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A New Concept In Dislocated and Dislocated Quasi Metric Spaces

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Abstract

In this paper we generalizes some new concepts and proved more than two fixed point theorems in complete dislocated quasi metric spaces and in complete dislocated metric spaces.

Keywords: dislocated quasi metric spaces , complete dislocated metric spaces , dq-limit , dq-Cauchy sequence, fixed point.

1 INTRODUCTION

The generalization of Banach Contraction Principle and notation of dislocated metric space[5,6] in quasi metric spaces are introduced by Hitzler and Seda . In logic programming,electronic engineering and topology this metrics, have a very important role The Banach contraction principle in complete metric space which generalized by fixed point theorem using rational type of contractive condition is proved by D.S. Jaggi[3]. Generalized the result of Hitzler and Seda in dislocated quasi-metric spaces and the concept of dislocated quasi-metric space is initiated by Zeyada et al.[4].Result on fixed points in dislocated and dislocated quasi metric spaces followed by Isufati[1],Aage and Ssalunke[2] and Srivastava, Ansari and Sharma [7] and very recently Zoto,Hoxha and A.Isufati [8].

In this paper we prove more than two fixed point theorems in the background of dislocated quasi metric space, which take a broad view and merge some old results.

2 PRELIMINARIES

Here we establish some necessary details and only some outcome in dislocated quasi- metric space.

Definition 2.1 Let X be a non empty and d: X . X → R^+ be a function , called a distance function if for all x, y Є X,satisfies:

d_1 : d(x,x) = 0

d_2 : d(x,y) = d(y,x) = 0 ⇒ x = y

d_3 : d(x,y) = d(y,x)

d_4 : d(x,y) ≤ d(x,z) + d(z,y)

If d satisfies the condition d_1 - d_4 then d is called a metric on X . If it satisfies on conditions d_1 , d_2 , d_4 it is called a quasi metric space. If satisfy condition d_2 , d_3 , d_4 it is called a dislocated metric space. If d satisfy condition d_2 , and d_4 it is called a dislocated quasi metric space on X. A nonempty set X with dq metric d, i.e (X,d) is calle,Type equation here.d a dislocated quasi metric space.

Definition 2.2 A sequence $(x_n)_{n\in N}$ in dq metric space (X,d) is called Cauchy if for all ε > 0 ;

∃ n_0 Є N such that ∀ m, n ≥ n_0 , $d(x_m ,x_n) < ε$ or $d(x_n ,x_m) < ε$

Definition 2.3 A sequence $(x_n)_{n\in N}$ dislocated quasi converges to x if $\lim_{n\to\infty} d(x,x_n) = 0$.

Definition 2.4 A dq –metric space(X,d) is complete if every Cauchy sequence in it is dq- convergent.

Definition 2.3 Let (X,d) be a complete dislocated quasi metric space. Let T: X → X be continuious mapping satisfies the condition:

$$d(Tx,Ty) \le \alpha d(x,y) + \beta \frac{d(x,Tx)d(y,Ty)}{d(x,y)} + \gamma[d(x,Tx) + d(y,Ty)] + \delta[d(x,Ty) + d(y,Tx)$$

For all x.y Є X , *and* α ,β,γ,δЄ[0,1] and $0 \le \alpha + \beta + 2\gamma + 2\delta \le 1$

3 Main Results:

Let (A,d) be a complete dq-metric space. Let T:A→Abe continuous mapping satisfies the condition:

d(Ta,Tb) ≤

$$\alpha d(a,b) + \beta \frac{d(a,Ta)d(b,Tb)}{d(a,b)} + \gamma[d(a,Ta) + d(b,Tb)] + \delta[\{d(a,Tb) + d(b,Ta)] + \eta\{d(a,Ta) + d(a,b)]$$

---(1)

for all a,b∈ A and $\alpha, \beta, \gamma, \delta, \eta$ non negative with $0 \le \alpha + \beta + 2\gamma + 2\delta + 2\eta < 1$

then T has a unique point.

