Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 13, Number 12 (2017), pp. 8361–8372

http://www.ripublication.com/gjpam.htm

Statistical Inferences With Jointly Type-II Censored Samples From Two Rayleigh Distributions

Bekheet N. Al-Matrafi and G.A. Abd-Elmougod
Department of Mathematics,
Faculty of Sciences, Taif University, Saudi Arabia.

Abstract

Practically, joint censoring scheme is applied in conducting comparative life tests of products from different lines within the same facility. The estimation problem of life time Rayleigh distributions (RD) based on joint Type-II censoring scheme is developed in this paper. The maximum likelihood estimators (MLEs) as well as the approximate confidence intervals based on the asymptotic normality of the MLEs are presented. The two parametric bootstrap confidence intervals for the distribution parameters are also used. The point and probability intervals estimation with Bayes method are adopted with different loss functions. The obtaining results are assessment and compare through Monte Carlo studies. An analysis of RD life time sample is presented for illustrative.

AMS subject classification:
Keywords: Bayesian estimation, Bootstrap confidence intervals, Maximum likelihood estimation, Joint Type-II censoring, Rayleigh distribution.

1. Introduction

In the manufactured products, when the aim of a study is to measure the relative merits of two competing duration of life products, comparative lifetime tests play important role. The joint type-II censoring can be applied in conducting comparative life-tests of products under different lines of production. To be more precise, suppose manufactured products obtain from two different lines ω_1 and ω_2 with the same conditions and that two independent samples of sizes M and N are selected from these two lines, respectively, to placed on a life-testing experiment. Then, for consideration of cost and time, the experimenter may be terminate the test after fixed number of failures occur. Each of failure times and the corresponding product types will be recorded. This type of censsoring scheme discussed earlier by several authers, more details see [[1], [2], [3], [4], [5], and [6]]. Exact likelihood inference compared with bootstrap approch under joint Type-II censoring of exponential populations in [7]. Generalization results in progressive Type-II censoring presented in [8]. [9] considered Bayesian inference of exponential populations under three types of loss functions, squared error (SE) , linear exponential (LINEX) and general entropy (GE) loss functions. Recently inferences of two Pareto distributions based jointly type-II censored samples in [10].

Suppose that ω_1 and ω_2 two line of production have the same possibilities. From a line ω_1, M unties with i.i.d (identical independent distributed) lifetimes random variables $T_1, T_2, \ldots, T_M$ have the population with cdf (cumulative distribution function) $F_1(t)$ and pdf (probability distribution) function $f_1(t)$. Also, N unties with $Y_1, Y_2, \ldots, Y_N$ (i.i.d.) lifetimes random variables have the cdf $F_2(y)$ and pdf $f_2(y)$ of a line ω_2. For given $Z_1 \leq Z_2 \leq \ldots \leq Z_m$ the order sample of size m of $\{T_1, T_2, \ldots, T_{M_m}, Y_1, Y_2, \ldots, Y_{N_m}\}$ where $m = M_m + N_m \leq M + N$. Hence, under the above sample, the observed jointly set (Z, ξ) consist of $Z = (Z_1, Z_2, \ldots, Z_m)$ with $1 \leq m < M + N$ and $\boldsymbol{\xi} = (\xi_1, \xi_1, \ldots, \xi_m)$ with $\left(\xi_i = 1 \text{ if } Z_i = T_i \text{ and } \xi_i = 0 \text{ if } Z_i = Y_i\right)$. Let $M_m = \sum_{i=1}^{m} \xi_i$ are T's failures among $(Z_1, Z_2, \ldots, Z_m)$ and $N_m = \sum_{i=1}^{m}(1 - \xi_i)$ are Y's. The joint likelihood function of $(Z, \boldsymbol{\xi})$ is constructed as follows

$$f(z, \boldsymbol{\xi}) = \frac{M!N![1 - F_1(z_m)]^{M-M_m}[1 - F_2(z_m)]^{N-N_m}}{(M - M_m)!(N - N_m)!} \prod_{i=1}^{m} [f_1(z_i)]^{\xi_i} [f_2(z_i)]^{1-\xi_i} . \tag{1}$$

RD is a special case of the Weibull distribution for more detial see [11, 12]. Also RD has different applications in several branches such as, communication, engineering and life tests experiment. Applications in life testing of electrovacum devices see [13], Statistical inference for the RD based on progressive Type-II fuzzy censored data see [14].

