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Pancharatnam Phase and Information Entropy Squeezing of Superconducting Qubits Coupled with An LC-Resonator

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Pancharatnam phase, field purity, and entropy squeezing are an important features in performing different tasks in quantum information processing such as quantum cryptography and teleportation. These quantum information tasks depend on finding the states in which squeezing in entropy can be created. In this paper, we investigate the evolution of the Pancharatnam phase, and field purity, for a system of coupled-charge qubits interacting with an LC-resonator. The initial state of the coupled-charge qubits are taken to be in the excited and superposition states. The dependence of the field purity, entropy squeezing and Pancharatnam phase on the coupling strength between the Josephson charge qubit are discussed. Furthermore, we examine the effects of the initial state and different system parameters on the evolution of entropy squeezing and Pancharatnam phase properties. Finally, we have found the relationship between the entropy squeezing and Pancharatnam phase of the system state during the time evolution. The results shown that the coupling strength plays a central role in the dynamical properties of different physical quantities, where, a high purity of the field and squeezing in entropy can be obtained to be a suitable choice of the coupling strength.

Keywords: Pancharatnam phase, quantum information, entropy squeezing, superconducting,

INTRODUCTION:

Solid-state superconducting circuits [1,2] consisting of Josephson-junction charge qubit(s), known as artificial atom(s), are versatile systems in which macroscopic quantum states can be engineered and controlled. Due to their low loss, low noise, high coherence, etc., much attention has been drawn on their study. An artificial atoms show macroscopic quantum properties [3-7]. It is also possible to study artificial atom-cavity or circuit-quantum electrodynamics (circuit QED), when the artificial atoms are coupled with resonator(s). Such a system can be used as an element of quantum information and quantum computation [8]. Recently, experiments have shown that many important optical phenomena can be replicated in such artificial-atom-cavity systems, such as generating a single microwave photons in a circuit [9], artificial atom lasing [10], artificial atom resonance fluorescence [11], coherent population trapping [12], optical nonlinearity [13,14], electromagnetically induced transparency [15], and so on.

The time evolution of the quantum entropy and nonlocal correlation in a system of multi-mode coherent light field resonantly interacting with a two-level atom by degenerating the multi-photon process is studied [16, 17]. It is found that the quantum entropy is strongly dependent on the atomic distribution angle. Inevitably, the entanglement of a multi-partite quantum state becomes degraded with time due to experimental and environmental noise. The influence of the noise on bipartite entanglement is a problem in the theory of open systems [18].

As known that the investigation of quantum entanglement outdated its context of bipartite and multipartite pure quantum states, considering the case of mixed states as well as investigates the entanglement in large scale many particle quantum systems. Whereas, a complete investigation of entanglement in complex systems is still absent until present. The quantification of this kind correlation in both discrete and continuous variables in the framework of theoretical approaches [19-21]. A measure of entanglement must be invariant under the local operations and classical communication (LOCC). The understanding derivable measures of quantum entanglement is account bearing in mind when searching to detect entanglement in large dimensional complex systems such as is considered in biological systems. Until present, the quantum concepts of nonlocal correlation were related to entangled states [22]. The separation between classical and quantum correlations in composite systems has attracted recently much attention in different branches of physics.

The quantum geometric phase is a basic intrinsic feature in quantum mechanics that has been investigated by almost two generations of physicists [23-25]. Berry proved that the solution of the quantum object (wave function) retains the memory of its evolution in its complex phase argument (known as the geometric phase factor), which, when observed from the perspective of its dynamical contribution, depends only on the geometry of the path traversed by the system [26, 27]. It is robust against environmental perturbations and imperfections in control. This explains why it has gained attention in the implementation of fault-tolerant quantum computation. More recently, the Pancharatnam phase, and physical properties of the field mode that is initially in a coherent state associated with a generalized Heisenberg algebra (CSGHA) have been studied [28]. Also, it is shown that the (CSGHA) strength and time-dependent coupling based on the atomic speed and acceleration have the potential to affect the time evolution of the entanglement, the Pancharatnam phase, and the Mandel parameter [29].