Proof: Let any a_0∈ A and define the sequence as follows: $T(a_0) = a_1$., $T(a_1) = a_2$, $T(a_2) = a_3$.,-----

$T(a_n) = a_{n+1}$.,---

Putting $a = a_{n-1}$ and $b = a_n$ in (1) ,we have,

$d(Ta,Tb) = d(Ta_{n-1}, Ta_n) = d(a_n, a_{n+1})$

$\le \alpha\, d(a_{n-1}, a_n) + \beta\, [d(a_{n-1}, T a_{n-1})\, d(a_n, T a_n)\_] / [d(a_{n-1}, a_n)] ++ \gamma[d(a_{n-1}, T a_{n-1}) + d(a_n, T a_n)] +$

$\delta\, [d(a_{n-1}, T a_n) + d(a_n, T a_{n-1})] + \eta\, [d(a_{n-1}, T a_{n-1}) + d(a_{n-1}, a_n)]$

$\le \alpha\, d(a_{n-1}, a_n) + \beta\, [d(a_{n-1}, a_n)\, d(a_n, a_{n+1})\_] / [d(a_{n-1}, a_n)] ++ \gamma[d(a_{n-1}, a_n) + d(a_n, a_{n+1})] +$

$\delta\, [d(a_{n-1}, a_{n+1}) + d(a_n, a_n)] + \eta\, [d(a_{n-1}, a_n) + d(a_{n-1}, a_n)]$

$\le \alpha\, d(a_{n-1}, a_n) + \beta\, [d(a_n, a_{n+1})\_] + \gamma[d(a_{n-1}, a_n)] + \gamma\, [d(a_n, a_{n+1})] + \delta\, [d(a_{n-1}, a_{n+1})] + \delta[d(a_n, a_n)] + \eta\, [d(a_{n-1}, a_n)] + \eta[d(a_{n-1}, a_n)]$

$\le \alpha\, d(a_{n-1}, a_n) + \beta\, [d(a_n, a_{n+1})\_] + \gamma[d(a_{n-1}, a_n)] + \gamma\, [d(a_n, a_{n+1})] + \delta\, [d(a_{n-1}, a_n)] + \delta[d(a_n, a_{n+1})] +$

$2\eta\, [d(a_{n-1}, a_n)]$

So, $d(a_n, a_{n+1}) \le (\alpha + \gamma + \delta + 2\eta)\, d(a_{n-1}, a_n) + (\beta + \gamma + \delta)\, d(a_n, a_{n+1})$

$= d(a_n, a_{n+1}) - (\beta + \gamma + \delta)\, d(a_n, a_{n+1}) \le (\alpha + \gamma + \delta + 2\eta)\, d(a_{n-1}, a_n)$

$= [1 - (\beta + \gamma + \delta)]\, d(a_n, a_{n+1}) \le (\alpha + \gamma + \delta + 2\eta)\, d(a_{n-1}, a_n)$

$= d(a_n, a_{n+1}) \le \{[(\alpha + \gamma + \delta + 2\eta)] / [1 - (\beta + \gamma + \delta)]\}\, d(a_{n-1}, a_n)$

Or $d(a_n, a_{n+1}) \le \{\lambda\}\, d(a_{n-1}, a_n)$ where $\lambda = \{[(\alpha + \gamma + \delta + 2\eta)] / [1 - (\beta + \gamma + \delta)]\}$…..(2)

Similarly $d(a_{n-1}, a_n) \le \{\lambda\}\, d(a_{n-2}, a_{n-1})$ …….(3)

Put the value of (3) in (2)

$d(a_n, a_{n+1}) \le \{\lambda\}\, [\{\lambda\}\, d(a_{n-2}, a_{n-1})] = \{\lambda^2\}\, d(a_{n-2}, a_{n-1})$ …….(4)

Continuing in this way , we have,

$d(a_n, a_{n+1}) \le \{\lambda^n\}\, d(a_{n-n}, a_{n-(n-1)})$

$d(a_n, a_{n+1}) \le \{\lambda^n\}\, d(a_0, a_1)$ ……..(5)

since $0 < \lambda < 1$ for $n \to \infty$ we have $d(a_n, a_{n+1}) \leq \{\lambda^\infty\} d(a_0, a_1)$

Then $d(a_n, a_{n+1}) \to 0$………(6)

Similarly we show that $d(a_{n+1}, a_n) \to 0$…..(7)

From equ. (6) and (7) hence $(a_n)_{n\in N}$ is a Cauchy Sequence in complete dislocated quasi metric space (A,d). So there exist u∈ A such that $(a_n)_{n\in N}$ quasi converge to u.

Since T is continuous therefore $T(u) = T(\lim_{n\to\infty} a_n) = \lim_{n\to\infty} T(a_n) = \lim_{n\to\infty} (a_{n+1}) = u$

Or we can say $T(u) = u$.

Thus u is a fixed point of T.

