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On Quasi soft sets in soft topology

Evanzalin Ebenanjar P
Department of Mathematics,
Karunya University, Coimbatore, India-641114.

Thangavelu P
Former Professor, Department of Mathematics,
Karunya University, Coimbatore, India-641114.

Abstract

In this paper, a link between general topological space and soft topological space is identified using the parameterized family of topologies induced by the soft topology. The concept of quasi soft set is introduced to characterize certain soft topological concepts using the parameterized family of topologies induced by the soft topology.

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1. Introduction

Molodstov [8] originate soft sets for dealing with uncertainities in many practical problems in science and Engineering. Maji et al. [7], [6] studied the theoretical concepts of soft sets and applied it in decision making problems. Ali et al. [1] presented algebraic operations for soft sets. Shabir et al. [9] studied soft topological spaces. He established the concept of parameterized family of topologies induced by the soft topology. Soft topological spaces are also studied in [2, 4, 10, 11, 13, 14, 15]. The mappings in soft topological spaces were studied in [5] and it is applied in Medical expert system. Soft continuous maps were studied in [14]. The concepts of soft connectedness and soft Hausdroff spaces have been studied in [10]. The aim of this paper is to characterize some soft topological concepts using the analogous concepts of the parameterized family of topologies induced by the soft topology. The basic concepts of soft sets and soft topology are given in section 2. In section 3, the soft concepts, like soft interior, soft closure, soft continuity and soft separation axioms are characterized by using the analogous concepts in the parameterized family of topologies induced by the soft topology.

2. Preliminaries

Throughout this paper X,Y are universal sets and E,K are parameter spaces.

Definition 2.1. ([8]) A pair (F, E) is called a soft set over X where $F : E \to 2^X$ is a function.

$S(X, E)$ denotes the collection of all soft sets over X with parameter space E. We denote (F, E) by $\widetilde{F}$ in which case we write $\widetilde{F} = \{(e, F(e)) : e \in E\}$. In some occasions, we use $\widetilde{F}(e)$ for $F(e)$.

The following terms are defined in [7], For any two soft sets $\widetilde{F}$ and $\widetilde{G}$ in $S(X, E)$, $\widetilde{F}$ is a **soft subset** of $\widetilde{G}$ (in brief $\widetilde{F} \subseteq \widetilde{G}$) if $F(e) \subseteq G(e)$ for all $e \in E$ and $\widetilde{F}$ and $\widetilde{G}$ are **soft equal** if and only if $F(e) = G(e)$ for all $e \in E$. That is $\widetilde{F} = \widetilde{G}$ if $\widetilde{F} \subseteq \widetilde{G}$ and $\widetilde{G} \subseteq \widetilde{F}$. The **soft null** and **soft absolute** sets are defined as $\widetilde{\Phi} = \{(e, \phi) : e \in E\} = \{(e, \Phi(e)) : e \in E\} = (\Phi, E)$. $\widetilde{X} = \{(e, X) : e \in E\} = \{(e, X(e)) : e \in E\} = (X, E)$. The **union of two soft sets** $\widetilde{F}$ and $\widetilde{G}$ is $\widetilde{F} \cup \widetilde{G} = (F \cup G, E)$ where $(F \cup G)(e) = F(e) \cup G(e)$ for all $e \in E$ and the **intersection of two soft sets** $\widetilde{F}$ and $\widetilde{G}$ is $\widetilde{F} \cap \widetilde{G} = (F \cap G, E)$ where $(F \cap G)(e) = F(e) \cap G(e)$ for all $e \in E$. If $\{\widetilde{F_\alpha} : \alpha \in \Delta\}$ is a collection of soft sets in $S(X, E)$ then the arbitrary union and the arbitrary intersection of soft sets are defined as

$$\bigcup\{\widetilde{F_\alpha} : \alpha \in \Delta\} = (\bigcup\{F_\alpha : \alpha \in \Delta\}, E)$$

and

$$\bigcap\{\widetilde{F_\alpha} : \alpha \in \Delta\} = (\bigcap\{F_\alpha : \alpha \in \Delta\}, E)$$

where

$$(\bigcup\{F_\alpha : \alpha \in \Delta\})(e) = \bigcup\{F_\alpha(e) : \alpha \in \Delta\}$$

and

$$(\bigcap\{F_\alpha : \alpha \in \Delta\})(e) = \bigcap\{F_\alpha(e) : \alpha \in \Delta\}, \text{ for all } e \in E.$$

The **complement of a soft set** $\widetilde{F}$ is denoted by $(\widetilde{F})' = (F', E)$ (relative complement in the sense of Ifran Ali et al. ([1]) where $F' : E \to 2^X$ is a mapping given by $F'(e) = X - F(e)$ for all $e \in E$.

It is noteworthy to see that with respect to above complement De Morgan's laws hold for soft sets as stated below.

Lemma 2.2. ([14]) Let I be an arbitrary index set and $\{\widetilde{F_i} : i \in I\} \subseteq S(X, E)$. Then

$$(\cup\{\widetilde{F_i} : i \in I\})' = \cap\{(\widetilde{F_i})' : i \in I\}$$

and

$$(\cap\{\widetilde{F_i} : i \in I\})' = \cup\{(\widetilde{F_i})' : i \in I\}.$$

Definition 2.3. ([9]) Let $\widetilde{\tau}$ be a collection of soft subset of $\widetilde{X}$. Then $\widetilde{\tau}$ is said to be a soft topology on X with parameter space E if

(i) $\widetilde{\Phi}, \widetilde{X} \in \widetilde{\tau}$,

(ii) $\widetilde{\tau}$ is closed under arbitrary union, and

(iii) $\widetilde{\tau}$ is closed under finite intersection.

If $\widetilde{\tau}$ is a soft topology on X with a parameter space E then the triplet $(X, E, \widetilde{\tau})$ is called a soft topological space over X with parameter space E. Identifying (X, E) with $\widetilde{X}$, $(\widetilde{X}, \widetilde{\tau})$ is a soft topological space. The soft open sets and soft closed sets are defined in a usual way.

Lemma 2.4. ([9]) Let $(X, E, \widetilde{\tau})$ be a soft space over X. Then the collection $\widetilde{\tau}_e = \{F(e) : \widetilde{F} \in \widetilde{\tau}\}$ is a topology on X for each $e \in E$.

It is clear that a soft topology on X gives a parameterized collection of topologies on X but the converse is not true.

Definition 2.5. ([5]) Let X and Y be any two universal sets. The functions $p : E \to K$ and $g : X \to Y$ induce the function $(g, p) : S(X, E) \to S(Y, K)$ defined as below:
For each $\widetilde{F}$ in $S(X, E)$ the image $(g, p)(\widetilde{F})$ is defined as,

$$(g, p)(\widetilde{F})(k) = \begin{cases} \cup\{g(F(e)) : e \in p^{-1}(k)\}, & if \quad p^{-1}(k) \neq \phi \\ \phi, & \text{otherwise.} \end{cases}$$

Let $\widetilde{G} \in S(Y, K)$. Then the inverse image of $\widetilde{G}$ under the soft function (g, p) is the soft set over X denoted by $(g, p)^{-1}(\widetilde{G})$, where $(g, p)^{-1}(\widetilde{G})(e) = g^{-1}(G(p(e)))$ for all $e \in E$.

Definition 2.6. ([15]) (g, p) is soft continuous from $(X, E, \widetilde{\tau})$ to $(Y, K, \widetilde{\sigma})$ if $(g, p)^{-1}(\widetilde{G}) \in \widetilde{\tau}$ for every $\widetilde{G} \in \widetilde{\sigma}$.

Definition 2.7. [14] A soft set $\widetilde{F}$ is called a soft point if for the element $e \in E$, $\widetilde{F}(e) \neq \phi$ and $\widetilde{F}(e') = \phi$ for all $e' \in E - \{e\}$.