In this paper, we present a detailed study on the evolution of the Pancharatnam phase, entropy squeezing, and purification properties for a solid state system of two coupled-charge qubits interacting with an LC-resonator. The physical properties of squeezing in entropy, field purity, and Pancharatnam phase when the resonator starts the interaction from the vacuum state will be discussed. The dependence of a Pancharatnam phase and field purity on the coupling strength between the two Josephson charge qubits will be studied.

2. QUANTUM SYSTEM AND DYNAMICS:

Here, we consider a model of a two superconducting qubits (TSQs) interacting with an LC-resonator, since the model is supposed to be an ideal one without regarding loss and decoherence. We assume that the charging energy to be smaller than the superconducting gap, such that no quasi particles are present in the Cooper-pair box. In this case, two Cooper-pair boxes have the same properties. At the degeneracy point, a single Cooper-pair box will be referred to a Josephson charge qubit. The LC-resonator, which, is the analogue of a high-Q cavity, can be described by a quantum harmonic oscillator with a characteristic frequency ω_R. The circuit has a three coupling capacitances as the capacitance C_ℓ couples the TSQs to each other, and C_{Q1}, C_{Q2} couple the TSQ with the resonator. The Hamiltonian of the system can be written as:

$$H = H_O + H_I, \qquad (1)$$

where, H_O represents the non-interaction Hamiltonian given by

$$H_O = \omega_R a^+ a + \sum_{k=1}^{2} \left\{ \wp_k (n_k e - n_g e)^2 - E_{\ell k} \cos\theta_\ell \right\}, \qquad (2)$$

where, ω_R is the frequency of the LC resonator and $a^+(a)$, is the creation (annihilation) operator for photons in the resonator. Also, n_k is the operator associated to the number of excess Cooper-pairs on the box, $n_g = n_{g_k} = C_{g_k} V_{g_k} / 2\varepsilon$ is the reduced gate charge and it can be controlled by a voltage in series with a gate capacitance C_{g_k}, $E_{\ell k}$ is the Josephson energy, θ_j denotes the phase of the superconducting current, n_k and θ_k are treated as operators obeying $[\theta_k, n_k] = k$ and $\wp_k = \frac{L_k}{C_{Q_k} + C_{g_k} + C_{R_k}}$ with $L_k = (C_{Q_k} + C_{g_k})(C_{R_k} + C_{\ell k}) + C_{R_k} C_{\ell k}$.

The electrostatic energy of the two charge states $|n=0\rangle$ and $|n=1\rangle$ are equal near to the charge degeneracy point (i.e. $n_g = 1/2$), then H_O can be rewritten in spin boson model notation [27] as:

$$H_O = \omega_R a^+ a - \sum_{k=1}^{2} \left\{ 2E_{C_k}(1-2n_g)\hat{\sigma}_{z_k} + \frac{E_{k\ell}\hat{\sigma}_{x_k}}{2} \right\}, \qquad (3)$$

$\hat{\sigma}_{z_k}$ and $\hat{\sigma}_{x_k}$ are referred to the Pauli spin operators in the basis $|0\rangle, |1\rangle$.

The interaction Hamiltonian including two parts, one is the interaction between the TCQs and the other is interaction between the LC resonator and the charge qubit, such as:

$$H_I = \sum_{k=1}^{2} \left\{ (2-k) J \hat{s}_{z_k} \hat{s}_{z_{k+1}} - i\lambda_k (n_k - n_g)(a^+ - a) \right\}, \qquad (4)$$

where,

$$\lambda_k = e\left(\wp_k - \frac{C_{ki}}{L_k} \right) \sqrt{2\omega_R \wp_k}, \qquad (5)$$