UNIQUENESS :Suppose u and v are two fixed point of T ; ($u \neq v$, $Tu= u$, $Tv = v$)….(8)

Let u be a fixed point. From equation (1) for u we have

$$d(u,u) = d(Tu,Tu) \leq \alpha\, d(u,u) + \beta\, [d(u,Tu)\, d(u,Tu)] / d(u,u) + \gamma\, [d(u,Tu) + d(u,Tu)] + \delta\, [d(u,Tu) + d(u,Tu)] + \eta\, [d(u,Tu) + d(u,u)]$$

$$d(Tu,Tu) \leq \alpha\, d(u,u) + \beta\, [d(u,u)\, d(u,u)] / d(u,u) + \gamma\, [d(u,u) + d(u,u)] + \delta\, [d(u,u) + d(u,u)] + \eta\, [d(u,u) + d(u,u)] \quad \text{.......from (8)}$$

$$\leq \alpha\, d(u,u) + \beta\, [d(u,u)] + 2\gamma\, [d(u,u)] + 2\delta\, [d(u,u)] + 2\eta\, [d(u,u)]$$

$$\leq (\alpha + \beta + 2\gamma + 2\delta + 2\eta)\, d(u,u)$$

Which implies that $d(u,u) = 0$. since $0 \leq \alpha + \beta + 2\gamma + 2\delta + 2\eta < 1$

Thus $d(u,u) = 0$ for a fixed point u of T .Similarly we can prove $d(v, v) = 0$ for a fixed point v of T. ………(9)

Now from equation (1)

$$d(u,v) = d(Tu,Tv) \leq \alpha\, d(u,v) + \beta\, [d(u,Tu)\, d(v,Tv)] / d(u,v) + \gamma\, [d(u,Tu) + d(v,Tv)] + \delta\, [d(u,Tv) + d(v,Tu)] + \eta\, [d(u,Tu) + d(u,v)]$$

$$\leq \alpha\, d(u,v) + \beta\, [d(u,u)\, d(v,v)] / d(u,v) + \gamma\, [d(u,u) + d(v,v)] + \delta\, [d(u,v) + d(v,u)] + \eta\, [d(u,u) + d(u,v)]$$

$$\leq \alpha\, d(u,v) + \beta\, [0 \times 0] / d(u,v) + \gamma\, [0 + 0] + \delta\, [d(u,v) + d(v,u)] + \eta\, [0 + d(u,v)] \quad \text{.........from equation (9)}$$

$$\leq \alpha\, d(u,v) + \delta\, [d(u,v) + d(v,u)] + \eta\, [d(u,v)]$$

So, $d(u,v) \leq (\alpha + \delta + \eta)d(u,v) + \delta [d(v,u)]$ … (10)

Similarly $d(v,u) \leq (\alpha + \delta + \eta)d(v,u) + \delta [d(u,v)]$ ……. .(11)

From equation (10) and (11)

$| d(u,v) - d(v,u) | \leq | (\alpha + \delta + \eta)d(u,v) + \delta [d(v,u)] - [(\alpha + \delta + \eta)d(v,u) + \delta [d(u,v)] |$

$= | \alpha d(u,v) + \delta d(u,v) + \delta d(u,v) + \eta d(u,v) - \alpha d(v,u) - \delta d(v,u) - \eta d(v,u) - \delta d(u,v) |$

$= | (\alpha + \eta) d(u,v) - (\alpha + \eta) d(v,u) |$

$= (\alpha + \eta) |d(u,v) - d(v,u)|$ $0 < \alpha, \eta < 1$

Since $0 < \alpha + \eta < 1$ so $| d(u,v) - d(v,u) | \leq (\alpha + \eta) |d(u,v) - d(v,u)|$ and $d(u,v) - d(v,u) \to 0$

So $d(u,v) = d(v,u)$ … (12)

From equation (10) , (11) , (12)

$$d(u,v) \leq (\alpha + \delta + \eta)d(u,v) + \delta [d(u,v)]$$

$d(u,v) \leq (\alpha + 2\delta + \eta)d(u,v)$ which gives $d(u,v) = 0$ since $0 < (\alpha + 2\delta + \eta) < 1$

So $d(u,v) = d(v,u) = 0$ which implies **u = v ………(I)**

Hence fixed point is unique

Again,

Let (E,d) be a complete dq-metric space. Let T:E→E be continuous mapping satisfies the condition:

d(Te,Tf) ≤

$$\alpha_1 d(e,f) + \beta_1 \frac{d(e,Te)(f,Tf)}{d(e,f)} + \gamma_1[d(e,Te) + d(f,Tf)] + \delta_1 [d(e,Tf) + d(f,Te)] + \eta_1 [d(e,Te) + d(e,f)]$$

. **………(i)**

for all e,f€ E and $\alpha_1, \beta_1, \gamma_1, \delta_1, \eta_1$ **non negative with** $0 < \alpha_1 + \beta_1 + \gamma_1 + \delta_1 + \eta_1 < 1$

then T has a unique point.