Definition 2.8. Let $(X, E, \widetilde{\tau})$ be a soft topological space over X. Then $(X, E, \widetilde{\tau})$ is said to be

1. soft T_o-space [9] $x, y \in X$ such that $x \neq y$. If there exist soft open sets $\widetilde{F}$ and $\widetilde{G}$ such that $x \in \widetilde{F}$ and $y \notin \widetilde{F}$ or $y \in \widetilde{G}$ and $x \notin \widetilde{G}$.
2. soft T_1-space [9] $x, y \in X$ such that $x \neq y$. If there exist soft open sets $\widetilde{F}$ and $\widetilde{G}$ such that $x \in \widetilde{F}$ and $y \notin \widetilde{F}$ and $y \in \widetilde{G}$ and $x \notin \widetilde{G}$.
3. soft T_2-space [9] $x, y \in X$ such that $x \neq y$. If there exist soft open sets $\widetilde{F}$ and $\widetilde{G}$ such that $x \in \widetilde{F}$, $y \in \widetilde{G}$ and $\widetilde{F} \cap \widetilde{G} = \widetilde{\Phi}$.
4. soft regular-space [9] $\widetilde{G}$ be a soft closed set in X and $x \in X$ such that $x \notin \widetilde{G}$. If there exist soft open sets $\widetilde{F_1}$ and $\widetilde{F_2}$ such that $x \in \widetilde{F_1}$, $\widetilde{G} \subseteq \widetilde{F_2}$ and $\widetilde{F_1} \cap \widetilde{F_2} = \widetilde{\Phi}$.
5. soft normal-space [9] $\widetilde{F}$ and $\widetilde{G}$ soft closed sets over X such that $\widetilde{F} \cap \widetilde{G} = \widetilde{\Phi}$. If there exist soft open sets $\widetilde{F_1}$ and $\widetilde{F_2}$ such that $\widetilde{F} \subseteq \widetilde{F_1}$, $\widetilde{G} \subseteq \widetilde{F_2}$ and $\widetilde{F_1} \cap \widetilde{F_2} = \widetilde{\Phi}$.

3. Characterizations

3.1. Soft interior operator

In this section, soft open and the soft interior operator are characterized by using the parameterized family of topologies induced by the soft topology.

Proposition 3.1. Let $(X, \widetilde{\tau}, E)$ be a soft topological space over X. Let $\widetilde{F} \in S(X, E)$ and $e \in E$. If $\widetilde{F}$ is soft open in $(X, \widetilde{\tau}, E)$ then $\widetilde{F}(e)$ is open in $(X, \widetilde{\tau}_e)$. Conversely if G is open in $(X, \widetilde{\tau}_e)$ then $G = \widetilde{F}(e)$ is soft open in $(X, \widetilde{\tau}, E)$ for some soft open set $\widetilde{F}$ in $(X, \widetilde{\tau}, E)$.

Proof. Suppose $\widetilde{F} \in \widetilde{\tau}$. Then by using Lemma 2.4, $\widetilde{F}(e) \in \widetilde{\tau}_e$. Conversely suppose $G \in \widetilde{\tau}_e$. Again by using same Lemma 2.4, $G = \widetilde{F}(e)$ for some $\widetilde{F} \in \widetilde{\tau}$. ■

Proposition 3.2. Let $(X, \widetilde{\tau}, E)$ be a soft topological space. Let $\widetilde{F} \in S(X, E)$. Then $(\widetilde{sint}\widetilde{F})(e) \subseteq int(\widetilde{F}(e))$ in $(X, \widetilde{\tau}_e)$ for every $e \in E$.

Proof. By using definition of the soft interior operator we have,

$$(\widetilde{sint}\widetilde{F}) = \bigcup\{\widetilde{H} : \widetilde{H} \subseteq \widetilde{F}, \widetilde{H} \in \widetilde{\tau}\}.$$

That is

$$(\widetilde{sint}\widetilde{F})(e) = \bigcup\{\widetilde{H}(e) : \widetilde{H} \subseteq \widetilde{F}, \widetilde{H} \in \widetilde{\tau}\}.$$

Also

$$int(\widetilde{F}(e)) = \bigcup\{G : G \subseteq \widetilde{F}(e), G \in \widetilde{\tau}_e\}.$$

$\widetilde{H} \subseteq \widetilde{F}, \widetilde{H} \in \widetilde{\tau}$. That implies $\widetilde{H}(e) \subseteq \widetilde{F}(e)$,
$\widetilde{H}(e) \in \widetilde{\tau}_e$. That is $\widetilde{H}(e) \subseteq int(\widetilde{F}(e))$. Therefore $(\widetilde{sint}\widetilde{F})(e) \subseteq int(\widetilde{F}(e))$ in $(X, \widetilde{\tau}_e)$ for every $e \in E$. ■

The inclusion in Proposition 3.2 may be proper as shown in the following examples.

Example 3.3. Let $X = \{x, y, z\}$, $E = \{e_1, e_2, e_3\}$ and

$$\widetilde{\tau} = \{\widetilde{\Phi}, \widetilde{X}, \widetilde{F}_1, \widetilde{F}_2, \widetilde{F}_3, \widetilde{F}_4\}$$

where

$$\widetilde{F}_1 = \{(e_1, \{x\}), (e_2, \{y\}), (e_3, X)\}$$
$$\widetilde{F}_2 = \{(e_1, \{y\}), (e_2, \{x\}), (e_3, X)\}$$
$$\widetilde{F}_3 = \{(e_1, \{x, y\}), (e_2, \{x, y\}), (e_3, X)\}$$
$$\widetilde{F}_4 = \{(e_1, \phi), (e_2, \phi), (e_3, X)\}$$

Then $\widetilde{\tau}$ defines the soft topology on X. Let

$$\widetilde{F} = \{(e_1, \{x, z\}), (e_2, \{x, y\}), (e_3, \{z\})\}$$

Then

$$\widetilde{s}int\widetilde{F} = \widetilde{\Phi}$$
$$(\widetilde{s}int\widetilde{F})(e_1) = \phi;$$
$$(\widetilde{s}int\widetilde{F})(e_2) = \phi;$$
$$(\widetilde{s}int\widetilde{F})(e_3) = \phi.$$

Now

$$\widetilde{\tau}_{e_1} = \{X, \phi, \{x\}, \{y\}, \{x, y\}\}$$
$$\widetilde{\tau}_{e_2} = \{X, \phi, \{x\}, \{y\}, \{x, y\}\}$$
$$\widetilde{\tau}_{e_3} = \{X, \phi\}$$
$$\widetilde{F}(e_1) = \{x, z\};\ \ \widetilde{F}(e_2) = \{x, y\};\ \ \widetilde{F}(e_3) = \{z\}$$

Then

$$int(\widetilde{F}(e_1)) = \{x\}; int(\widetilde{F}(e_2)) = \{x, y\}; int(\widetilde{F}(e_3)) = \phi.$$

It follows that

$$(\widetilde{s}int\widetilde{F})(e_1) \subseteq int(\widetilde{F}(e_1)),$$
$$(\widetilde{s}int\widetilde{F})(e_2) \subseteq int(\widetilde{F}(e_2)),$$
$$(\widetilde{s}int\widetilde{F})(e_3) = int(\widetilde{F}(e_3)).$$

Example 3.4. Let $X = \{x, y, z\}$, $E = \{e_1, e_2, e_3\}$ and $\widetilde{\tau} = \{\widetilde{\Phi}, \widetilde{X}, \widetilde{F}_1, \widetilde{F}_2, \widetilde{F}_3\}$ where

$$\widetilde{F}_1 = \{(e_1, \{x\}), (e_2, \{y\}), (e_3, X)\}$$
$$\widetilde{F}_2 = \{(e_1, \{y\}), (e_2, \{x\}), (e_3, \Phi)\}$$
$$\widetilde{F}_3 = \{(e_1, \{x, y\}), (e_2, \{x, y\}), (e_3, X)\}$$

Then $\widetilde{\tau}$ defines the soft topology on X. Let

$$\widetilde{F} = \{(e_1, \{x, z\}), (e_2, \{y, z\}), (e_3, X)\}$$

Then

$$\widetilde{s}int\widetilde{F} = \{(e_1, \{x\}), (e_2, \{y\}), (e_3, X)\}$$
$$(\widetilde{s}int\widetilde{F})(e_1) = \{x\};$$
$$(\widetilde{s}int\widetilde{F})(e_2) = \{y\};$$
$$(\widetilde{s}int\widetilde{F})(e_3) = X.$$

Now

$$\widetilde{\tau}_{e_1} = \{X, \phi, \{x\}, \{y\}, \{x, y\}\}$$
$$\widetilde{\tau}_{e_2} = \{X, \phi, \{x\}, \{y\}, \{x, y\}\}$$
$$\widetilde{\tau}_{e_3} = \{X, \phi\}$$
$$\widetilde{F}(e_1) = \{x, z\};\ \ \widetilde{F}(e_2) = \{y, z\};\ \ \widetilde{F}(e_3) = X$$

Then

$$int(\widetilde{F}(e_1)) = \{x\};\ int(\widetilde{F}(e_2)) = \{y\};\ int(\widetilde{F}(e_3)) = X.$$

It follows that

$$(\widetilde{s}int\widetilde{F})(e_1) = int(\widetilde{F}(e_1)),$$
$$(\widetilde{s}int\widetilde{F})(e_2) = int(\widetilde{F}(e_2)),$$
$$(\widetilde{s}int\widetilde{F})(e_3) = int(\widetilde{F}(e_3)).$$

Our aim is to find a suitable condition on $\widetilde{\tau}$ to get the equality in Proposition 3.2. For this we need the following definition.