The random variable T has the RD with positive parameter β and pdf, given by

$$f_i(t) = 2\beta_i t e^{-\beta_i t^2}, \quad t > 0,\ \beta_i > 0,\ i = 1, 2. \tag{2}$$

The cdf, reliability function $(S(t))$, and hazard rate function $(h(t))$ of the RD given, respectively by

$$F_j(x) = 1 - e^{-\beta_i t^2}, \tag{3}$$

$$R_j(t) = e^{-\beta_i t^2}, \tag{4}$$

and

$$H_j(t) = 2\beta_i t, \tag{5}$$

where β_i is the Rayleigh parameters. The reliability function of RD is decreases with higher rate than in the case of exponential distribution see [15]. Also The RD has linear failure rate function increasing of time. This means that when the failure times are distributed according to the Rayleigh law, an intense aging of the equipment takes place. Then as time increases Inferences for the Rayleigh distribution were discussed by several authors. [16] derived an explicit form for the maximum likelihood estimator based on Type II censored data. [11, 12] provided the best linear unbiased estimator based on complete sample, censored sample and selected order statistics. Doubly censored samples were considered, among other authors, by [17], and [18]. Bayesian estimation and prediction problems are also important and have been investigated, among others, by [19], and [20]. In addition, [21, 22] studied Bayesian predictive densities and prediction bounds of generalized order statistics and future records

This paper is organized as follows, in Section 2, The MLEs of the model parameters as well as the approximate confidences intervals are presented. In Section 3, The parametric bootstrap confidence intervals are applied. With respected to SE and LINEX loss function the Bayes estimates of life time RD are developed in Section 4. The simulated life time data set are analyzed in Section 5. In Section 6, the numerical results are assessment and compared through Monte Carlo simulation studies. In Section 7, some brief comments about numerical studies are discussed.

2. Maximum Likelihood Estimation

Let $\underline{Z}$=$(Z_1, Z_2, \ldots, Z_m)$ be the m-first order statistics of $\{T_1, T_2, \ldots, T_{M_m}, Y_1, Y_2, \ldots, Y_{N_m}\}$, with (i.i.d.) samples T and Y taken from the two RD (2). From equations (1), (2) and (3) the likelihood function under jointly type-II censoring sample $\underline{Z}$=$(Z_1, Z_2, \ldots, Z_m)$, is written as

$$L\left(\beta_1, \beta_2|\underline{z}\right) \propto \beta_1^{M_m}\beta_2^{(m-M_m)} \exp\left\{-\beta_1\sum_{i=1}^{m}\xi_i z_i^2 - \beta_2\sum_{i=1}^{m}\left(1-\xi_i\right)z_i^2 \right.$$
$$\left. -M_m\beta_1 z_m^2 - (m-M_m)\beta_2 z_m^2\right\}. \tag{6}$$

The logarithm of the likelihood function given by

$$\ell\left(\beta_1, \beta_2|\underline{z}\right) \propto M_m \log\beta_1 + (m-M_m)\log\beta_2 - \beta_1\sum_{i=1}^{m}\xi_i z_i^2 - \beta_2\sum_{i=1}^{m}\left(1-\xi_i\right)z_i^2$$
$$-M_m\beta_1 z_m^2 - (m-M_m)\beta_2 z_m^2\}. \tag{7}$$

2.1. The point MLE's

To obtained the likelihood equations, calculate the first partial derivatives of (7) with respect to β_1 and β_2 as

$$\frac{\partial\ell\left(\beta_1, \beta_2|\underline{z}\right)}{\partial\beta_1} = \frac{M_m}{\beta_1} - \sum_{i=1}^{m}\xi_i z_i^2 - M_m z_m^2 = 0, \tag{8}$$

and

$$\frac{\partial\ell\left(\beta_1, \beta_2|\underline{z}\right)}{\partial\beta_2} = \frac{(m-M_m)}{\beta_2} - \sum_{i=1}^{m}\left(1-\xi_i\right)z_i^2 - (m-M_m)z_m^2 = 0, \tag{9}$$

hence

$$\hat{\beta}_1 = \frac{M_m}{\sum_{i=1}^{m} \xi_i z_i^2 + M_m z_m^2}, \tag{10}$$

and

$$\beta_2 = \frac{(m - M_m)}{\sum_{i=1}^{m} \left(1 - \xi_i\right) z_i^2 + (m - M_m) z_m^2}, \tag{11}$$