Proof: Let any e_0€ E and define the sequence as follows: $T(e_0) = e_1$., $T(e_1) = e_2$, $T(e_2) = e_3$.,-----

$T(e_n) = e_{n+1}$.,---

Putting $e = e_{n-1}$ and $f = e_n$ in (i) ,we have,

$d(Te,Tf) = d(Te_{n-1}, Te_n) = d(e_n, e_{n+1})$

$\leq \alpha_1\, d(e_{m-1}, e_m) + \beta_1\, [d(e_{m-1}, T e_{m-1})\, d(e_m, T e_m)] / [d(e_{m-1}, e_m)] + \gamma_1\, [d(e_{m-1}, T e_{m-1}) + d(e_m, T e_m)] +$

$\delta_1\, [d(e_{m-1}, T e_m) + d(e_m, T e_{m-1})] + \eta_1\, [d(e_{m-1}, T e_{m-1}) + d(e_{m-1}, e_m)]$

$\leq \alpha_1 d(e_{m-1}, e_m) + \beta_1\, [d(e_{m-1}, e_m)\, d(e_m, e_{m+1})] / [d(e_{m-1}, e_m)] + \gamma_1\, [d(e_{m-1}, e_m) + d(e_m, e_{m+1})] +$

$\delta_1\, [d(e_{m-1}, e_{m+1}) + d(e_m, e_m)] + \eta_1\, [d(e_{m-1}, e_n) + d(e_{m-1}, e_m)]$

$\leq \alpha_1 d(e_{m-1}, e_m) + \beta_1\, [d(e_m, e_{m+1})] + \gamma_1\, [d(e_{m-1}, e_m) + d(e_m, e_{m+1})] + \delta_1\, [d(e_{m-1}, e_{m+1}) + d(e_m, e_m)] +$

$\eta_1\, [d(e_{m-1}, e_n) + d(e_{m-1}, e_m)]$

$\leq \alpha_1 d(e_{m-1}, e_m) + \beta_1\, [d(e_m, e_{m+1})] + \gamma_1\, [d(e_{m-1}, e_m)] + \gamma_1\, [d(e_m, e_{m+1})] + \delta_1\, [d(e_{m-1}, e_m)] + \delta_1\, [d(e_m, e_{m+1})]$

$+2\eta_1\, [d(e_{m-1}, e_m)]$

So, $d(e_m, e_{m+1}) \leq (\alpha_1 + \gamma_1 + \delta_1 + 2\eta_1)\, d(e_{m-1}, e_m) + (\beta_1 + \gamma_1 + \delta_1)\, d(e_m, e_{m+1})$

$= d(e_m, e_{m+1}) - (\beta_1 + \gamma_1 + \delta_1)\, d(e_m, e_{m+1}) \leq (\alpha_1 + \gamma_1 + \delta_1 + 2\eta_1)\, d(e_{m-1}, e_m)$

$= [1 - (\beta_1 + \gamma_1 + \delta_1)]\, d(e_m, e_{m+1}) \leq (\alpha_1 + \gamma_1 + \delta_1 + 2\eta_1)\, d(e_{m-1}, e_m)$

$= d(e_m, e_{m+1}) \leq \{ [(\alpha_1 + \gamma_1 + \delta_1 + 2\eta_1)] / [1 - (\beta_1 + \gamma_1 + \delta_1)] \}\, d(e_{m-1}, e_m)$

Or $d(e_m, e_{m+1}) \leq \{\lambda_1\}\, d(e_{m-1}, e_m)$ where $\lambda_1 = \{ [(\alpha_1 + \gamma_1 + \delta_1 + 2\eta_1)] / [1 - (\beta_1 + \gamma_1 + \delta_1)] \}$…..(ii)

Similarly $d(e_{m-1}, e_m) \leq \{\lambda 1\} d(e_{m-2}, a_{m-1})$ …….(iii)

Put the value of (iii) in (ii)

$d(e_m, e_{m+1}) \leq \{\lambda_1\}[\{\lambda_1\}(e_{m-2}, a_{m-1})] = \{\lambda_1^{\,2}\}\, d(e_{m-2}, a_{m-1})$ …….(4)

