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Max-Min Method for Solving Transshipment Problem with Mixed Constraints

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Abstract

Transshipment problem is a generalization of the transportation problem in which the origin and destination constraints consist not only of equality but also of greater than or equal to or less than or equal to type constraints. A simple method has been developed to find the optimal solution for transshipment problem with mixed constraints. Transshipment problem is converted into an equivalent transportation problem with mixed constraints, we proposed a new method for solving transshipment problem with mixed constraints and in the form of algorithm to find an optimal solution from max-min method. The optimal max-min solution procedure is illustrated with numerical example and also computer programming.

Keywords: Transportation problem, Transshipment problem, Mixed constraints, Optimal solution.

INTRODUCTION

The transportation problem is a special class of linear programming problem, which deals with shipping commodities from sources to destinations. The objective of the transportation problem is to determine the shipping schedule that minimizes the total shipping cost while satisfying supply and demand limits. That is, the problem is to find the amount of a uniform commodity which should be transported from each m sources to each n destination satisfying all the supply and demand limits of sources and destinations respectively so that the overall transporting cost is minimum. But instead of direct shipments from sources to destinations, the commodity can be transported to a particular destination through one or more intermediate source and destination. Each of these supply in turn supply to other points. Thus when the shipments pass from destination to destination and from source to source also, then minimizing the overall transshipment cost satisfying the demand and supply limits of sources and destinations.

In the transshipment problem all the sources and destinations can function in any direction, i.e., from destination to destination, source to source, source to destination, destination to source also.

Since transshipment problem is a particular case of transportation problem hence to solve transshipment problem we firstly convert transshipment problem into equivalent transportation problem and then solve it to obtain optimal solution using max-min method.

Bridgen (1974)[1], considered the transportation problem with mixed constraints. He solved this problem by considering a related standard transportation problem having two additional supply points and two additional destinations.

Khurana, Arora(2011)[2] , considered the transshipment problem with mixed constraints. They change it to transportation problem with mixed constraints.

We propose a method for getting the optimal solution for the transshipment problem with mixed constraints.

Mathematical formulation of the Transshipment problem

To formulate the transshipment problem we consider a transportation table given below:

<u>Transshipment Table</u>

		1	2	...	i	...	m	$m+1$	$m+2$	...	$m+j$	...	$m+n$	
		O_1	O_2	...	O_i	...	O_m	D_1	D_2	...	D_j	...	D_n	supply
1	O_1	x_{11}	x_{12}	...	x_{1i}	...	x_{1m}	$x_{1,m+1}$	$x_{1,m+2}$		$x_{1,m+j}$	...	$x_{1,m+n}$	a_1
2	O_2	x_{21}	x_{22}	...	x_{2i}	...	x_{2m}	$x_{2,m+1}$	$x_{2,m+2}$	...	$x_{2,m+j}$	...	$x_{2,m+n}$	a_2
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
i	O_i	x_{i1}	x_{i2}	...	x_{ii}	...	x_{im}	$x_{i,m+1}$	$x_{i,m+2}$	...	$x_{i,m+j}$	...	$x_{i,m+n}$	a_i
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
m	O_m	x_{m1}	x_{m2}	...	x_{mi}	...	x_{mm}	$x_{m,m+1}$	$x_{m,m+2}$	...	$x_{m,m+j}$	...	$x_{m,m+n}$	a_m
$m+1$	D_1	$x_{m+1,1}$	$x_{m+1,2}$	...	$x_{m+1,i}$	...	$x_{m+1,m}$	$x_{m+1,m+1}$	$x_{m+1,m+2}$	...	$x_{m+1,m+j}$	...	$x_{m+1,m+n}$	...
$m+2$	D_2	$x_{m+2,1}$	$x_{m+2,2}$	...	$x_{m+2,i}$	...	$x_{m+2,m}$	$x_{m+2,m+1}$	$x_{m+2,m+2}$	...	$x_{m+2,m+j}$	...	$x_{m+2,m+n}$	...
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
$m+j$	D_j	$x_{m+j,1}$	$x_{m+j,2}$	...	$x_{m+j,i}$	...	$x_{m+j,m}$	$x_{m+j,m+1}$	$x_{m+j,m+2}$	...	$x_{m+j,m+j}$	...	$x_{m+j,m+n}$	...
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
$m+n$	D_n	$x_{m+n,1}$	$x_{m+n,2}$	...	$x_{m+n,i}$	...	$x_{m+n,m}$	$x_{m+n,m+1}$	$x_{m+n,m+2}$	...	$x_{m+n,m+j}$	...	$x_{m+n,m+n}$	...
Demand		...	...	...	...	...	...	b_1	b_2	...	b_j	...	b_n	