2.2. Asymptotic distribution

In this section the asymptotic distribution of parameters vector $\left(\beta_1,\ \beta_2\right)$ are obtained. Based on the asymptotic distribution of $\left(\beta_1,\ \beta_2\right)$, the asymptotic confidence interval is constructed. The expected Fisher information can be defined by $\Upsilon\left(\beta_1,\ \beta_2\right) = -\mathrm{E}(I\left(\beta_1,\ \beta_2\right))$ where $I\left(\beta_1,\ \beta_2\right)$ is the observed information matrix written as

$$I\left(\beta_1,\ \beta_2\right) = \begin{bmatrix} \dfrac{\partial^2 \ell\left(\beta_1, \beta_2 | \underline{z}\right)}{\partial \beta_1^2} & \dfrac{\partial^2 \ell\left(\beta_1, \beta_2 | \underline{z}\right)}{\partial \beta_1 \partial \beta_2} \\ \dfrac{\partial^2 \ell\left(\beta_1, \beta_2 | \underline{z}\right)}{\partial \beta_2 \partial \beta_1} & \dfrac{\partial^2 \ell\left(\beta_1, \beta_2 | \underline{z}\right)}{\partial \beta_2^2} \end{bmatrix} = \begin{bmatrix} \dfrac{-M_m}{\beta_1^2} & 0 \\ 0 & \dfrac{-(m - M_m)}{\beta_2^2} \end{bmatrix}. \tag{12}$$

Then

$$\Upsilon_{1,1} = \frac{M_m}{\beta_1^2},\ \Upsilon_{1,2} = \Upsilon_{2,1} = 0 \text{ and } \Upsilon_{2,2} = \frac{(m - M_m)}{\beta_2^2}, \tag{13}$$

and

$$\Upsilon^{-1}\left(\beta_1,\ \beta_2\right) = \begin{bmatrix} \dfrac{\beta_1^2}{M_m} & 0 \\ 0 & \dfrac{\beta_2^2}{(m - M_m)} \end{bmatrix}_{\text{at } (\beta_1,\ \beta_2) = \left(\hat{\beta}_1,\ \hat{\beta}_2\right)}. \tag{14}$$

The normal approximation of MLE is used as

$$(\hat{\beta}_1,\ \hat{\beta}_2) \twoheadrightarrow N\left(\left(\beta_1,\ \beta_2\right), \Upsilon^{-1}(\hat{\beta}_1,\ \hat{\beta}_2)\right), \tag{15}$$

The MLEs is distributed as bivariate normal with mean $\left(\beta_1,\ \beta_2\right)$ and covariance matrix $\Upsilon^{-1}(\hat{\beta}_1, \hat{\beta}_2)$, then the approximate confidence intervals of model parametrs β_1 and β_2 can be obtained. Thus, the 100(1-2α)% approximate confidence intervals for β_1 and β_2 given by

$$\hat{\beta}_1\left(1 \mp \frac{z_\alpha}{\sqrt{M_m}}\right) \text{ and } \hat{\beta}_2\left(1 \mp \frac{z_\alpha}{\sqrt{(m - M_m)}}\right), \tag{16}$$

respectively, in (16) and z_α is the tabulated value of the standard normal distribution with significant level equal 2α.

3. Bootstrap Confidence Intervals

When the objective of my studying is determined estimation, confidence intervals, bias, variance of an estimator or calibrate hypothesis tests the bootstrap technique plays important role. Different types of bootstrap technique available, parametric bootstrap technique [23] and nonparametric bootstrap technique [24]. In this section the parametric bootstrap technique are adopted to constructed the percentile bootstrap confidence interval more details see [25] and bootstrap-t confidence interval see [26]. the following algorithm are used to expose the two type of bootstrap technique:

1 Under consideration the original jointly type-II censored sample $(Z, \boldsymbol{\xi})$ the MLEs of parameters β_1 and β_2 say $\hat{\beta}_1$ and $\hat{\beta}_2$ are obtain.

2 The values $\hat{\beta}_1$, $\hat{\beta}_2$ and m are used to generate from RD jointly type-II bootstrap censored sample $(Z^*, \boldsymbol{\xi}^*)$.

3 New sample $(Z^*, \boldsymbol{\xi}^*)$ and m are used to obtained the bootstrap sample estimates $\hat{\beta}^*_1, \hat{\beta}^*_2$.

4 The Steps 2 and 3 are repeated S times to presented different samples of bootstrap estimates.

5 The bootstrap samples estimates put in an ascending order as $(\hat{\beta}_i^{*[1]}, \hat{\beta}_i^{*[2]}, \ldots, \hat{\beta}_i^{*[S]})$, $i = 1, 2$.