Definition 3.5. Let $A \subseteq X$ and $e \in E$. The soft set $\widetilde{A}_e$ is called a quasi soft set if

$$\widetilde{A}_e(\alpha) = \begin{cases} A & \text{if } \alpha = e \\ \phi & \text{otherwise} \end{cases}$$

Then every soft point is a quasi soft set. The converse is not true. Let $QS(\widetilde{\tau}_e) = \{\widetilde{G}_e : G \in \widetilde{\tau}_e\}$ for every $e \in E$.

Proposition 3.6. Let $(X, \widetilde{\tau}, E)$ be a soft topological space with the condition: $QS(\widetilde{\tau}_e) \subseteq \widetilde{\tau}$ for every $e \in E$. Then $(\widetilde{s}int\widetilde{F})(e) = int(\widetilde{F}(e))$ in $(X, \widetilde{\tau}_e)$.

Proof. By Proposition 3.2, we have $(\widetilde{s}int\widetilde{F})(e) \subseteq int(\widetilde{F}(e))$ in $(X, \widetilde{\tau}_e)$. Now we have to prove the reverse inclusion.

$$int(\widetilde{F}(e)) = \bigcup\{G : G \subseteq \widetilde{F}(e), G \in \widetilde{\tau}_e\}.$$

Let $G \in \widetilde{\tau}_e$, $G \subseteq \widetilde{F}(e)$. Since $QS(\widetilde{\tau}_e) \subseteq \widetilde{\tau}$, $G = \widetilde{G}_e(e) \subseteq \widetilde{F}(e)$. That implies $\widetilde{G} \subseteq \widetilde{F}$, $\widetilde{G} \in \widetilde{\tau}$. That is $\widetilde{G} \subseteq (\widetilde{s}int\widetilde{F})$. Therefore $int(\widetilde{F}(e)) \subseteq (\widetilde{s}int\widetilde{F})(e)$. This proves

$$(\widetilde{s}int\widetilde{F})(e) = int(\widetilde{F}(e)) \text{ in } (X, \widetilde{\tau}_e).$$

■

Hussain and Ahmad [11] discussed the following properties of soft interior operators in soft topological spaces. In this section, we characterize these properties of soft interior operator by using the analogous concepts in the parameterized family of topologies induced by the soft topology with the condition that $\widetilde{\tau}$ always contains $QS(\widetilde{\tau}_e)$.

Proposition 3.7. Let $(X, \widetilde{\tau}, E)$ be a soft topological space over X. Let $\widetilde{F}$ and $\widetilde{G} \in S(X, E)$. Then

(i) $\widetilde{s}int\widetilde{\Phi} = \widetilde{\Phi}$ and $\widetilde{s}int\widetilde{X} = \widetilde{X}$

(ii) $\widetilde{s}int\widetilde{F} \subseteq \widetilde{F}$

(iii) $\widetilde{s}int(\widetilde{s}int(\widetilde{F})) = \widetilde{s}int\widetilde{F}$

(iv) $\widetilde{F} \subseteq \widetilde{G} \Rightarrow \widetilde{s}int\widetilde{F} \subseteq \widetilde{s}int\widetilde{G}$

(v) $\widetilde{s}int\widetilde{F} \cap \widetilde{s}int\widetilde{G} = \widetilde{s}int(\widetilde{F} \cap \widetilde{G})$

(vi) $\widetilde{s}int\widetilde{F} \cup \widetilde{s}int\widetilde{G} \subseteq \widetilde{s}int(\widetilde{F} \cup \widetilde{G})$.

Proof. Let $\widetilde{\Phi}$ be the soft null set. Then

$$[\widetilde{s}int\widetilde{\Phi}](e) = int(\widetilde{\Phi}(e)) = (\widetilde{\Phi}(e)) = (\widetilde{\Phi})(e)$$

by using Proposition 3.6. Therefore $\widetilde{s}int\widetilde{\Phi} = \widetilde{\Phi}$.
Also let $\widetilde{X}$ be the soft absolute set. Then

$$[\widetilde{s}int\widetilde{X}](e) = int(\widetilde{X}(e)) = (\widetilde{X}(e)) = (\widetilde{X})(e)$$

by using Proposition 3.6. Therefore $\widetilde{s}int\widetilde{X} = \widetilde{X}$. This proves (i).

Let $\widetilde{F} \in S(X, E)$. Then $[\widetilde{s}int\widetilde{F}](e) = int(\widetilde{F}(e))$ by using Proposition 3.6. Since $int(\widetilde{F}(e)) \subseteq (\widetilde{F}(e))$, we have $[\widetilde{s}int\widetilde{F}](e) \subseteq (\widetilde{F}(e)) = (\widetilde{F})(e)$. Therefore $\widetilde{s}int\widetilde{F} \subseteq \widetilde{F}$. This proves (ii).

Let $\widetilde{F} \in S(X, E)$. Then $(\widetilde{s}int\ (\widetilde{s}int\ (\widetilde{F})))\ (e) = int\ [(\widetilde{s}int\ (\widetilde{F}))(e)]$ by using Proposition 3.6. Again by using the same proposition we have, $(\widetilde{s}int(\widetilde{s}int(\widetilde{F})))(e) = int\ int\ (\widetilde{F}(e))$. That is equal to $int\ (\widetilde{F}(e))$. That implies $(\widetilde{s}int(\widetilde{s}int(\widetilde{F})))(e) = (\widetilde{s}int\ \widetilde{F})(e)$ for all $e \in E$ by using Proposition 3.6. Therefore $\widetilde{s}int(\widetilde{s}int(\widetilde{F})) = \widetilde{s}int\widetilde{F}$. This proves (iii).

Let $\widetilde{F}$ and $\widetilde{G} \in S(X, E)$. Then $\widetilde{F} \subseteq \widetilde{G}$. That implies $\widetilde{F}(e) \subseteq \widetilde{G}(e)$ for every e. That is $int\ \widetilde{F}(e) \subseteq int\ \widetilde{G}(e)$. That implies $[\widetilde{s}int\ \widetilde{F}](e) \subseteq [\widetilde{s}int\ \widetilde{G}](e)$ by using Proposition 3.6.
That is $\widetilde{s}int\ \widetilde{F} \subseteq \widetilde{s}int\ \widetilde{G}$. This proves (iv).

Let $\widetilde{F}$ and $\widetilde{G} \in S(X, E)$. Then $(\widetilde{s}int(\widetilde{F} \cap \widetilde{G}))(e) = int[(\widetilde{F} \cap \widetilde{G})(e)]$ by using Proposition 3.6. That is equal to $int(\widetilde{F}(e) \cap \widetilde{G}(e)) = int\ \widetilde{F}(e) \cap int\ \widetilde{G}(e)$. Again by using Proposition 3.6 we have, $int\ \widetilde{F}(e) \cap int\ \widetilde{G}(e) = (\widetilde{s}int\ \widetilde{F})(e) \cap (\widetilde{s}int\ \widetilde{G})(e)$ for all $e \in E$. Therefore $\widetilde{s}int\widetilde{F} \cap \widetilde{s}int\widetilde{G} = \widetilde{s}int(\widetilde{F} \cap \widetilde{G})$. This proves (v).

Let $\widetilde{F}$ and $\widetilde{G} \in S(X, E)$. Then $(\widetilde{s}int(\widetilde{F} \cup \widetilde{G}))(e) = int[(\widetilde{F} \cup \widetilde{G})(e)]$ by using Proposition 3.6. That is $int(\widetilde{F}(e) \cup \widetilde{G}(e)) \supseteq int\ \widetilde{F}(e) \cup int\ \widetilde{G}(e)$. That is equal to $(\widetilde{s}int\ \widetilde{F})(e) \cup (\widetilde{s}int\ \widetilde{G})(e)$ for all $e \in E$ by using Proposition 3.6. Therefore we have $\widetilde{s}int\widetilde{F} \cup \widetilde{s}int\widetilde{G} \supseteq \widetilde{s}int(\widetilde{F} \cap \widetilde{G})$. This proves (vi). ■

4. Soft closure operator

In this section, soft closed and the soft closure operator are characterized by using the parameterized family of topologies induced by the soft topology.