In the transshipment table $O_1, O_2, ...O_i, ...O_m$ are sources from where goods are to be transported to destinations $D_1, D_2, ...D_j, ...D_n$. Any of the sources can transport to any of the destinations. C_{ij} is per unit transporting cost of goods from i^{th} source O_i to j^{th} destination D_j for all i=1,2,..m and j=1,2,..n. x_{ij} is the amount of goods transporting from i^{th} source O_i to j^{th} destination D_j. a_i be the amount of goods available at the origins O_i and b_j the demand at the destinations D_j. The corresponding transportation problem is

$$\min z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$s.t. \quad \sum_{j=1}^{n} x_{ij} = a_i, i = 1, 2, ...m$$

$$\sum_{i=1}^{m} x_{ij} = b_j, j = 1, 2, ...n$$

$$x_{ij} \geq 0, \forall i = 1, 2, ...m \; \& \; j = 1, 2, ...m.$$

Since in a transshipment problem, any origin or destination can ship to any other origin or destination it would be convenient to number them successively so that the origins are numbered from 1 to m and the destinations from m + 1 to m + n.

We now extend this transportation problem to permit transshipment with the additional feature that shipments may go via any sequence of points rather than being restricted to direct connections from one origin to one of the destination. The unit cost of shipment from a point considered as a shipper to the same point considered as receiver is set equal to zero.

$$x_{i1} + x_{i2} + \ldots + x_{i\ m+n} = a_i + (x_{1i} + x_{2i} + \ldots x_{m+n\ i})$$

$$x_{i1} + x_{i2} + \ldots + x_{i,i-1} \ldots x_{i,i+1} + \ldots x_{i,m+n}$$

$$= a_i + (x_{1i} + x_{2i} + \ldots x_{i-1,i} + x_{i+1,i} + \ldots + x_{m+n,i})$$

$$i.e. \sum_{j=1, j\neq i}^{n} x_{ij} = a_i + \sum_{j=1, j\neq i}^{m+n} x_{ji}, i = 1, 2, \ldots m$$

$$i.e. \sum_{j=1, j\neq i}^{m+n} x_{ij} - \sum_{j=1, j\neq i}^{m+n} x_{ji} = a_i, i = 1, 2, \ldots m$$

Similarly the total amount received at a destination D_j must be equal to its demand plus what it transships.

$$x_{1,m+j} + x_{2,m+j} + \ldots + x_{m+j-1,m+j} + x_{m+j+1,m+j} + \ldots + x_{m+n,m+j}$$

$$= b_{m+j} + (x_{m+j,1} + x_{m+j,2} + \ldots + x_{m+j,m+j-1} + x_{m+j,m+j+1} + \ldots + x_{m+j,m+n}$$

$$i.e.\ x_{1\ \mathrm{m+j}} + x_{2\ \mathrm{m+j}} + \ldots + x_{m+n\ m+j}$$

$$= b_{m+j} + (x_{m+j\ 1} + x_{m+j\ 2} + \ldots + x_{m+j\ m} + x_{m+j\ m+j} + \ldots + x_{m+j\ m+n})$$

$$i.e. \sum_{i=1, i\neq j}^{m+n} x_{i\ m+j} = b_{m+j} + \sum_{i=1, \neq j}^{m+n} x_{m+j\ i}, j = 1, 2, \ldots n$$

$$i.e. \sum_{i=1, i\neq j}^{m+n} x_{i\ m+j} - \sum_{i=1, i\neq j}^{m+n} x_{m+j\ i} = b_{m+j}, j = 1, 2, \ldots n$$

$$i.e. \sum_{i=1, i\neq j}^{m+n} x_{ij} - \sum_{i=1, i\neq j}^{m+n} x_{ji} = b_j, j = m+1, m+2, \ldots m+n.$$

$$and\ x_{ij} \geq 0, i = 1, 2, \ldots m+n,\ \mathrm{j} \neq \mathrm{i}$$

Thus the transshipment problem may be written as

$$\text{Minimize } z = \sum_{i=1}^{m+n} \sum_{j=1, j\neq i}^{m+n} c_{ij} x_{ij}$$

subject to constraints

$$\sum_{j=1, j\neq i}^{m+n} x_{ij} - \sum_{j=1, j\neq i}^{m+n} x_{ji} = a_i, i = 1, 2, \ldots m$$

$$\sum_{i=1, i\neq j}^{m+n} x_{ij} - \sum_{i=1, i\neq j}^{m+n} x_{ji} = b_j, j = m+1, m+2, \ldots m+n.$$

$$x_{ij} \geq 0, i = 1, 2, \ldots m+n, \ \mathrm{j} \neq \mathrm{i}$$

The above formulation is a linear programming problem, which is similar to a transportation problem but not exactly since the coefficients of x_{ij}'s are -1. The problem however easily be converted to a standard transportation problem.