Bootstrap technique 1: p-b ⟨percentile bootstrap⟩:
Let $K(x) = P(\hat{\beta}^*_i \leqslant x)$ be a cumulative distribution function of $\hat{\beta}^*_i$, $i = 1, 2$. The p-b values ${}_{\text{p-b}}\hat{\beta}^*_i = K^{-1}(x)$ for given x. Then the approximate bootstrap $100(1-2\alpha)\%$ confidence interval of $\hat{\beta}^*_i$ given by

$$\left[{}_{\text{p-b}}\hat{\beta}^*_i(\alpha),\ {}_{\text{p-b}}\hat{\beta}^*_i(1-\alpha)\right],\ i = 1, 2. \tag{17}$$

Bootstrap technique 1: b-t ⟨bootstrap-t⟩:
Under definition the order statistics $\eta_i^{*[1]} < \eta_i^{*[2]} < \cdots < \eta_i^{*[\mathbf{M}]}$, as

$$\eta_i^{*[l]} = \frac{\hat{\beta}_i^{*[l]} - \hat{\beta}_i}{\sqrt{\operatorname{var}\left(\hat{\beta}_i^{*[l]}\right)}},\ l = 1, 2, \ldots, \mathbf{S},\ i = 1,\ 2. \tag{18}$$

Let $K(x) = P(\eta^*_i < z)$ be the cumulative distribution function of η^*_i. For a given x, define

$$_{\mathbf{b-t}}\hat{\beta}_i = \hat{\beta}_i + \sqrt{\operatorname{Var}(\hat{\beta}_i)}K^{-1}(x). \tag{19}$$

The approximate $100(1-2\alpha)\%$ confidence interval of $\hat{\psi}_k$ is given by

$$\left({}_{\mathbf{b-t}}\hat{\beta}_i(\alpha), {}_{\mathbf{b-t}}\,\hat{\beta}_i(1-\alpha)\right). \tag{20}$$

4. Bayes estimation of the model parameters

In this section, we adopt the Bayessian estimation of the two unknown parameters β_1 and β_2 under consideration independent gamma priors with prior density as

$$\pi_1^*(\beta_1) = \frac{b^a}{\Gamma(a)}\beta_1^{a-1}\exp(-b\beta_1),\ \beta_1 > 0,\ (a,\ b > 0), \tag{21}$$

and

$$\pi_2^*(\beta_2) = \frac{d^c}{\Gamma(c)}\beta_2^{c-1}\exp(-d\beta_2),\ \beta_2 > 0,\ (c,\ d > 0). \tag{22}$$

Then the joint prior density of β_1 and β_2, written by

$$\pi^*(\beta_1,\ \beta_2) \propto \beta_1^{a-1}\beta_2^{c-1}\exp(-b\beta_1 - d\beta_2). \tag{23}$$

The joint posterior density under joint prior density (23) and the likelihood function (6), given by

$$\begin{aligned}\pi(\beta_1,\ \beta_2|\underline{z}) &= \frac{L\left(\beta_1, \beta_2|\underline{z}\right) \times \pi^*(\beta_1,\ \beta_2)}{\int_{\beta_1}\int_{\beta_2} L\left(\beta_1, \beta_2|\underline{z}\right) \times \pi^*(\beta_1,\ \beta_2)}.\\ &= \left(\frac{B^A}{\Gamma(A)}\beta_1^{A-1}\exp(-B\beta_1)\right)\left(\frac{D^C}{\Gamma(C)}\beta_2^{C-1}\exp(-D\beta_2)\right),\end{aligned} \tag{24}$$

where

$$\begin{aligned}A &= a + M_m,\ B = b + \sum_{i=1}^{m}\xi_i z_i^2 + M_m z_m^2,\ \text{and}\\ C &= c + m - M_m,\ D = d + \sum_{i=1}^{m}\left(1 - \xi_i\right) z_i^2 + (m - M_m)\beta_2 z_m^2.\end{aligned} \tag{25}$$

In Bayesian estimation procedure, we can adopt different type of loss functions. The most popular loss function is symmetric SEL ⟨square error loss⟩ function which widely employed in inference. In some situations where over and under estimation can lead to different consequences. The SEL function equally penalizes over- and under-estimation of the same magnitude. A useful asymmetric loss function known can be applied the LINEX function. In the flowering, we adopt the symmetric SEL function and asymmetric LINEX function