Is the coupling strength between the charge qubit and the cavity field, while J is the coupling strength between the TSQs [27]. Under the (RWA) rotating wave approximation, the interaction Hamiltonian [27] can be written as:

$$H_I = \sum_{k=1}^{2} \left\{ (2-k) J \hat{s}_{z_k} \hat{s}_{z_{k+1}} - i\lambda_k (a\hat{s}_k^+ - a^+ \hat{s}_k^-) \right\}, \qquad (6)$$

where, s_k^+ (s_k^-) is the raising (lowering) operator. In this manner, the Hamiltonian (6) is similar to the Jaynes-Cummings model (JCM). The charged qubits can resonated with LC oscillator $\omega_r = E_k$.

Let us assume that the initial state vector of the system prepared as:

$$|\psi(t)\rangle = X_1(t)|e_1, e_2, 0\rangle + X_2(t)|e_1, g_2, 1\rangle + X_3(t)|g_1, e_2, 1\rangle + X_4(t)|g_1, g_2, 2\rangle. \qquad (7)$$

The Schrödinger equation of the considered system in the interaction picture is:

$$i\frac{d|\psi(t)\rangle}{dt} = H_I|\psi(t)\rangle, \qquad (8)$$

By solving the Schrödinger equation, the amplitudes of the wave function $|\psi(t)\rangle$ ($X_\ell(t),\ \ell = 1,2,3,4$) are given by:

$$X_1(t) = \frac{1}{3}\left[2X_1(0) + \sqrt{2}X_4(0)\right] - 2gX_2(0)\frac{\sin(ft)}{f}$$
$$+\frac{1}{3}\left(X_1(0) - \sqrt{2}X_4(0)\right)\left\{\cos tf - igJ\,\frac{\sin(tf)}{f}\right\},$$

$$X_2(t) = X_3(t) = X_2(0)\left\{\cos tf + igJ\,\frac{\sin(ft)}{f}\right\} + \lambda\left(X_1(0) - \sqrt{2}X_4(0)\right)\frac{\sin(ft)}{f},$$

$$X_4(t) = \frac{1}{\sqrt{2}}\left\{X_1(t) - \left(X_1(0) - \sqrt{2}X_4(0)\right)\cos(ft)\right.$$
$$\left. + \left[6gX_2(0) + iJ\left(X_1(0) - \sqrt{2}X_4(0)\right)\frac{\sin(tf)}{f}\right]\right\}, \tag{9}$$

Where,

$$f = \sqrt{J^2 + 6g^2}$$

Now, the amount of purity of the field or resonator is defined as follows:

$$\xi_F = 1 - \mathbf{Tr}\left(\rho_R^2\right) \tag{10}$$

Where, $\rho_{Q_1Q_2[R]} = \mathbf{Tr}_{R[Q_1Q_2]}\left(|\psi(t)\rangle\langle\psi(t)|\right)$ is the reduced SC-qubits [field] density matrix, and $\mathbf{Tr}_{R[Q_1Q_2]}$ means that the trace is taken over the resonator(SC-qubits) variables.

3. The single charge qubit entropy squeezing (SCQES):

Let us consider the evolution of the reduced density matrix operator of qubit A then,

$$\rho_{Q_1}(t) = Tr_{Q_2}\left(\rho_{Q_1Q_2}(t)\right),$$

so, one can calculate the usual entropy squeezing for a single TSQ (e.g. qubit Q_1) as [30,31],

In this case the atomic reduced density operator $\rho_{Q_1}(t)$, we can obtain the information entropies of the atomic operators S_x , S_y and S_z, can be obtained such as in the form [30,31].

$$H(S_\gamma) = -(\frac{1}{2} + \langle S_\gamma\rangle)\ln(\frac{1}{2} + \langle S_\gamma\rangle) - (\frac{1}{2} - \langle S_\gamma\rangle)\ln(\frac{1}{2} - \langle S_\gamma\rangle), \quad \gamma = x, y, z. \tag{11}$$

The fluctuation of the component S_γ ($\gamma = x$ or y) of the atomic dipole is said to be "squeezed in entropy" if the information entropy $H(S_\gamma)$ of S_γ satisfies the condition [31,32,33]

$$E(S_\gamma) = \delta H(S_\gamma) - \sqrt{\frac{2}{|\delta H(S_z)|}} < 0, \qquad \gamma = x, y \tag{12}$$

Where,

$$\delta H(S_\gamma) = \exp[H(S_\gamma)]$$

Therefore, $E(S_\gamma)$ denotes the entropy squeezing.