Continuing in this way , we have,

$d(e_m, e_{m+1}) \leq \{\lambda_1^{\,m}\}\, d(e_{m-m}, e_{m-(m-1)})$

$d(e_m, e_{m+1}) \leq \{\lambda_1^{\,m}\}\, d(e_0, e_1)$ ……….(v)

since $0 < \lambda_1 < 1$ for $m \to \infty$ we have $d(e_m, e_{m+1}) \leq \{\lambda_1^{\infty}\}\, d(e_0, e_1)$

Then $d(e_m, e_{m+1}) \to 0$………(vi)

Similarly we show that $d(e_{m+1}, e_m) \to 0$…..(vii)

From equ. (vi) and (vii) hence $(e_m)_{m \in N}$ is a Cauchy Sequence in complete dislocated quasi metric space (E,d). So there exist g$\in$ E such that $(e_m)_{m\in N}$ quasi converge to g.

Since T is continuous therefore $T(g) = T(\lim_{m\to\infty} e_m) = \lim_{m\to\infty} T(e_m) = \lim_{m\to\infty} (e_{m+1}) = g$

Or we can say $T(g) = g$.

Thus g is a fixed point of T.

UNIQUENESS :Suppose g and h are two fixed point of E ; ($g \neq h$, $Tg = g$, $Th = h$)….(viii)

Let g be a fixed point. From equation (i)for u we have

$$d(g,g)= d(Tg,Tg) \leq \alpha_1 d(g,g)+ \beta_1 [d(g ,Tg) d(g,Tg)] / d(g,g)+ \gamma_1 [d(g,Tg)+ d(g,Tg)] + \delta_1 [d(u,Tu) + d(u,Tu)] + \eta_1 [d(u,Tu) + d(u,u)]$$

$$d(Tg,Tg) \leq \alpha_1 d(g,g)+ \beta_1 [d(g,g) d(g,g)] / d(g,g) + \gamma_1 [d(g,g) + d(g,g)] + \delta_1 [d(g,g) + d(g,g)] + \eta_1 [d(g,g) + d(g,g)] \quad \text{.......from (viii)}$$

$$\leq \alpha_1 d(g,g)+ \beta_1 [d(g,g)] + 2 \gamma_1 [d(g,g)] + 2 \delta_1 [d(g,g)] + 2 \eta_1 [d(g,g)]$$

$$\leq (\alpha_1 + \beta_1 + 2\gamma_1+ 2 \delta_1 +2 \eta_1) d(g,g)$$

Which implies that $d(g,g)= 0$. since $0 < (\alpha_1 + \beta_1 + 2\gamma_1+ 2 \delta_1 +2 \eta_1) < 1$

Thus $d(g,g)= 0$ for a fixed point g of T .Similarly we can prove $d(h,h)= 0$ for a fixed point h of T. ………(ix)

Now from equation (i)

$$d(g.h)= d(Tg,Th) \leq \alpha_1 d(g,h) + \beta_1 [d(g,Tg) d(h,Th)] / d(g,h) + \gamma_1 [d(g,Tg) + d(h,Th)] + \delta_1 [d(g,Th) + d(h,Tg)] + \eta_1 [d(g,Tg) + d(g,h)]$$

$$\leq \alpha_1 d(g,h) + \beta_1 [d(g,g) d(h,h)] / d(g,h) + \gamma_1 [d(g,g) + d(h,h)] + \delta_1 [d(g,h) + d(h,g)] + \eta_1 [d(g,g) + d(g,h)]$$

$$\leq \alpha_1 d(g,h) + \beta_1 [0 \times 0] / d(g,h) + \gamma_1 [0 + 0] + \delta_1 [d(g,h) + d(h,g)] + \eta_1 [0 +d(g,h)] \text{......from equation (ix)}$$

$$\leq \alpha_1 d(g,h) + \delta_1 [d(g,h) + d(h,g)] + \eta_1 [d(g,h)]$$

So, $d(g,h) \leq (\alpha_1+ \delta_1 + \eta_1)d(g,h) + \delta_1 [d(h,g)]$ ……..(x)

Similarly $d(h,g) \leq (\alpha_1+ \delta_1 + \eta_1)d(h,g) + \delta_1 [d(g,h)]$ ……..(xi)

From equation (x) and (xi)

$$| d(g,h)- d(h,g) | \leq (\alpha_1+ \delta_1 + \eta_1)d(g,h) + \delta_1 [d(h,g)] - [(\alpha_1+ \delta_1 + \eta_1)d(h,g) + \delta_1 [d(g,h)] |$$

$= | \alpha_1 d(g,h) + \delta_1 d(g,h) + \delta_1 d(h,g) + \eta_1 d(g,h) - \alpha_1 d(h,g) - \delta_1 d(h,g) - \eta_1 d(h,h) - \delta_1 d(g,h)$