4.1. Bayesian estimation under a SEL function

From the posterior distribution (24), SEL function and the original jointly type-II censoring sample, the Bayes estimator for the parameter β_1 and β_2, can be derived as

$$\hat{\beta}_{1\mathrm{B(SEL)}} = \frac{a + M_m}{b + \sum_{i=1}^{m}\xi_i z_i^2 + M_m z_m^2}, \tag{26}$$

and

$$\hat{\beta}_{2B(SEL)} = \frac{c + m - M_m}{d + \sum_{i=1}^{m} \left(1 - \xi_i\right) z_i^2 + (m - M_m)\beta_2 z_m^2}. \tag{27}$$

4.2. Bayesian estimation under a LINEX function

From the posterior distribution (24), LINEX function and the original jointly type-II censoring sample, Bayes estimator $\theta_{B(LINEX)}$ of θ under the LINEX function is given by

$$\hat{\theta}_{B(LINEX)} = -\frac{1}{\delta} \log(E_\theta[\exp(-\delta\theta)]), \tag{28}$$

then the Bayes estimator for the parameter β_1 and β_2, can be derived as

$$\hat{\beta}_{1B(LINEX)} = \frac{-(a + M_m)}{\delta} \log\left(\frac{b + \sum_{i=1}^{m} \xi_i z_i^2 + M_m z_m^2}{\delta + b + \sum_{i=1}^{m} \xi_i z_i^2 + M_m z_m^2}\right), \tag{29}$$

and

$$\hat{\beta}_{2B(LINEX)} = \frac{-(c + m - M_m)}{\delta} \log\left(\frac{d + \sum_{i=1}^{m} \left(1 - \xi_i\right) z_i^2 + (m - M_m)\beta_2 z_m^2.}{\delta + d + \sum_{i=1}^{m} \left(1 - \xi_i\right) z_i^2 + (m - M_m)\beta_2 z_m^2.}\right), \tag{30}$$

4.3. Bayesian probability intervals

The Bayesian probability intervals of a two-sided equitailed $100(1 - 2\alpha)\%$ interval for β_1 and β_2 can be obtained by solving the following equation, for the lower bound, L and upper bound, U

$$\int_0^{L_1} \pi(\beta_1|\underline{z})d\beta_1 = \int_{U_1}^{\infty} \pi(\beta_1|\underline{z})d\beta_1 = \alpha, \tag{31}$$

hence

$$\Gamma(a, bL_1) = \Gamma(a)\,(1 - \alpha) \text{ and } \Gamma(a, bu_1) = \Gamma(a)\alpha \tag{32}$$

where $\Gamma(\omega, \phi)$ is the incomplete gamma function satisfies $\Gamma(\omega, \phi) = \int_\phi^\infty t^{\omega-1} e^{-t} dt$ and $\Gamma(\omega)$ is the gamma function satisfies $\Gamma(\omega) = \int_0^\infty t^{\omega-1} e^{-t} dt$. Also

$$\int_0^{L_2} \pi(\beta_2|\underline{z})d\beta_2 = \int_{U_2}^{\infty} \pi(\beta_2|\underline{z})d\beta_2 = \alpha \tag{33}$$

hence

$$\Gamma(c, dL_2) = \Gamma(c)\,(1-\alpha) \text{ and } \Gamma(c, du_2) = \Gamma(c)\alpha. \tag{34}$$

The analytically solution in this case is not possible then, we need numerical technique for solving these non-linear equations (32) and (34) such as Newton Rafson.

Table 1: RD jointly type-II censored data with β_2=1.2, $\beta_2 = 1.5$ $M = N = 20$ and $m = 20$.

t_i	0.025032	0.341644	0.357557	0.431531	0.566832	0.593807	0.649684	0.688526	0.698121	0.815417
	0.840426	0.921828	0.984228	0.987759	0.998815	1.083400	1.178040	1.197470	1.461300	1.587830
y_i	0.168476	0.212767	0.227479	0.317529	0.354431	0.487878	0.493789	0.631015	0.697599	0.734696
	0.766991	0.769535	0.837876	0.847666	0.989218	1.113050	1.167950	1.241240	1.540880	1.540880
w_i	0.025032	0.168476	0.212767	0.227479	0.317529	0.341644	0.354431	0.357557	0.431531	0.487878
	0.493789	0.566832	0.593807	0.631015	0.649684	0.688526	0.697599	0.698121	0.734696	0.766991
z_i	1	0	0	0	0	1	0	1	1	0
	0	1	1	0	1	1	0	1	0	0

5. Illustrative Example

In this section, jointly Type-II censoring data are simulated from the two RDs to illustrate our obtaining theoretical results. The parameters vectors $(\beta_1,\ \beta_2)$=(1.2, 1.5) and censoring scheme $(M,\ N,\ m) = (20, 20, 20)$. The simulate samples T and Y, then the joint Type-II censoring samples with indicator type z say $(Z,\ \zeta)$ are given in Table 1.