$$t_i = \sum_{j=1, j\neq i}^{m+n} x_{ji}, i = 1, 2, \ldots m,$$

$$i.e. \qquad t_i + x_{ii} = \sum_{j=1}^{m+n} x_{ji}, i = 1, 2, \ldots m$$

$$and \ t_j = \sum_{i=1, i\neq j}^{m+n} x_{ji}, j = m+1, m+2, \ldots m+n$$

$$i.e. \qquad t_j + x_{jj} = \sum_{i=1}^{m+n} x_{ji}, j = m+1, m+2, \ldots m+n$$

Where t_i represents the total amount of transshipment through the i^{th} origin
and t_j represents the total amount shipped put from the j^{th} destination as transshipment.

Let T>0 be sufficiently large number so that $t_i \leq T$, for all i and $t_j \leq T$ for all j.

We now write $t_i + x_{ii} = T$,then the non negative slack variable x_{ii} represents the difference between T and the actual amount of transshipment through the i^{th} origin.

Similarly, if we let $t_j + x_{jj} = T$, then the non negative slack variable x_{jj} represents the difference between T and the actual amount of transshipment through the j^{th} destination.

Note that T can be interpreted as a buffer stock at each origin and destination. Since we assume that any amount of goods can be transshipped at each point, T should be large enough to take care of all transshipments. It is clear that the volume of goods transshipped at any point cannot exceed the amount produced or received and hence we take $T = \sum_{i=1}^{m} a_i$

The transshipment problem then reduces to

$$\min z = \sum_{i=1}^{m+n} \sum_{j=1, j \neq i}^{m+n} c_{ij} x_{ij}$$

$$\text{s.t.} \quad \sum_{j=1}^{m+n} x_{ij} = a_i + T, i = 1, 2, \ldots m,$$

$$\sum_{j=1}^{m+n} x_{ij} = T, i = m+1, m+2, \ldots m+n,$$

$$\sum_{i=1}^{m+n} x_{ij} = T, j = 1, 2, \ldots m,$$

$$\sum_{i=1}^{m+n} x_{ij} = b_j + T, j = m+1, m+2, \ldots m+n,$$

$$x_{ij} \geq 0, i = 1, 2, \ldots m+n \text{ and } j = 1, 2, \ldots m+n,$$

where $c_{ii} = 0, i = 1, 2, \ldots m+n.$

The above mathematical model represents a standard transportation problem with (m+n) origins and (m+n) destinations.

The solution of the problem contains 2m+2n-1 basic variables. However, m+n of these variables appearing in the diagonal cells represent the remaining buffer stock and if they are omitted. We have m+n-1 basic variables of our intrest.

Transshipment problem with mixed constraints

The substantially increase or decrease of the capacity of a factory will affect the overall production and transportation cost.

Similarly, the substantially increase or decrease of the demand of a destination will affect the overall production and transportation cost.
Suppose that the source $O_i, i \in \alpha_1$ supplies exactly fixed amount a_i, source $O_i, i \in \alpha_2$ supplies at least amount a_i and source $O_i, i \in \alpha_3$ supplies at most an amount a_i. Similarly, the destination $D_j, j \in \beta_1$ demands exactly the fixed amount b_j, the destination $D_j, j \in \beta_2$ demands at least an amount b_j, the destination $D_j, j \in \beta_3$ demands at most an amount b_j.

