similar images (human face) under Gaussian noise.

(c) Histograms, original and noisy, PSNR=28 dB.

(d) Symmetric joint histogram of identical images (noise-free).

Figure 2: Performance comparison of SSIM and SjhCorr2 using similar images (human face) under Gaussian noise.

Table 2: SJHCORR2 vs. SSIM under impulsive noise (PSNR = 10 dB).

<table>
<thead>
<tr>
<th>Image</th>
<th>Measure</th>
<th>SSIM</th>
<th>SJHCORR2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Landscape</td>
<td>SSIM</td>
<td>0.0450</td>
<td>0.6685</td>
</tr>
<tr>
<td>Human Face</td>
<td>SSIM</td>
<td>0.0960</td>
<td>0.6958</td>
</tr>
<tr>
<td>Geometric Shape</td>
<td>SSIM</td>
<td>0.0484</td>
<td>0.6700</td>
</tr>
</tbody>
</table>
Figure 3: Performance comparison of SSIM and Sjh-Corr2 using similar images (geometric image) under Gaussian noise.
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Figure 4: Performance of SSIM and SjhCorr2 using similar images (landscape) under impulsive noise.

(a) Tested images: original and noisy images, PSNR=20 dB.  
(b) Performance comparison of SSIM and SjhCorr2.

Figure 5: Performance of SSIM and SjhCorr2 using similar images (human face) under impulsive noise.

(a) Tested images: original and noisy images, PSNR=20 dB.  
(b) Performance comparison of SSIM and SjhCorr2.
6. Conclusions

An image hybrid similarity measure, called symmetric joint histogram with 2D correlation (SjhCorr2), has been proposed and tested versus the well-known structural similarity measure (SSIM) under Gaussian noise and impulsive noise (salt and pepper model from MATLAB). Design of the proposed method is based on a combination of information-theoretic and statistical features. A symmetric formula for the joint histogram has been used as information-theoretic tool, and image 2D correlation has been used as a statistical tool. The new measure gave better performance (more similarity) than the existing statistical-based SSIM under noisy conditions.

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References