Proposition 4.1. Let $(X, \widetilde{\tau}, E)$ be a soft topological space over X. Let $\widetilde{F} \in S(X, E)$ and $e \in E$. If $\widetilde{F}$ is soft closed in $(X, \widetilde{\tau}, E)$ then $\widetilde{F}(e)$ is closed in $(X, \widetilde{\tau}_e)$. Conversely if G is closed in $(X, \widetilde{\tau}_e)$ then $G = \widetilde{F}(e)$ is soft closed in $(X, \widetilde{\tau}, E)$ for some soft closed set $\widetilde{F}$ in $(X, \widetilde{\tau}, E)$.

Proof. Suppose $\widetilde{F}$ is soft closed. Then $(\widetilde{F})'$ is soft open. Then by using Lemma 2.4, $(\widetilde{F})'(e)$ is open in $(X, \widetilde{\tau}_e)$. That is $X - \widetilde{F}(e)$ is open in $(X, \widetilde{\tau}_e)$. That is $\widetilde{F}(e)$ is closed in $(X, \widetilde{\tau}_e)$ for all $e \in E$.
Conversely suppose G is closed in $(X, \widetilde{\tau}_e)$. Then G' is open in $(X, \widetilde{\tau}_e)$. Again by using same Lemma 2.4, $G' = (\widetilde{F})'(e)$ for some $\widetilde{F} \in \widetilde{\tau}$. That is $G = \widetilde{F}(e)$ is soft closed in $(X, \widetilde{\tau}, E)$ for some soft closed set $\widetilde{F}$ in $(X, \widetilde{\tau}, E)$. ■

Proposition 4.2. Let $(X, \widetilde{\tau}, E)$ be a soft topological space. Let $\widetilde{F} \in S(X, E)$. Then $(\widetilde{scl}\widetilde{F})(e) \supseteq cl(\widetilde{F}(e))$ in $(X, \widetilde{\tau}_e)$ for every $e \in E$.

Proof. By using definition of the soft closure operator we have,

$$(\widetilde{scl}\widetilde{F}) = \bigcap\{\widetilde{H} : \widetilde{H} \supseteq \widetilde{F}, (\widetilde{H})' \in \widetilde{\tau}\}.$$

That is

$$(\widetilde{scl}\widetilde{F})(e) = \bigcap\{\widetilde{H}(e) : \widetilde{H} \supseteq \widetilde{F}, (\widetilde{H})' \in \widetilde{\tau}\}.$$

Also

$$cl(\widetilde{F}(e)) = \bigcap\{G : G \supseteq \widetilde{F}(e), G' \in \widetilde{\tau}_e\}.$$

$\widetilde{H} \supseteq \widetilde{F}, (\widetilde{H})' \in \widetilde{\tau}$. That implies $\widetilde{H}(e) \supseteq \widetilde{F}(e), (\widetilde{H})'(e) \in \widetilde{\tau}_e$. That is $\widetilde{H}(e) \supseteq cl(\widetilde{F}(e))$.
Therefore $(\widetilde{scl}\widetilde{F})(e) \supseteq cl(\widetilde{F}(e))$ in $(X, \widetilde{\tau}_e)$ for every $e \in E$. ■

The inclusion in Proposition 4.2 may be proper as shown in the next examples.

Example 4.3. Let $(X, \widetilde{\tau}, E)$ be a soft topological space as in Example 3.3.
Let $\widetilde{F} = \{(e_1, \{x, z\}), (e_2, \{x, y\}), (e_3, \{z\})\}$
Then $\widetilde{scl}\widetilde{F} = \widetilde{X}$
$(\widetilde{scl}\widetilde{F})(e_1) = X$; $(\widetilde{scl}\widetilde{F})(e_2) = X$; $(\widetilde{scl}\widetilde{F})(e_3) = X$.
Now
The closed sets of $\widetilde{\tau}_{e_1} = \{X, \phi, \{y, z\}, \{x, z\}, \{z\}\}$
The closed sets of $\widetilde{\tau}_{e_2} = \{X, \phi, \{y, z\}, \{x, z\}, \{z\}\}$
The closed sets of $\widetilde{\tau}_{e_3} = \{X, \phi\}$
$\widetilde{F}(e_1) = \{x, z\}$; $\widetilde{F}(e_2) = \{x, y\}$; $\widetilde{F}(e_3) = \{z\}$
Then $cl(\widetilde{F}(e_1)) = \{x, z\}$; $cl(\widetilde{F}(e_2)) = X$; $cl(\widetilde{F}(e_3)) = X$.
It follows that $(\widetilde{scl}\widetilde{F})(e_1) \supseteq cl(\widetilde{F}(e_1))$, $(\widetilde{scl}\widetilde{F})(e_2) = cl(\widetilde{F}(e_2))$,
$(\widetilde{scl}\widetilde{F})(e_3) = cl(\widetilde{F}(e_3))$.

Example 4.4. Let $(X, \widetilde{\tau}, E)$ be a soft topological space as in Example 3.4.
Let $\widetilde{F} = \{(e_1, \{x, z\}), (e_2, \{y, z\}), (e_3, X)\}$
Then $\widetilde{scl}\widetilde{F} = \{(e_1, \{x, z\}), (e_2, \{y, z\}), (e_3, X)\}$
$(\widetilde{scl}\widetilde{F})(e_1) = \{x, z\}$; $(\widetilde{scl}\widetilde{F})(e_2) = \{y, z\}$; $(\widetilde{scl}\widetilde{F})(e_3) = X$.
Now

The closed sets of $\widetilde{\tau}_{e_1} = \{X, \phi, \{y, z\}, \{x, z\}, \{z\}\}$
The closed sets of $\widetilde{\tau}_{e_2} = \{X, \phi, \{y, z\}, \{x, z\}, \{x\}\}$
The closed sets of $\widetilde{\tau}_{e_3} = \{X, \phi\}$
$\widetilde{F}(e_1) = \{x, z\};\ \widetilde{F}(e_2) = \{y, z\};\ \widetilde{F}(e_3) = X$
Then $cl(\widetilde{F}(e_1)) = \{x, z\};\ cl(\widetilde{F}(e_2)) = \{y, z\};\ cl(\widetilde{F}(e_3)) = X$.
It follows that $(\widetilde{s}cl\widetilde{F})(e_1) = cl(\widetilde{F}(e_1)), (\widetilde{s}cl\widetilde{F})(e_2) = cl(\widetilde{F}(e_2))$,
$(\widetilde{s}cl\widetilde{F})(e_3) = cl(\widetilde{F}(e_3))$.

Proposition 4.5. Let $(X, \widetilde{\tau}, E)$ be a soft topological space with the condition: $QS(\widetilde{\tau}_e) \subseteq \widetilde{\tau}$ for every $e \in E$. Then $(\widetilde{s}cl\widetilde{F})(e) = cl(\widetilde{F}(e))$.

Proof. By Proposition 4.2, we have $(\widetilde{s}cl\widetilde{F})(e) \supseteq cl(\widetilde{F}(e))$ in $(X, \widetilde{\tau}_e)$. Now we have to prove the reverse inclusion.

$$cl(\widetilde{F}(e)) = \bigcap\{G : G \supseteq \widetilde{F}(e), G' \in \widetilde{\tau}_e\}.$$

Let $G \supseteq \widetilde{F}(e)$ and G be closed in $\widetilde{\tau}_e$. Then $G = X - A$ where $A \in \widetilde{\tau}_e$.
$G = X - A$, $\widetilde{A}_e \in \widetilde{\tau}$, $\widetilde{A}_e(e) = A$.
$\widetilde{A}_e \in \widetilde{\tau}$, $X - \widetilde{A}_e(e) = (\widetilde{A}_e)'(e)$.
Define $\widetilde{G} = (\widetilde{A}_e)'$.

$$\widetilde{G}(\alpha) = X - \widetilde{A}_e(\alpha) = \begin{cases} X - A & \text{for } \alpha = e \\ X - \phi & \text{otherwise.} \end{cases} = \begin{cases} G & \text{for } \alpha = e \\ X & \text{otherwise} \end{cases} \supseteq \widetilde{F}(\alpha)$$

for every α and $\widetilde{G}(e) = G \supseteq \widetilde{F}(e)$.
Then $\widetilde{G} \supseteq \widetilde{F}$ and $\widetilde{G}$ is soft closed in $(X, \widetilde{\tau}, E)$.
Since $\widetilde{G}$ is soft closed. $\widetilde{G} \supseteq \widetilde{s}cl(\widetilde{F})$. That is $\widetilde{G}(e) \supseteq \widetilde{s}cl(\widetilde{F})(e)$. Since this is true for every soft closed set $G \supseteq \widetilde{F}(e)$, $cl(\widetilde{F}(e)) \supseteq (\widetilde{s}cl\widetilde{F})(e)$. This proves $(\widetilde{s}cl\widetilde{F})(e) = cl(\widetilde{F}(e))$. ■

Muhammad Shabir and Munazza Naz [9] discussed the following properties of soft closure operators in soft topological spaces. In this section, we characterize the properties of soft closure operator by using the analogous concepts in the parameterized family of topologies induced by the soft topology with the condition that $\widetilde{\tau}$ always contains $QS(\widetilde{\tau}_e)$.