The point estimates based on MLE, bootstrap and Bayes methods are presented in Table 2. Also 95% confidence intervals based on MLE, bootstrap and Bayes methods are presented in Table 3. The informative gamma prior with selected hyper parameters as$\{a$ =3, b =2.5, c =3, $d = 2\}$ are used. In Bayesian approach under LINX loss function different value of δ are used.

6. Simulation Studies

In this section, the obtain theoretical results of point and interval estimation are assessment and compared through simulation studies. The simulation studies are considered with different RD sample sizes and different affect sample size m. The performance of results are measured with different measurement such as average (AVG) and mean square error (MSE) in the case of point estimation, where

$$\text{AVG}\left(\hat{\theta}\right) = \frac{1}{s}\sum_{i=1}^{s}\hat{\theta}^{(i)} \tag{35}$$

$$\text{MSE}\left(\hat{\theta}\right) = \sqrt{\frac{1}{S}\sum_{i=1}^{S}\left(\hat{\theta}^{(i)} - \theta\right)^2}, \tag{36}$$

Table 2: The parameters estimation with differnt methods

	MLE	Boot.	Bayes SE	Bayes $\delta = 1.0$	Bayes $\delta = -1.0$
β_1 =1.2	1.15632	1.16640	1.16690	1.11360	1.2988
β_2 =1.5	1.18056	1.23330	1.23700	1.18540	1.5521

Table 3: Bayesian probability, two bootstrap and asymptotic 95% confidence intervals.

	ML	Boot-p	Boot-t	Bayes
β_1=1.2	(0.4009, 1.9118)	(0.7095, 1.9255)	(0.4281, 1.6182)	(0.3271, 2.5183)
β_2=1.5	(0.4829, 1.8782)	(0.7526, 1.9222)	(0.5093, 1.6361)	(0.4089, 3.1479)

Table 4: The AVG and MSE values of 256 3-10 estimates with diferent methods and the parameters values $\beta_1 = 0.5$ and $\beta_2 = 1.0$

(M, N, m)	Pa.	$(.)_{ML}$		$(.)_{b\text{-}p}$		$(.)_{\frac{Bayes}{SE}}$		$(.)_{\frac{Bayes}{\delta=1.0}}$		$(.)_{\frac{Bayes}{\delta=1.0}}$	
		AVGs	MSEs	AVGs	MSEs	AVGs	MSEs	AVGs	MSEs	AVGs	MSEs
(15, 15,10)	β_1	0.528461	0.0731	0.5294	0.0854	0.50197	0.0684	0.5225	0.0621	0.5045	0.0452
	β_2	1.03734	0.1132	1.0589	0.1245	0.9938	0.1010	0.9885	0.0942	1.1008	0.0998
(15, 15,20)	β_1	0.529937	0.0630	0.5221	0.0777	0.4933	0.0613	0.5292	0.0609	0.4927	0.0410
	β_2	1.09485	0.1015	1.0992	0.1187	1.0143	0.0997	1.0019	0.0920	1.1099	0.0899
(25, 25,25)	β_1	0.499199	0.0601	0.4873	0.0695	0.4965	0.0600	0.5232	0.0590	0.5164	0.0400
	β_2	1.1402	0.1002	1.1141	0.1103	1.0815	0.0893	1.1493	0.090	1.0112	0.088
(25, 25,35)	β_1	0.5079	0.0589	0.4802	0.0603	0.4920	0.0579	0.5223	0.0566	0.4950	0.0397
	β_2	1.0386	0.0999	1.1484	0.1003	1.0220	0.0950	1.1382	0.0854	1.0913	0.0824
(40, 40,40)	β_1	0.4564	0.0571	0.4802	0.0587	0.4771	0.0561	0.5202	0.0551	0.4802	0.0380
	β_2	1.1406	0.0947	0.9999	0.0987	1.0573	0.0920	0.9901	0.0801	1.1467	0.0800
(40, 40,60)	β_1	0.5126	0.0557	0.5153	0.0581	0.5132	0.0555	0.4967	0.0542	0.4931	0.0372
	β_2	1.0189	0.0940	1.0989	0.0978	1.1184	0.0911	1.0540	0.0798	0.9972	0.0780