$= | (\alpha_1 + \eta_1) d(g,h) - (\alpha_1 + \eta_1) d(h,g) |$

$= (\alpha_1 + \eta_1) |d(g,h) - d(h,g)| \quad 0 < \alpha_1, \eta_1 < 1$

Since $0 < \alpha_1 + \eta_1 < 1$ so $| d(g,h) - d(h,g) | \leq (\alpha_1 + \eta_1) | d(g,h) - d(h,g) |$ and $d(g,h) - d(h,g) \to 0$

So $d(g,h) = d(h,g)$ ……(xii)

From equation (x), (xi) , (xii)

$$d(g,h) \leq (\alpha_1 + \delta_1 + \eta_1)d(g,h) + \delta_1 [d(g,h)]$$

$d(g,h) \leq (\alpha_1 + 2\delta_1 + \eta_1)d(g,h)$ which gives $d(g,h) = 0$ since $0 < (\alpha_1 + 2\delta_1 + \eta_1) < 1$

So $d(g,h) = d(h,g) = 0$ which implies **g = h ………(II)**

Hence fixed point is unique

Now we have to prove if u = v = g = h **then T has a unique point**

Let (A,d) and (E,d) be two complete dq-metric space. Let T:A→A ;T:E→E be continuous mapping satisfies the conditions:

d(Ta,Tf) ≤

$$\alpha_2 d(a,f) + \beta_2 \frac{d(a,Ta)d(f,Tf)}{d(a,f)} + \gamma_2[d(a,Ta) + (f,Tf)] + \delta_2[d(a,Tf) + d(f,Ta)] + \eta_2[d(a,Ta) + d(a,f)]$$

……….(13)

And d(Tb,Te) ≤

$$\alpha_3 d(b,e) + \beta_3 \frac{d(b,Tb)d(e,Te)}{d(b,e)} + \gamma_3[d(b,Tb) + (e,Te)] + \delta_3[d(a,Tb) + d(e,Tb)] + \eta_3[d(b,Tb) + d(b,e)$$

………(14)

for all a,b€ A, e,f € E and $(\alpha_2, \beta_2, \gamma_2, \delta_2, \eta_2)$ **,** $(\alpha_3, \beta_3, \gamma_3, \delta_3, \eta_3)$ **on negative with**

0< $\alpha_2 + \beta_2 + \gamma_2 + \delta_2 + \eta_2$ **< 1 and 0** $< \alpha_3 + \beta_3 + \gamma_3 + \delta_3 + \eta_3$ **< 1**

then T has a unique point.

Firstly we have to prove u = h then g = v

Proof: Let any a_0€ A and define the sequence as follows: $T(a_0) = a_{1.}$, $T(a_1) = a_2$, $T(a_2) = a_{3.}$,-----

$T(a_n) = a_{n+1.}$,---

Putting $a = y_{p-1}$ and $f = y_p$ in (13) ,we have,

$d(Ta,Tf) = d(Ty_{p-1} ,Ty_p) = d(y_p , y_{p+1})$

$\leq \alpha_2 d(y_{p-1} , y_p) + \beta_2 [d(y_{p-1} , Ty_{p-1}) d(y_p, Ty_p)\_] / [d(y_{p-1} , y_p)] + \gamma_2 [d(y_{p-1} , Ty_{p-1}) + d(y_p, Ty_p)]+$

$\delta_2 [d(y_{p-1} , Ty_p) + d(y_p, Ty_{p-1})] + \eta_2[d(y_{p-1} , Ty_{p-1}) + d(y_{p-1}, y_p)]$

$\leq \alpha_2 d(y_{p-1} , y_p) + \beta_2 [d(y_{p-1} , y_p) d(y_p, y_{p+1})\_] / [d(y_{p-1} , y_p)] + \gamma_2 [d(y_{p-1} , y_p) + d(y_p, y_{p+1})]+$

$\delta_2 [d(y_{p-1} , y_{p+1}) + d(y_p, y_p)] + \eta_2[d(y_{p-1} , y_p) + d(y_{p-1}, y_p)]$

$\leq \alpha_2 d(y_{p-1} , y_p) + \beta_2 [d(y_p, y_{p+1})] + \gamma_2 [d(y_{p-1} , y_p)]+ \gamma_2 [d(y_p, y_{p+1})]+ \delta_2 [d(y_{p-1} , y_{p+1})]+ \delta_2 [d(y_p, y_p)] +$