Proposition 4.6. Let $(X, \widetilde{\tau}, E)$ be a soft topological space over X. Let $\widetilde{F}$ and $\widetilde{G} \in S(X, E)$. Then

(i) $\widetilde{s}cl\widetilde{\Phi} = \widetilde{\Phi}$ and $\widetilde{s}cl\widetilde{X} = \widetilde{X}$

(ii) $\widetilde{F} \subseteq \widetilde{s}cl\widetilde{F}$

(iii) $\widetilde{s}cl(\widetilde{s}cl(\widetilde{F})) = \widetilde{s}cl\widetilde{F}$

(iv) $\widetilde{F} \subseteq \widetilde{G} \Rightarrow \widetilde{s}cl\widetilde{F} \subseteq \widetilde{s}cl\widetilde{G}$

(v) $\widetilde{s}cl\widetilde{F} \cap \widetilde{s}cl\widetilde{G} \supseteq \widetilde{s}cl(\widetilde{F} \cap \widetilde{G})$

(vi) $\widetilde{s}cl\widetilde{F} \cup \widetilde{s}cl\widetilde{G} = \widetilde{s}cl(\widetilde{F} \cup \widetilde{G})$.

Proof. Let $\widetilde{\Phi}$ be soft null set. Then $[\widetilde{s}cl\widetilde{\Phi}](e) = cl(\widetilde{\Phi}(e)) = (\widetilde{\Phi}(e)) = (\widetilde{\Phi})(e)$ by using Proposition 4.5. Therefore $\widetilde{s}cl\widetilde{\Phi} = \widetilde{\Phi}$. Also let $\widetilde{X}$ be soft absolute set. Then by using Proposition 4.5 we have, $[\widetilde{s}cl\widetilde{X}](e) = cl(\widetilde{X}(e)) = (\widetilde{X}(e)) = (\widetilde{X})(e)$. Therefore $\widetilde{s}cl\widetilde{X} = \widetilde{X}$. This proves (i).

Let $\widetilde{F} \in S(X, E)$. Then $[\widetilde{s}cl\widetilde{F}](e) = cl(\widetilde{F}(e))$ by using Proposition 4.5. Since $cl(\widetilde{F}(e)) \supseteq (\widetilde{F}(e))$, we have $[\widetilde{s}cl\widetilde{F}](e) \supseteq (\widetilde{F}(e)) = (\widetilde{F})(e)$. Therefore $\widetilde{s}cl\widetilde{F} \supseteq \widetilde{F}$. This proves (ii).

Let $\widetilde{F} \in S(X, E)$. Then $(\widetilde{s}cl(\widetilde{s}cl(\widetilde{F})))(e) = cl\ [(\widetilde{s}cl(\widetilde{F}))(e)]$ by using Proposition 4.5. Again using the same proposition we have, $(\widetilde{s}cl(\widetilde{s}cl(\widetilde{F})))(e) = cl\ (cl\ (\widetilde{F})(e))$ $= cl\ cl\ (\widetilde{F}(e)) = cl\ (\widetilde{F}(e))$. That is $(\widetilde{s}cl(\widetilde{s}cl(\widetilde{F})))(e) = (\widetilde{s}cl\ \widetilde{F})(e)$ for all $e \in E$ by using Proposition 4.5. Therefore $\widetilde{s}cl(\widetilde{s}cl(\widetilde{F})) = \widetilde{s}cl\widetilde{F}$. This proves (iii).

Let $\widetilde{F}$ and $\widetilde{G} \in S(X, E)$. Then $\widetilde{F} \subseteq \widetilde{G}$ that implies $\widetilde{F}(e) \subseteq \widetilde{G}(e)$ for every e. That is
$cl\ \widetilde{F}(e)\ \subseteq cl\ \widetilde{G}(e)$. That implies $[\widetilde{s}cl\ \widetilde{F}](e)\ \subseteq [\widetilde{s}cl\ \widetilde{G}](e)$ by using Proposition 4.5. That is $\widetilde{s}cl\ \widetilde{F}\ \subseteq \widetilde{s}cl\ \widetilde{G}$. This proves (iv).

Let $\widetilde{F}$ and $\widetilde{G} \in\ S(X, E)$. Then $(\widetilde{s}cl\ (\widetilde{F} \cap \widetilde{G}))\ (e) = cl\ [(\widetilde{F} \cap \widetilde{G})(e)]$ by using Proposition 4.5. That is equal to $cl(\widetilde{F}(e) \cap \widetilde{G}(e)) \supseteq cl\ \widetilde{F}(e) \cap cl\ \widetilde{G}(e) = (\widetilde{s}cl\ \widetilde{F})(e) \cap (\widetilde{s}cl\ \widetilde{G})(e)$ for all $e \in E$ by using Proposition 4.5. Therefore $\widetilde{s}int\widetilde{F} \cap \widetilde{s}int\widetilde{G} \supseteq \widetilde{s}int(\widetilde{F} \cap \widetilde{G})$. This proves (v).

Let $\widetilde{F}$ and $\widetilde{G} \in\ S(X, E)$. Then $(\widetilde{s}cl\ (\widetilde{F} \cup \widetilde{G}))\ (e) = cl\ [(\widetilde{F} \cup \widetilde{G})(e)]$ by using Proposition 4.5. That is equal to $cl(\widetilde{F}(e) \cup \widetilde{G}(e)) = cl\ \widetilde{F}(e) \cup cl\ \widetilde{G}(e) = (\widetilde{s}cl\ \widetilde{F})(e) \cup (\widetilde{s}cl\ \widetilde{G})(e)$ for all $e \in E$ by using Proposition 4.5. Therefore $\widetilde{s}cl\widetilde{F} \cup \widetilde{s}cl\widetilde{G} = \widetilde{s}cl(\widetilde{F} \cap \widetilde{G})$. This proves (vi). ■

5. Soft continuity

Zorlutuna, Akdag, Min and Atmaca [14] studied about soft continuous functions in soft topological spaces. In this section, we characterize this using the analogous concepts in the parameterized family of topologies induced by the soft topology.

The standard soft topology in R: Let σ denote the standard topology in R, the set of real numbers and $E = \{0\}$ be the parameter space. Define $\widetilde{\sigma} = \{\widetilde{G} : G \in \sigma\}$ where $\widetilde{G}(0) = G$. It is easy to see that $\widetilde{\sigma}$ is a soft topology on R and $(R, \widetilde{\sigma}, \{0\})$ is a soft topological space. $\widetilde{\sigma}$ is called the standard soft topology on R.

The next proposition shows that $\widetilde{\tau}$ is induced by τ.

Proposition 5.1. For a given parameter space E every topological space (X, τ) induces a soft topological space $(X, \widetilde{\tau}, E)$.

Proof. Suppose (X, τ) is a topological space. Let $O \in \tau$. Let E be a parameter set. Define $\widetilde{O}(e) = O$ for every $e \in E$. Then $\widetilde{\tau} = \{\widetilde{O} : O \in \tau\}$ is a soft topology over X.

For

(i) $\phi \in \tau$, $\widetilde{\Phi}(e) = \phi$ for every $e \in E$ and $X \in \tau$, $\widetilde{X}(e) = X$ for every $e \in E$.