where $\theta = \beta_1$ or β_2 and S is the simulation number. Also in the case of interval estimation presented of the MLEs, the two different bootstrap confidence intervals and probability intervals presented by Bayes method are measured with average of interval widths (AW) and probability coverage (PC). In simulation studies the parameters β_1 and β_2 selected as $(\beta_1, \ \beta_2) = (0.5, 1.0)$. In Bayesian approach the informative gamma priors are selected to satisfies E(θ)=$\frac{a}{b}$ as $(a = 1, \ b = 2.0, \ c = 2.0, \ d = 2.0)$. In all cases, the squared error LINX loss function are used to compute the Bayes estimates. The results of AVG values and MSE of estimates are presented in Table 4. Also the results of CP and the AW of 95% CIs for all the methods presented in Table 5.

Table 5: Diferent values CPs and the AWs of confidence interval with values $\beta_1 = 0.5$ and $\beta_2 = 1.0$

(M, N, m)	Pa.	$(.)_{ML}$		$(.)_{b\text{-}p}$		$(.)_{b\text{-}t}$		$(.)_{Bayes}$	
		CP	AW	CP	AW	CP	AW	CP	AW
(15, 15,10)	β_1	0.90	1.2124	0.89	2.2417	0.92	1.2471	0.92	1.1792
	β_2	0.91	3.9547	0.89	5.2532	0.91	3.1942	0.90	3.1308
(15, 15,20)	β_1	0.91	1.1472	0.90	2.1472	0.93	1.2256	0.93	1.1092
	β_2	0.92	3.7495	0.91	4.0598	0.92	3.1207	0.93	3.1001
(25, 25,25)	β_1	0.94	1.1185	0.93	2.1333	0.97	1.1124	0.93	1.1022
	β_2	0.90	3.5221	0.92	4.0417	0.94	3.1045	0.93	2.9008
(25, 25,35)	β_1	0.92	1.1140	0.92	2.1309	0.96	1.1108	0.95	1.1008
	β_2	0.93	3.5200	0.91	4.0310	0.93	3.1029	0.95	2.8011
(40, 40,40)	β_1	0.93	1.1090	0.93	2.1209	0.95	1.1086	0.95	1.0985
	β_2	0.91	3.5125	0.93	4.0285	0.96	3.1001	0.97	2.7329
(40, 40,60)	β_1	0.92	1.1007	0.94	2.1189	0.96	1.1008	0.95	1.0903
	β_2	0.93	3.5071	0.92	4.0204	0.97	2.9954	0.94	2.7227

7. Concluding Remarks

In the manufactured products, the comparative lifetime experiments are used to determined the relative merits of two competing duration of different life products. So my object of this study is to discuss different estimation problems as MLE, bootstrap and Bayes estimation of unknown parameters of two RD under joint Type-II censoring data. To discussed and assessment obtained results, we are adopted numerical example and simulation study. From these results, we observe the following.

1 Tables 4 and 5, showed that the obtaining results are acceptable for estimation problem of two RD under jointly type II censored samples .

2 Informative Bayesian estimations perform better compared with the MLEs and bootstrap.

3 The MSEs estimators decreases when the effective sample size m increases.

4 Bayesian interval estimation are closed to bootstrap-t confidence interval estimations.

References

[1] Rao U. V. R., Savage I. R. and Sobel M. (1960). Contributions to the theory of rank order statistics: the two-sample censored case. Ann Math Stat, 31, 415–426.

[2] Basu A. P. (1968). On a generalized savage statistic with applications to life testing. Ann Math Stat, 39, 1591–1604.

[3] Johnson R. A. and Mehrotra K. G. (1972). Locally most powerful rank tests for the two-sample problem with censored data. Ann Math Stat, 43, 823–831.

[4] Mehrotra K. G. and Johnson R. A. (1976). Asymptotic sufficiency and asymptotically most powerful tests for the two sample censored situation. Ann Stat, 4, 589–596.

[5] Bhattacharyya G. K. and Mehrotra K. G. (1981). On testing equality of two exponential distributions under combined type-IIcensoring. J Am Stat Assoc, 76, 886–894.