4. Quantum phase:

For the quantum system evolving from an initial wave function to a final wave function, if the final wave function cannot be obtained from the initial wave function by a multiplication with a complex number, therefore the initial and final states are distinct and the evolution is noncyclic. Suppose state $|\psi(0)\rangle$ evolves to a state $|\psi(t)\rangle$ after a certain time t, if the scalar product [24] is given by:

$$F(t) = \langle \psi(0) | \exp\{-iH_I t\} | \psi(0)\rangle \tag{13}$$

the equation (13) can be written as $F(t) = \Gamma \exp(i\phi)$, where, Γ is a real number, then the noncyclic phase due to the evolution from $|\psi(0)\rangle$ to $|\psi(t)\rangle$ is the angle ϕ. This noncyclic phase generalizes the cyclic geometric phase since the latter can be regarded as a special case of the former in which $\Gamma = 1$.

Determination of the phase between the two states for such an evolution is nontrivial. Pancharatnam prescribed the phase acquired during an arbitrary evolution of a wave function from the vector $|\psi(0)\rangle$ to $\exp\{-iH_I t\}|\psi(0)\rangle$ as **arg** $[F(t)]$.

Then the Pancharatnam phase can be written as [24,29] by the following formula:

$$\phi_P(t) = -\sin^{-1}\left\{\frac{y(t)}{\sqrt{x^2(t) + y^2(t)}}\right\}, \tag{14}$$

Where, $x(t) = \text{Re}[F(t)]$ and $y(t) = \text{Im}[F(t)]$.

In the next section we discuss the numerical results of the field purity, entropy squeezing and Pancharatnam phase which are given by equations (10), (12) and (14) respectively.

5. Results and Discussion:

In Figures (1–4), we compare between the weak and strong coupling strength regimes between the TSQs and study how the behavior of the Pancharatnam phase, the field purity and the SCQES components $E(S_x)$ and $E(S_y)$ during the time evolution. The figures present the evolution of the physical quantities which depends on the coupling strength and initial SC-qubit state preparation. We first start the discussion by Fig. 1, which depicts the effect of the weak coupling between the TSQs (i.e. $J/g=0.2$), where, the TSQs are prepared in the disjoint states, and all the quantities of the system varies periodically. It is observed that the Pancharatnam phase oscillates between a maximum and minimum values and has zero vale at $gt=(2\text{m}-1)5\pi$, where $m=1,2,3,...$. Also, the minimum value of the field purity corresponds to zero value of the geometric phase or the value $\pm\pi$. The squeezing only appears in the component $E(S_x)$ in a periodic manner. Interestingly, the appearance of the squeezing corresponds to the case which the field loss its purity and with no phase at the periodic $gt=(2\text{m}-1)5\pi$. On the other hand the evolution of $E(S_y)$ does not presents any squeezing at all and have zero value at the periodic time. From the discussion of Fig. 1, we can reach to conclude that there is a clear correlation between the field purity, geometric and the appearance of the entropy squeezing $E(S_x)$, since this connection is clearly appeared at the periodic time.