Considering this fact, the standard transportation problem may be written as

$$Min z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$s.t. \quad \sum_{j=1}^{n} x_{ij} = a_i \forall i \in \alpha_1$$

$$\sum_{j=1}^{n} x_{ij} \geq a_i \forall i \in \alpha_2$$

$$\sum_{j=1}^{n} x_{ij} \leq a_i \forall i \in \alpha_3$$

$$\sum_{i=1}^{m} x_{ij} = b_j \forall j \in \beta_1$$

$$\sum_{i=1}^{m} x_{ij} \geq b_j \forall j \in \beta_2$$

$$\sum_{i=1}^{m} x_{ij} \leq b_j \forall j \in \beta_3$$

$$where \ \mathrm{I}_1 = \{1, 2, \ldots m\} = \alpha_1 \cup \alpha_2 \cup \alpha_3\}$$

$$I_2 = \{1, 2, \ldots n\} = \beta_1 \cup \beta_2 \cup \beta_3\}$$

The corresponding transshipment problem then according is as follows
find the values of x_{ij} such that

$$\text{Minimize } z=\sum_{i=1}^{m+n}\sum_{j=1, j\neq i}^{m+n} c_{ij}x_{ij}$$

Subject to constraints

$$\sum_{i=1,i\neq j}^{m+n} x_{ij}-\sum_{i=1,i\neq j}^{m+n} x_{ji}=a_i,\forall i\in\alpha_1$$

$$\sum_{i=1,i\neq j}^{m+n} x_{ij}-\sum_{i=1,i\neq j}^{m+n} x_{ji}\geq a_i,\forall i\in\alpha_2$$

$$\sum_{j=1,j\neq i}^{m+n} x_{ij}-\sum_{j=1,j\neq i}^{m+n} x_{ji}\leq a_i, i\in\alpha_3$$

$$\sum_{i=1,i\neq j}^{m+n} x_{ij}-\sum_{i=1,i\neq j}^{m+n} x_{ji}=b_j, j\in\beta_1$$

$$\sum_{i=1,i\neq j}^{m+n} x_{ij}-\sum_{i=1,i\neq j}^{m+n} x_{ji}\geq b_j, j\in\beta_2$$

$$\sum_{i=1,i\neq j}^{m+n} x_{ij}-\sum_{i=1,i\neq j}^{m+n} x_{ji}\leq b_j, j\in\beta_3$$

$$x_{ij}\geq 0, i, j=1,2,...m+n, i\neq j$$

$$where\ \ \alpha_1\cup\alpha_2\cup\alpha_3=I_1=\{1,2,...m\},$$

$$\beta_1\cup\beta_2\cup\beta_3=I_2=\{m+1,m+2,...m+n\}.$$

The problem is said to be the transshipment problem with mixed constraints.

If $\sum_{i=1}^{m}a_i=\sum_{j=1}^{n}b_j$ then the problem is said to be a balanced transshipment problem with mixed constraints.

If $\sum_{i=1}^{m}a_i\neq\sum_{j=1}^{n}b_j$ then the problem is said to be an unbalanced transshipment problem with mixed constraints. In this case a dummy origin/destination can be introduced to make it a balanced transshipment problem with mixed constraints.

Now supposing large number T for $\sum_{j=1}^{m+n}x_{ji},\forall i\in I_1$ and also for $\sum_{i=1}^{m+n}x_{ij},\forall j\in I_2$,the above transshipment problem can be reduced to

$$\text{Minimize } z = \sum_{i=1}^{m+n} \sum_{j=1, j\neq i}^{m+n} c_{ij} x_{ij}$$

Subject to constraints

$$\sum_{i=1, i\neq j}^{m+n} x_{ij} = a_i + T, \forall i \in \alpha_1$$

$$\sum_{i=1, i\neq j}^{m+n} x_{ij} \geq a_i + T, \forall i \in \alpha_2$$

$$\sum_{j=1, j\neq i}^{m+n} x_{ij} \leq a_i + T, i \in \alpha_3$$

$$\sum_{i=1, i\neq j}^{m+n} x_{ij} = T, \forall i \in \alpha_1 \cup \alpha_2 \cup \alpha_3$$

This comes out to be the transportation problem with mixed constraints.

Proposed Algorithm-

Step1. Change transshipment problem into equivalent transportation problem with mixed constraints.

Step2. Select the least element in the greatest cost cell row/column and assign these two cells.

Step3. Find the cost occurred corresponding to these two cells.

Step4. The cell corresponding to lowest cost is remained assign and the other one assignment is vacated.