(ii) Let $\widetilde{O}_1, \widetilde{O}_2 \in \widetilde{\tau}$ then $(\widetilde{O}_1 \cap \widetilde{O}_2)(e) = \widetilde{O}_1(e) \cap \widetilde{O}_2(e) = O_1 \cap O_2 = \widetilde{O_1 \cap O_2}(e)$. Therefore $\widetilde{O}_1 \cap \widetilde{O}_2 \in \widetilde{\tau}$.

(iii) Let $\{\widetilde{O}_i \in \widetilde{\tau}\}$ be a family of soft sets in $\widetilde{\tau}$. Let $O = \{\cup O_i : i \in \Delta\}$. $\widetilde{O}(e) = \{\cup \widetilde{O}_i(e) : i \in \Delta\} = \{\cup O_i : i \in \Delta\} = O$ for every $e \in E$. since $O \in \tau$, $\widetilde{O} = \{\cup \widetilde{O}_i : i \in \Delta\} \in \widetilde{\tau}$. Therefore $(X, \widetilde{\tau}, E)$ is a soft topological space. ■

Proposition 5.2. If (g, p) is soft continuous from $(X, \widetilde{\tau}, E)$ to $(R, \widetilde{\sigma}, \{0\})$ then g is continuous from $(X, \widetilde{\tau}_e)$ to (R, σ) , where $p(e) = 0$ for all e.

Proof. Suppose (g, p) is soft continuous from $(X, \widetilde{\tau}, E)$ to $(R, \widetilde{\sigma}, \{0\})$. That is $(g, p)^{-1}(\widetilde{G})$ is soft open in $(X, \widetilde{\tau}, E)$ for every soft open set $\widetilde{G}$ in $(R, \widetilde{\sigma}, \{0\})$. By using Proposition 3.1, that implies $(g, p)^{-1}(\widetilde{G})(e)$ is open in $(X, \widetilde{\tau}_e)$ for every e whenever $\widetilde{G}(p(e))$ is open in (R, σ). That is $g^{-1}(\widetilde{G}(p(e)))$ is open in $(X, \widetilde{\tau}_e)$ for every e whenever $\widetilde{G}(p(e))$ is open in (R, σ) for every e. That implies $g^{-1}(\widetilde{G}(0))$ is open in $(X, \widetilde{\tau}_e)$ for every e whenever $\widetilde{G}(0)$ is open in (R, σ) for every e. Therefore g is continuous from $(X, \widetilde{\tau}_e)$ to (R, σ) for every e. ■

The following example shows that the converse of the above proposition is not true.

Example 5.3. Let $X = R$, $E = \{e_1, e_2\}$ and $A \subseteq X$. $\widetilde{\tau} = \{\widetilde{\Phi}, \widetilde{X}, \widetilde{F}_1, \widetilde{F}_2\}$ where $\widetilde{F}_1$ and $\widetilde{F}_2$ are soft sets over X, defined as follows
$\widetilde{F}_1 = \{(e_1, A), (e_2, X - A)\}$
$\widetilde{F}_2 = \{(e_1, X - A), (e_2, A)\}$. Then $(X, \widetilde{\tau}, E)$ is a soft topological space over X. It can be easily seen that
$\widetilde{\tau}_{e_1} = \{\phi, X, A, X - A\}$
$\widetilde{\tau}_{e_2} = \{\phi, X, X - A, A\}$
Define $g : X \to R$ by

$$g(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$$

let G be an open set in R.

$$g^{-1}(G) = \begin{cases} X & \text{if } 0 \in G, 1 \in G \\ A & \text{if } 0 \notin G, 1 \in G \\ X - A & \text{if } 0 \in G, 1 \notin G \\ \phi & \text{if } 0 \notin G, 1 \notin G \end{cases}$$

Therefore g is continuous from $(X, \widetilde{\tau}_e)$ to R. Now

$$(g, p)^{-1}(\widetilde{G}(e)) = g^{-1}(G(p(e))) = g^{-1}(\widetilde{G}(0)) = g^{-1}(G)$$

Therefore,

$$(g, p)^{-1}(\widetilde{G}) = \{(e_1, X), (e_2, X)\}$$

or
$\{(e_1, A), (e_2, A)\}$
or
$\{(e_1, X - A), (e_2, X - A)\}$
or $\{(e_1, \phi), (e_2, \phi)\}$
If we choose a soft open set $\widetilde{G}$ in R such that $0 \in G$, $1 \notin G$ then
$(g, p)^{-1}(\widetilde{G}) = \{(e_1, (X - A)), (e_2, X - A)\}$ is not soft open. Therefore $(g, p)^{-1}(\widetilde{G})$ is not soft open. Therefore (g, p) is not soft continuous.

Proposition 5.4. If $(g, p) : S(X, E) \to S(Y, K)$ is soft continuous from $(X, \widetilde{\tau}, E)$ to $(Y, \widetilde{\sigma}, K)$ then $g : X \to Y$ is continuous from $(X, \widetilde{\tau}_e)$ to $(Y, \widetilde{\sigma}_{p(e)})$ for all $e \in E$.

Proof. Suppose (g, p) is soft continuous from $(X, \widetilde{\tau}, E)$ to $(Y, \widetilde{\sigma}, K)$. Fix $e \in E$. Let V be open in $(Y, \widetilde{\sigma}_{p(e)})$. Then $V = \widetilde{G}(p(e)) \in \widetilde{\sigma}_{p(e)}$ where $\widetilde{G} \in \widetilde{\sigma}$. Since $(g, p)^{-1}(\widetilde{G})$ is soft open in $(X, \widetilde{\tau}, E)$ then by using Proposition 3.1 we have, $((g, p)^{-1}(\widetilde{G}))(e)$ is open in $\widetilde{\tau}_e$. That implies $g^{-1}(\widetilde{G}(p(e)))$ is open in $\widetilde{\tau}_e$. That is $g^{-1}(V)$ is open in $\widetilde{\tau}_e$. Therefore g is continuous from $(X, \widetilde{\tau}_e)$ to $(Y, \widetilde{\sigma}_{p(e)})$ for all $e \in E$. ∎

The next example shows that the converse of the above proposition is not true.

Example 5.5. Let $X = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2\}$ and

$$\widetilde{\tau} = \{\widetilde{\Phi}, \widetilde{X}, \widetilde{F}_1, \widetilde{F}_2, \widetilde{F}_3, \widetilde{F}_4, \widetilde{F}_5\}$$

where

$$\widetilde{F}_1, \widetilde{F}_2, \widetilde{F}_3, \widetilde{F}_4, \widetilde{F}_5$$

are soft sets over X, defined as follows

$$\widetilde{F}_1 = \{(e_1, \{x_2\}), (e_2, \{x_1\})\}$$
$$\widetilde{F}_2 = \{(e_1, \{x_2, x_3\}), (e_2, \{x_1, x_2\})\}$$
$$\widetilde{F}_3 = \{(e_1, \{x_1, x_2\}), (e_2, \widetilde{X})\}$$
$$\widetilde{F}_4 = \{(e_1, \{x_1, x_2\}), (e_2, \{x_1, x_3\})\}$$
$$\widetilde{F}_5 = \{(e_1, \{x_2\}), (e_2, \{x_1, x_2\})\}$$

$\widetilde{\tau}$ defines a soft topology on X and hence $(X, \widetilde{\tau}, E)$ is a soft topological space over X. It can be easily seen that

$$\widetilde{\tau}_{e_1} = \{\phi, X, \{x_2\}, \{x_2, x_3\}, \{x_1, x_2\}\}$$
$$\widetilde{\tau}_{e_2} = \{\phi, X, \{x_1\}, \{x_1, x_2\}, \{x_1, x_3\}\}$$

Let $Y = \{y_1, y_2, y_3\}$, $K = \{k_1, k_2\}$ and $\widetilde{\sigma} = \{\widetilde{\Phi}, \widetilde{Y}, \widetilde{G}\}$ where

$$\widetilde{G} = \{(k_1, \{y_1, y_2\}), (k_2, \{y_1, y_2\})\}$$

Then $(Y, \widetilde{\sigma}, K)$ is a soft topological space. It can be easily seen that

$$\widetilde{\sigma}_{k_1} = \{\phi, Y, \{y_1, y_2\}\}$$
$$\widetilde{\sigma}_{k_2} = \{\phi, Y, \{y_1, y_2\}\}$$

Define $g : X \to Y$ as
$g(x_1) = y_1$; $g(x_2) = y_2$; $g(x_3) = y_3$. and $p : E \to K$ by
$p(e_1) = k_1$; $p(e_2) = k_2$. Therefore $\widetilde{\sigma}_{p(e_1)} = \widetilde{\sigma}_{k_1}$; $\widetilde{\sigma}_{p(e_2)} = \widetilde{\sigma}_{k_2}$.
Then $g : (X, \widetilde{\tau}_{e_1}) \to (Y, \widetilde{\sigma}_{p(e_1)})$ and $g : (X, \widetilde{\tau}_{e_2}) \to (Y, \widetilde{\sigma}_{p(e_2)})$ are continuous because $g^{-1}(\{y_1, y_2\}) = \{x_1, x_2\}$ is open in $\widetilde{\tau}_{e_1}$ and $\widetilde{\tau}_{e_2}$. But $(g, p)^{-1}(\widetilde{G}) = \{(e_1, \{x_1, x_2\}), (e_2, \{x_1, x_2\})\}$ is not soft open in $\widetilde{\tau}$. Therefore (g, p) is not soft continuous from $(X, \widetilde{\tau}, E)$ to $(Y, K, \widetilde{\sigma})$.