$2\eta_2[d(y_{p-1} , y_p)]$

$\leq \alpha_2 d(y_{p-1} , y_p) + \beta_2 [d(y_p, y_{p+1})] + \gamma_2 [d(y_{p-1} , y_p)]+ \gamma_2 [d(y_p, y_{p+1})]+ \delta_2 [d(y_{p-1} , y_p)]+ \delta_2 [d(y_p, y_{p+1})] +$

$2\eta_2[d(y_{p-1} , y_p)]$

So, $d(y_p , y_{p+1}) \leq (\alpha_2 + \gamma_2 + \delta_2 + 2 \eta_2) d(y_{p-1} , y_p) + (\beta_2 + \gamma_2 + \delta_2) d(y_p, y_{p+1})$

$= d(y_p , y_{p+1}) - (\beta_2 + \gamma_2 + \delta_2) d(y_p, y_{p+1}) \leq (\alpha_2 + \gamma_2 + \delta_2 + 2 \eta_2) d(y_{p-1} , y_p)$

$= [1 - (\beta_2 + \gamma_2 + \delta_2)] d(y_p , y_{p+1}) \leq (\alpha_2 + \gamma_2 + \delta_2 + 2 \eta_2) d(y_{p-1} , y_p)$

$= d(y_p , y_{p+1}) \leq \{[(\alpha_2 + \gamma_2 + \delta_2 + 2 \eta_2) / [1 - (\beta_2 + \gamma_2 + \delta_2)] \} d(y_p , y_{p+1})$

Or , $d(y_p , y_{p+1}) \leq \{ \lambda_3\} d(y_p , y_{p-1})$ where $\lambda_3 = \{[(\alpha_2 + \gamma_2 + \delta_2 + 2 \eta_2) / [1 - (\beta_2 + \gamma_2 + \delta_2)] \}$…..(15)

Similarly $d(y_{p-1} ,y_p) \leq \{\lambda_3\} d(y_{p-2} , y_{p -1})$ … (16)

Put the value of (16)in (15)

$d(y_p, y_{p+1}) \leq \{\lambda_3\} [\{\lambda_3\} d(y_{p-2} , y_{p -1})] = \{ \{\lambda_3{}^2\} d(y_{p-2} , y_{p -1})$ … .(17)

Continuing in this way , we have,

$d(y_p, y_{p+1}) \leq \{\lambda_3{}^p\} d(y_{p-p} , y_{p-(p -1)})$

$d(y_p, y_{p+1}) \leq \{\lambda_3{}^p\} d(y_0 , y_1)$ … .(18)

since $0 < \{\lambda_3 \} < 1$ for $p\to\infty$ we have $d(y_p, y_{p+1}) \leq \{ \lambda_3{}^{\infty} \} d(y_0 , y_1)$

Then $d(y_p, y_{p+1}) \to 0$…… (19)

Similarly we show that $d(y_{p+1}, y_p) \to 0$… (20)

From equ. (19) and (20) hence $(y_p)_{p\in N}$ is a Cauchy Sequence in complete dislocated quasi metric space (A,d). So there exist u∈ A such that $(y_p)_{p\in N}$ quasi converge to u.

Since T is continuous therefore $T(u) = T(\lim_{p\to\infty} y_p) = \lim_{p\to\infty} T(y_p) = \lim_{p\to\infty} (a_{p+1}) = u$

Or we can say T(u) = u .

Thus u is a fixed point of T.

UNIQUENESS :Suppose u and h are two fixed point of T ; ($u \neq h$, Tu= u, Th = h)….(21)

Let u be a fixed point. From equation (13) for u we have

$d(u,u) = d(Tu,Tu) \le \alpha_2\, d(u,u) + \beta_2 [d(u,Tu)\, d(u,Tu)] / d(u,u) + \gamma_2 [d(u,Tu) + d(u,Tu)] +$

$\delta_2 [d(u,Tu) + d(u,Tu)] + \eta_2 [d(u,Tu) + d(u,u)]$

$d(Tu,Tu) \le \alpha_2\, d(u,u) + \beta_2 [d(u,u)\, d(u,u)] / d(u,u) + \gamma_2 [d(u,u) + d(u,u)] + \delta_2 [d(u,u) + d(u,u)] +$

$\eta_2 [d(u,u) + d(u,u)]$ …….from (21)

$\le \alpha_2\, d(u,u) + \beta_2 [d(u,u)] + 2\gamma_2 [d(u,u)] + 2\delta_2 [d(u,u)] + 2\eta_2 [d(u,u)]$