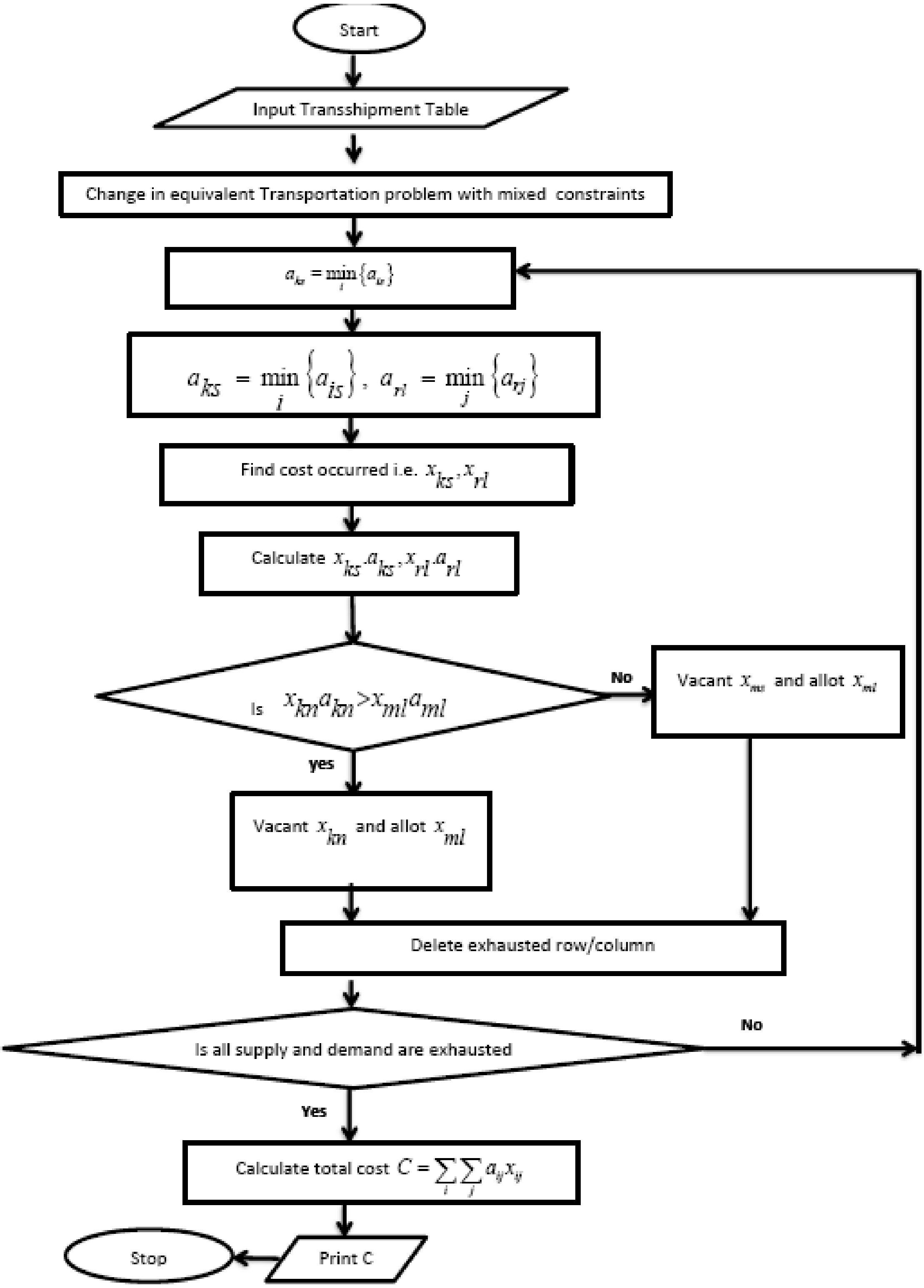
Step5. Delete the exhausted row/column of the assigned to get the reduced table.

Step6. Repeat the same process for the reduced table till all the supplies/demands are exhausted.

Step7. Collecting all the assign cells from before table we have optimal solution of balanced transshipment problem with mixed constraints.

Supply	**Demand**	**Assignment**
a_i	b_j	
=	=	$\min(a_i, b_j)$
=	≤	$\min(a_i, b_j)$
=	≥	a_i
≤	≤	0
≤	=	$\min(a_i, b_j)$
≤	≥	a_i
≥	≥	$\min(a_i, b_j)$
≥	=	b_j
≥	≤	b_j

Flow chart of proposed method



Balanced transhipment problem with mixed constraints.

To illustrate the max-min method we consider the balanced transshipment problem involving two origins and two destinations. The availabilities at the origins, the requirements at the destinations and the costs are given below in the Table.

Table 1

	O_1(j=1)	O_2(j=2)	D_1(j=3)	D_2(j=4)	
O_1(i=1)	0	12	9	6	=5
O_2(i=2)	7	0	7	7	≥ 6
D_1(i=3)	6	5	0	12	...
D_2(i=4)	6	8	11	0	...
	...	...	=4	≥ 7	

since $T = \sum_{i=1}^{2} a_i = \sum_{j=3}^{4} b_j = 11$, we convert the problem into a linear transportation problem with mixed constraints by adding 11 units to each a_i and b_j as shown in table 2.