6. Soft separation axioms

Muhammad Shabir and Munazza Naz [9] studied soft separation axioms. In this section, we characterize soft separation axioms by using the analogous concepts in the parameterized family of topologies induced by the soft topology.

Suppose $(X, \widetilde{\tau}, E)$ is a soft topological space. Then $\{(X, \widetilde{\tau}_e) : e \in E\}$ is the parameterized family of topological spaces induced by the soft topology $\widetilde{\tau}$. Let $\widetilde{\tau}_E$ be the soft topology generated by $\bigcup_{e \in E} \widetilde{\tau}_e$ that is generated by $\{\widetilde{F}(e) : \widetilde{F} \in \widetilde{\tau}, e \in E\}$. This topology is finer than $\widetilde{\tau}_e$ for all e.

Proposition 6.1. Let $(X, \widetilde{\tau}, E)$ be a soft topological space. If $(X, \widetilde{\tau}, E)$ is a soft T_0-space then $(X, \widetilde{\tau}_E)$ is a T_0-space.

Proof. Let $(X, \widetilde{\tau}, E)$ is a soft T_0-space. Then by using Definition 2.8(i) we have, for $x, y \,\widetilde{\in}\, X$ with $x \neq y$, there exists soft open sets $\widetilde{F}$ and $\widetilde{G}$ such that ($x \in \widetilde{F}$ and $y \,\widetilde{\notin}\, \widetilde{F}$) or ($y \in \widetilde{G}$ and $x \,\widetilde{\in}\, \widetilde{G}$). That is ($x \in \widetilde{F}(e)$ for all e and $y \notin \widetilde{F}(e)$ for some e) or ($y \in \widetilde{G}(e)$ for all e and $x \notin \widetilde{G}(e)$ for some e). Therefore there exist e_1, e_2 in E with (($x \in \widetilde{F}(e_1)$) and ($y \notin \widetilde{F}(e_1)$)) or (($y \in \widetilde{G}(e_2)$) and ($x \notin \widetilde{G}(e_2)$)). Since $\widetilde{F}(e_1)$ and $\widetilde{G}(e_2)$ are open sets in $\widetilde{\tau}_E$, Then $(X, \widetilde{\tau}_E)$ is a T_0-space. ■

The converse of the above proposition is not true as proved in the following example.

Example 6.2. Let $X = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2\}$ and $\widetilde{\tau} = \{\widetilde{\Phi}, \widetilde{X}, \widetilde{F}_1, \widetilde{F}_2, \widetilde{F}_3, \widetilde{F}_4, \widetilde{F}_5, \widetilde{F}_6, \widetilde{F}_7\}$

where $\widetilde{F}_1, \widetilde{F}_2, \widetilde{F}_3, \widetilde{F}_4, \widetilde{F}_5, \widetilde{F}_6, \widetilde{F}_7$ are soft sets over X, defined as follows

$$\begin{aligned}
\widetilde{F}_1 &= \{(e_1, \{x_1\}), (e_2, \phi)\}\\
\widetilde{F}_2 &= \{(e_1, \phi), (e_2, \{x_2\})\}\\
\widetilde{F}_3 &= \{(e_1, \{x_3\}), (e_2, \phi)\}\\
\widetilde{F}_4 &= \{(e_1, \{x_1\}), (e_2, \{x_2\})\}\\
\widetilde{F}_5 &= \{(e_1, \{x_3\}), (e_2, \{x_2\})\}\\
\widetilde{F}_6 &= \{(e_1, \{x_1, x_3\}), (e_2, \phi)\}\\
\widetilde{F}_7 &= \{(e_1, \{x_1, x_3\}), (e_2, \{x_2\})\}
\end{aligned}$$

Then $(X, \widetilde{\tau}, E)$ is a soft topological space over X. We have

$$\begin{aligned}
\widetilde{\tau}_{e_1} &= \{\phi, X, \{x_1\}, \{x_3\}, \{x_1, x_3\}\}\\
\widetilde{\tau}_{e_2} &= \{\phi, X, \{x_2\}\}\\
\widetilde{\tau}_E &= \{\phi, X, \{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_2\}, \{x_2, x_3\}, \{x_1, x_3\}\}
\end{aligned}$$

Clearly $(X, \widetilde{\tau}_E)$ is a T_0-space, but $(X, \widetilde{\tau}, E)$ is not a soft T_0-space.

Proposition 6.3. Let $(X, \widetilde{\tau}, E)$ be a soft topological space. If $(X, \widetilde{\tau}, E)$ is a soft T_1-space then $(X, \widetilde{\tau}_E)$ is a T_1-space.

Proof. Let $(X, \widetilde{\tau}, E)$ is a soft T_1-space. Then by using Definition 2.8(ii), we have for $x, y \in X$ with $x \neq y$, there exists soft open sets $\widetilde{F}$ and $\widetilde{G}$ such that $(x \in \widetilde{F}$ and $y \notin \widetilde{F})$ and $(y \in \widetilde{G}$ and $x \notin \widetilde{G})$. That is $(x \in \widetilde{F}(e)$ for all e and $y \notin \widetilde{F}(e)$ for some $e)$ and $(y \in \widetilde{G}(e)$ for all e and $x \notin \widetilde{G}(e)$ for some $e)$. Therefore there exists e_1, e_2 in E with $(x \in \widetilde{F}(e_1)$ and $y \notin \widetilde{F}(e_1))$ and $(y \in \widetilde{G}(e_2)$ and $x \notin \widetilde{G}(e_2))$. Since $\widetilde{F}(e_1)$ and $\widetilde{G}(e_2)$ are open sets in $\widetilde{\tau}_E$, Then $(X, \widetilde{\tau}_E)$ is a T_1-space. ■

The following example proves that the converse of the above proposition is not true.

Example 6.4. Let $X = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2\}$ and $\widetilde{\tau} = \{\widetilde{\Phi}, \widetilde{X}, \widetilde{F}_1, \widetilde{F}_2, \widetilde{F}_3\}$ where $\widetilde{F}_1, \widetilde{F}_2, \widetilde{F}_3$ are soft sets over X, defined as follows

$$\begin{aligned}
\widetilde{F}_1 &= \{(e_1, \{x_1\}), (e_2, \{x_3\})\}\\
\widetilde{F}_2 &= \{(e_1, \{x_2\}), (e_2, \{x_1, x_2\})\}\\
\widetilde{F}_3 &= \{(e_1, \{x_1, x_2\}), (e_2, \widetilde{X})\}
\end{aligned}$$

Then $(X, \widetilde{\tau}, E)$ is a soft topological space over X. We have

$$\begin{aligned}
\widetilde{\tau}_{e_1} &= \{\phi, X, \{x_1\}, \{x_2\}, \{x_1, x_2\}\}\\
\widetilde{\tau}_{e_2} &= \{\phi, X, \{x_3\}, \{x_1, x_2\}\}\\
\widetilde{\tau}_E &= \{\phi, X, \{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_2\}, \{x_2, x_3\}, \{x_1, x_3\}\}
\end{aligned}$$

Clearly $(X, \widetilde{\tau}_E)$ is a T_1-space, but $(X, \widetilde{\tau}, E)$ is not a soft T_1-space.

Proposition 6.5. Let $(X, \widetilde{\tau}, E)$ be a soft topological space. If $(X, \widetilde{\tau}, E)$ is a soft T_2-space then $(X, \widetilde{\tau}_e)$ is a T_2-space for every e.