[6] Mehrotra K. G. and Bhattacharyya G. K. (1982). Confidence intervals with jointly type-II censored samples from two exponential distributions. J Am Stat Assoc, 77, 441–446.

[7] Balakrishnan N. and Rasouli A. (2008). Exact likelihood inference for two exponential populations under joint type-II censoring. Comput Stat Data Anal, 52, 2725–2738.

[8] Rasouli A. and Balakrishnan N. (2010). Exact likelihood inference for two exponential populations under joint progressive type-II censoring. Commun Stat Theory Methods, 39, 2172–2191.

[9] Shafaya A. R., Balakrishnanbc N. and Abdel-Atyd Y. (2014). Bayesian inference based on a jointly type-II censored sample from two exponential populations. Journal of Statistical Computation and Simulation, 84, 2427–2440.

[10] Abu-Zinadah H. H. (2017). Statistical inferences with jointly type-II censored samples from two Pareto distributions. Open Phys., 15, 557–565.

[11] Dyer D. D. and Whisenand C. W. (1973). Best linear unbiased estimator of the parameter of the Rayleigh distribution}Part I: small sample theory for censored order statistics. IEEE Transactions on Reliability 1973; 22:27–34.

[12] Dyer D. D. and Whisenand C. W. (1973). Best linear unbiased estimator of the parameter of the Rayleigh distribution}Part II: optimum theory for selected order statistics. IEEE Transactions on Reliability 1973; 22:229–231.

[13] Polovko A. M. (968). Fundamentals of Reliability Theory. Academic Press: New York.

[14] Pak A., Parham G. A. and Saraj M. (2014). Inference for the Rayleigh distribution based on progressive Type-II fuzzy censored data. Journal of Modern Applied Statistical, 13, 287–304.

[15] Polovko A. M. (968). Fundamentals of Reliability Theory. Academic Press: New York.

[16] Harter H. L. and Moore A. H. (1965). Point and interval estimators, based on m order statistics, for the scale parameter of a Weibull population with known shape parameter. Technometrics; 7, 405–422.

[17] Lalitha S and Mishra A. (1996). Modified maximum likelihood estimation for Rayleigh distribution. Communications in Statistics} Theory and Methods, 25, 389–401.

[18] Kong F and Fei H. (1996) Limits theorems for the maximum likelihood estimate under general multiply type II censoring. Annals of the Institute of Statistical Mathematics, 48, 731–755.

[19] Howlader H.A. and Hossain A. (1995). On Bayesian estimation and prediction from Rayleigh based on type II censored data. Communications in Statistics} Theory and Methods, 24, 2249–2259.

[20] Fernandez A. J. (2000). Bayesian inference from type II doubly censored Rayleigh data. Statistics and Probability Letters, 48, 393–399.

[21] AL-Hussaini E. K. and Ahmad A. A. (2003a). On Bayes predictive distributions of generalized order statistics. Metrika, 57, 165–176.

[22] AL-Hussaini E.K. and Ahmad A. A. (3003b). On Bayes interval prediction of future records. Test, 12, 79–99.

[23] Efron B. and Tibshirani R. J. (1993). An introduction to the bootstrap. New York Chapman and Hall.

[24] Davison A. C. and Hinkley D. V. (1997). *Bootstrap Methods and their Applications*, 2^{nd}., Cambridge University Press, Cambridge United Kingdom.

[25] Hall P. (1988). Theoretical comparison of bootstrap confidence intervals. Annals of Statistics, 16, 927–953.

[26] Efron B. (1981). Censored data and bootstrap. *Journal of the American Statistical Association*, 76, 312–319.

[27] Soliman A. A.,Abd Ellah A. H., Abou-Elheggag N. A and Abd-Elmougod G. A. (2011). A simulation-based approach to the study of coefficient of variation of Gompertz distribution under progressive first-failure censoring. *Indian Journal of Pure and Applied Mathematics*, 42, 335 (2011).

[28] Abd-Elmougod G. A., El-Sayed M. A. and Abdel-Rahman E. O. (2015). Coefficient of variation of Topp-Leone distribution under adaptive Type-II progressive censoring scheme: Bayesian and non-Bayesian approach. *Journal of Computational and Theoretical Nanoscience* 12, 1–8.

[29] Metropolis N., Rosenbluth A. W., Rosenbluth M. N., Teller A. H. and E. Teller (1953). Equations of state calculations by fast computing machines, *Journal Chemical Physics*, 21, 1087–1091.