$\le (\alpha_2 + \beta_2 + 2\gamma_2 + 2\delta_2 + 2\eta_2)\, d(u,u)$

Which implies that d(u ,u) = 0 . ……since $0 \le \alpha_2 + \beta_2 + 2\gamma_2 + 2\delta_2 + 2\eta_2 < 1$

Thus d(u,u) = 0 for a fixed point u of T .Similarly we can prove d(h , h) = 0 for a fixed point v of T. ………(22)

$d(u,h) = d(Tu,Th) \le \alpha_2\, d(u,h) + \beta_2 [d(u,Tu)\, d(h,Th)] / d(u,h) + \gamma_2 [d(u,Tu) + d(h,Th)] +$

$\delta_2 [d(u,Th) + d(h,Tu)] + \eta_2 [d(u,Tu) + d(u,h)]$

$\le \alpha_2\, d(u,h) + \beta_2 [d(u,u)\, d(h,h)] / d(u,h) + \gamma_2 [d(u,u) + d(h,h)] + \delta_2 [d(u,h) + d(h,u)] + \eta_2 [d(u,u) + d(u,h)]$

$d(u,h) \le (\alpha_2 + \delta_2 + \eta_2)\, d(u,h) + \delta_2 [d(h,u)]$ ……. .(23)

Similarly $d(h,u) \le (\alpha_2 + \delta_2 + \eta_2) d(h,u) + \delta_2 [d(u,h)]$ ……. .(24)

From equation (23)

$\le \alpha_2\, d(u,h) + \beta_2 [0 \times 0] / d(u,h) + \gamma_2 [0 + 0] + \delta_2 [d(u,h) + d(h,u)] + \eta_2 [0 + d(u,h)]$….from equation (22)

$\le \alpha_2\, d(u,h) + \delta_2 [d(u,h) + d(h,u)] + v [d(u,h)]$

So,) and (24)

$| d(u,h) - d(h,u) | \leq | (\alpha_2 + \delta_2 + \eta_2)d(u,h) + \delta_2 [d(h,u)] - [(\alpha_2 + \delta_2 + \eta_2)d(h,u) + \delta_2 [d(u,h)] |$

$= | \alpha_2 d(u,h) + \delta_2 d(u,h) + \delta_2 d(u,h) + \eta_2 d(u,h) - \alpha_2 d(h,u) - \delta_2 d(h,u) - \eta_2 d(h,u) + \delta_2 d(u,h)|$

$= | (\alpha_2 + \eta_2) d(u,h) - (\alpha_2 + \eta_2) d(h,u) |$

$= (\alpha_2 + \eta_2) |d(u,h) - d(h,u) | \quad 0 < \alpha_2, \eta < 1$

Since $0 < \alpha_2 + \eta_2 < 1$ so $| d(u,h) - d(h,u) | \leq (\alpha_2 + \eta_2) |d(u,h) - d(h,u) |$ and $d(u,h) - d(h,u) \to 0$

So $d(u,h) = d(h,u)$ … (25)

From equation (25) we can say **u= h……(III)**

Similarly we can prove **g = v…….(IV)**

From (I) ,(II) ,(III) and (IV) we prove u = v = g = h **then T has a unique point.**

References

[1] D.S. Jaggi, Some unique fixed point theorems, *Indian J.Pure Appl.Math*,8(2) :223-230, 1977.

[2] P.Hitzler and A.K.Seda, Dislocated topologies.,*J.Electr.Engin.*,51(12/S):3:7,2000.

[3] P.Hitzler, Generalized Metrics andTopology in Logic Programming Semantics, Ph.d. thesis ,*National University of Ireland, University College Cork*, 2001

[4] F.M.Zeyada, G.H.Hassan,And M.A.Ahmed. A generalization of a fixed point theorem due to P. Hitzler and A.K.Seda, Dislocated quasi metric spaces. *The Arabian j. for Sci.and Eng.,* 31(1A):111:114,2005.

[5] C.T.Aage and JN.Salunke. The results on fixed points in dislocated and dislocated quasi-metric space. *Appl.Math.Sci.,* 2(59):2941-2948,2008.

[6] A. Isufati , Fixed points in dislocated quasi-metric space, *Appl.Math.Sci.,* 4(5):217-223,2010.

[7] R.srivastava, Z.k.Ansari and M. Sharma. Some results on fixed points in dislocated and dislocated quasi- metric space. *Journal of Advanced Studies in Topology;* Vol.3,No. 1 ,2012.

[8] Kastroit Zoto, Elida Hoxha, and A. Isufati , Some new results in dislocated and dislocated quasi- metric spaces. *Appl.Math.Sci.,*Vol.6,No.71, 3519-3526,2012