Proof. Let $(X, \widetilde{\tau}, E)$ is a soft T_2-space. Then by using Definition 2.8(iii), we have for $x, y \in X$ with $x \neq y$, there exists soft open sets $\widetilde{F}$ and $\widetilde{G}$ such that $x \in \widetilde{F}$, $y \in \widetilde{G}$ and $\widetilde{F} \cap \widetilde{G} = \widetilde{\Phi}$. That is $x \in \widetilde{F}(e)$, $y \in \widetilde{G}(e)$ and $\widetilde{F}(e) \cap \widetilde{G}(e) = \phi$ for all $e \in E$. Since $\widetilde{F}(e)$ and $\widetilde{G}(e)$ are open sets in $\widetilde{\tau}_e$, then $(X, \widetilde{\tau}_e)$ is a T_2-space. ■

Proposition 6.6. Let $(X, \widetilde{\tau}, E)$ be a soft topological space. If $(X, \widetilde{\tau}, E)$ is soft regular then $(X, \widetilde{\tau}_e)$ is regular for every e.

Proof. Let $(X, \widetilde{\tau}, E)$ is soft regular. Then by using Definition 2.8(iv), we have for $x \in X$, $\widetilde{G}$ is soft closed in X with $x \neq \widetilde{G}$, there exists soft open sets $\widetilde{F_1}$ and $\widetilde{F_2}$ such that $x \in \widetilde{F_1}$, $\widetilde{G} \subseteq \widetilde{F_2}$ and $\widetilde{F_1} \cap \widetilde{F_2} = \widetilde{\Phi}$. That is $x \in \widetilde{F}(e)$, $\widetilde{G}(e) \subseteq \widetilde{F_2}(e)$ and $\widetilde{F_1}(e) \cap \widetilde{F_2}(e) = \phi$ for all $e \in E$. Since $\widetilde{F_1}(e)$ and $\widetilde{F_2}(e)$ are open sets in $\widetilde{\tau}_e$, then $(X, \widetilde{\tau}_e)$ is regular. ■

The converses of the above 2 propositions are not true as shown in the following example.

Example 6.7. Let $X = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2\}$ and $\widetilde{\tau} = \{\widetilde{\Phi}, \widetilde{X}, \widetilde{F_1}, \widetilde{F_2}, \widetilde{F_3}, \widetilde{F_4}, \widetilde{F_5}, \widetilde{F_6}\}$ where $\widetilde{F_1}, \widetilde{F_2}, \widetilde{F_3}, \widetilde{F_4}, \widetilde{F_5}, \widetilde{F_6}$ are soft sets over X, defined as follows

$$\widetilde{F_1} = \{(e_1, \{x_1\}), (e_2, \{x_2\})\}$$
$$\widetilde{F_2} = \{(e_1, \{x_2\}), (e_2, \{x_3\})\}$$
$$\widetilde{F_3} = \{(e_1, \{x_3\}), (e_2, \{x_1\})\}$$
$$\widetilde{F_4} = \{(e_1, \{x_1, x_2\}), (e_2, \{x_2, x_3\})\}$$
$$\widetilde{F_5} = \{(e_1, \{x_1, x_3\}), (e_2, \{x_1, x_2\})\}$$
$$\widetilde{F_6} = \{(e_1, \{x_2, x_3\}), (e_2, \{x_1, x_3\})\}$$

Then $(X, \widetilde{\tau}, E)$ is a soft topological space over X. We have

$$\widetilde{\tau}_{e_1} = \{\phi, X, \{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_2\}, \{x_1, x_3\}, \{x_2, x_3\}\}$$
$$\widetilde{\tau}_{e_2} = \{\phi, X, \{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_2\}, \{x_1, x_3\}, \{x_2, x_3\}\}$$

For x_1, x_3 in X, $\widetilde{F_5}$, $\widetilde{F_6}$ and $\widetilde{X}$ are the only soft open sets with $x_1 \in \widetilde{F_5}$, $x_3 \in \widetilde{F_6}$, $x_1, x_3 \in \widetilde{X}$. Since $\widetilde{F_5} \cap \widetilde{F_6} = \widetilde{F_3} \neq \widetilde{\Phi}$. Therefore $(X, \widetilde{\tau}, E)$ is not soft T_2. Now $x_1 \notin (\widetilde{F_6})' = \widetilde{F_1}$. $\widetilde{F_5}$ and $\widetilde{X}$ are the only soft open sets with $x_1 \in \widetilde{F_5}$ and $x_1 \in \widetilde{X}$. Since $\widetilde{F_5} \cap \widetilde{F_1} = \widetilde{F_1} \neq \widetilde{\Phi}$. Therefore $(X, \widetilde{\tau}, E)$ is not soft regular. Clearly $(X, \widetilde{\tau}_e)$ is T_2 and regular but $(X, \widetilde{\tau}, E)$ is not soft T_2 and not soft regular.

Proposition 6.8. Let $(X, \widetilde{\tau}, E)$ be a soft topological space. If $(X, \widetilde{\tau}, E)$ is soft normal then $(X, \widetilde{\tau}_e)$ is normal for every e.

Proof. Let $(X, \widetilde{\tau}, E)$ is soft normal. Then by using Definition 2.8(v), we have $\widetilde{F}$ and $\widetilde{G}$ are soft closed sets over X with $\widetilde{F} \cap \widetilde{G} = \widetilde{\Phi}$ there exists soft open sets $\widetilde{U}$ and $\widetilde{V}$ such that $\widetilde{F} \subseteq \widetilde{U}$, $\widetilde{G} \subseteq \widetilde{V}$ and $\widetilde{U} \cap \widetilde{V} = \widetilde{\Phi}$. That is $\widetilde{F}(e) \subseteq \widetilde{U}(e)$, $\widetilde{G}(e) \subseteq \widetilde{V}(e)$ and $\widetilde{U}(e) \cap \widetilde{V}(e) = \phi$. Since $\widetilde{F}(e)$ and $\widetilde{G}(e)$ are closed sets over X with $\widetilde{F}(e) \cap \widetilde{G}(e) = \phi$ and $\widetilde{U}(e)$ and $\widetilde{V}(e)$ are open sets in $(X, \widetilde{\tau}_e)$, then $(X, \widetilde{\tau}_e)$ is normal for every $e \in E$. ■

The next example shows that the converse of the above proposition is not true.

Example 6.9. Let $X = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2\}$ and $\widetilde{\tau} = \{\widetilde{\Phi}, \widetilde{X}, \widetilde{F}_1, \widetilde{F}_2, \widetilde{F}_3, \widetilde{F}_4, \widetilde{F}_5, \widetilde{F}_6\}$ are soft sets over X, defined as follows

$$\begin{aligned}
\widetilde{F}_1 &= \{(e_1, \{x_1, x_2\}), (e_2, \widetilde{X})\},\\
\widetilde{F}_2 &= \{(e_1, \{x_1, x_3\}), (e_2, \widetilde{X})\},\\
\widetilde{F}_3 &= \{(e_1, \{x_1\}), (e_2, \widetilde{X})\},\\
\widetilde{F}_4 &= \{(e_1, \{x_2, x_3\}), (e_2, \widetilde{X})\}\\
\widetilde{F}_5 &= \{(e_1, \{x_2\}), (e_2, \widetilde{X})\}\\
\widetilde{F}_6 &= \{(e_1, \{x_3\}), (e_2, \widetilde{X})\} \text{ and}
\end{aligned}$$

Then $(X, \widetilde{\tau}, E)$ is a soft topological space over X. We have

$$\begin{aligned}
\widetilde{\tau}_{e_1} &= \{\phi, X, \{x_1\}, \{x_2\}, \{x_3\}, \{x_2, x_3\}, \{x_1, x_2\}, \{x_1, x_3\}\}\\
\widetilde{\tau}_{e_2} &= \{\phi, X\}
\end{aligned}$$

For $(\widetilde{F}_3)'$ and $(\widetilde{F}_4)'$ are soft closed sets in X such that $(\widetilde{F}_3)' \cap (\widetilde{F}_4)' = \Phi$ but there do not exist soft open set $\widetilde{F}$ and $\widetilde{G}$ in X such that $(\widetilde{F}_3)' \subseteq \widetilde{F}$ and $(\widetilde{F}_4)' \subseteq \widetilde{G}$ and $\widetilde{F} \cap \widetilde{G} = \widetilde{\Phi}$. Therefore $(X, \widetilde{\tau}, E)$ is not soft normal. Clearly $(X, \widetilde{\tau}_e)$ is normal but $(X, \widetilde{\tau}, E)$ is not soft normal.

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