Table-2

	O_1(j=1)	O_2(j=2)	D_1(j=3)	D_2(j=4)	
O_1(i=1)	0 **11**	12	9	6	=16
O_2(i=2)	7	0	7	7	≥ 17
D_1(i=3)	6	5	0	12	=11
D_2(i=4)	6	8	11	0	=11
	=11	=11	=15	≥ 18	

Maximum cost in table 2 is 12 corresponding to the cell (O_1, O_2) and (D1,D2) . Breaking ties arbitrarily the cell (O_1, O_2) is selected. The minimum cost cell in its row is (O_1, O_1) and the minimum cost cell in its column is (O_2, O_2). The possible assignments in the cell (O_1, O_1) and (O_2, O_2) are respectively 11 and 11. Corresponding costs after assignment in these cells are 0, 0. Breaking ties arbitrarily the minimum cost cell is selected as (O_1, O_1) and assigned it. The demand D_1 is satisfied. Deleting O_1 column the reduced table 3 is given below.

Table-3

	O_2 (j=2)	D_1 (j=3)	D_2 (j=4)	
O_1 (i=1)	12	9	6	=5
O_2 (i=2)	0 **11**	7	7	≥ 17
D_1 (i=3)	5	0	12	=11
D_2 (i=4)	8	11	0	=11
	=11	=15	≥ 18	

Repeating the same process in table 3. The assignment is 11 in cell (O_2, O_2). The demand corresponding to O_2 is exhausted. Deleting the O_2 column the reduced table 4 is given below.

Table 4

	D_1 (j=3)	D_2 (j=4)	
O_1 (i=1)	9	6	=5
O_2 (i=2)	7	7	≥ 6
D_1 (i=3)	0 **11**	12	=11
D_2 (i=4)	11	0	=11
	=15	≥ 18	

in table 4 cell (D_1, D_1) is assigned and row D_1 is deleted. The reduced table 5 is given below.

Table 5

	D_1 (j=3)	D_2 (j=4)	
O_1 (i=1)	9	6	=5
O_2 (i=2)	7	7	≥ 6
D_2 (i=4)	11	0	=11
	=4	≥ 18	

Assignment 11 is done in the cell (D_2, D_2). Row D_2 is exhausted . Deleting it the reduced table 6 is given below.

Table 6

	D_1 (j=3)	D_2 (j=4)	
O_1 (i=1)	9	6	=5
O_2 (i=2)	7	7	≥ 6
	=4	≥ 7	

In table 6 cell (O_2, D_1) is assigned. D_1 column is exhausted. Deleting it the reduced table 7 is given below.

Table 7

	D_2 (j=4)	
O_1 (i=1)	6	=5
O_2 (i=2)	7	≥ 2
	≥ 7	

Being only one column in table 7 it is assigned directly as 5 unit in cell (O_1, D_2) and 2 unit in cell (O_2, D_2). The assigned and all the demands and supplies exhausted table 8 is given below.

Table-8

	D_2 (j=4)	
O_1 (i=1)	6 **5**	=0
O_2 (i=2)	7 **2**	≥ 0
	≥ 0	

Summarizing all the assignments done above the full assigned table 9 is given below.

Table 9

	O_1 (j=1)	O_2 (j=2)	D_1 (j=3)	D_2 (j=4)	
O_1 (i=1)	0 **11**	12	9	6 **5**	=16
O_2 (i=2)	7	0 **11**	7 **4**	7 **2**	≥ 17
D_1 (i=3)	6	5	0 **11**	12	=11
D_2 (i=4)	6	8	11	0 **11**	=11
	=11	=11	=15	≥ 18	

From the table 9 optimal solution is

Min Z=72 for $x_{11} = 11$, $x_{14} = 5$, $x_{22} = 11$, $x_{23} = 4$, $x_{24} = 2$, $x_{33} = 11$, $x_{44} = 11$.

CONCLUSION

We have developed a simple algorithm for solving a linear transshipment problem with mixed constraints. The cases of a balanced as well as unbalanced transshipment problem have been discussed.

In this paper, a new and simple method Max-Min method for solving transshipment problem with mixed constraints is proposed. This method is useful for all type of transshipment problem maximization or minimization, balanced or unbalanced and restricted. The algorithm of the method has been presented.

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