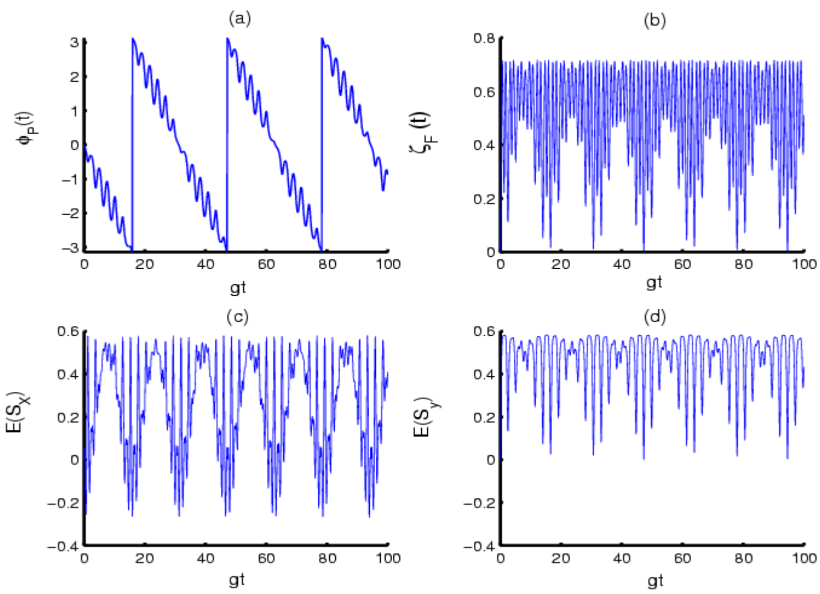

Figure-1: The evolution of: (a) the Pancharatnam phase ϕ_P (b)the field purity ξ_F, (c) the **SCQES** component $E(S_x)$, (d) the **SCQES** component $E(S_y)$ for weak coupling between the two charge qubits $J/g = 0.2$, and $\theta = 0$.

Now we are in a position to examine the effect of the initial atomic state preparation and weak strength coupling regime on the dynamical properties of different quantities. It is seen that all the quantum quantifiers are affected by strength coupling regime (see Fig. 2 and 3). It is seen that the Pancharatnam phase and the field purity have the same behavior with increasing the intensity of oscillations during the time evolution. Interestingly, the **SCQES** components $E(S_y)$ and $E(S_x)$ are strongly affected by the change of the initial state from the excited state to the superposition state. In this case there no squeezing at all on the **SCQES** components $E(S_y)$ and $E(S_x)$. Also, $E(S_y)$ has the same behavior of $E(S_x)$.

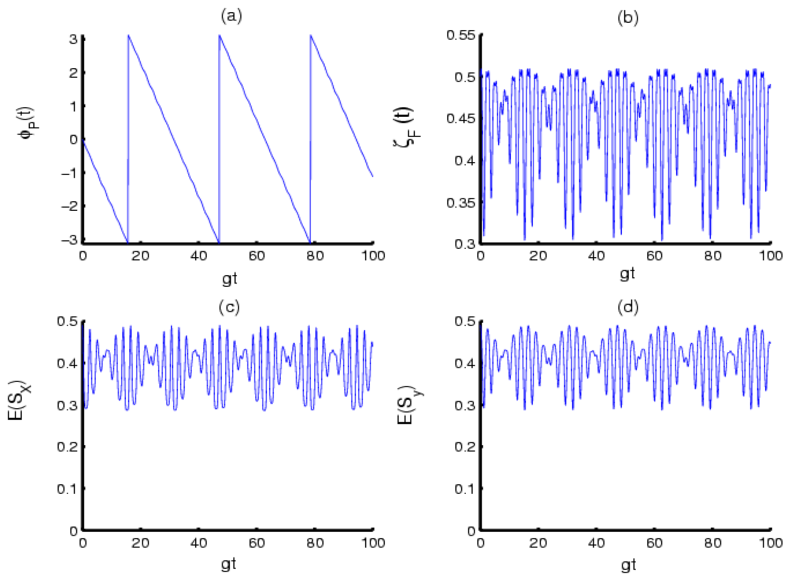

Figure-2: The evolution of: (a) the Pancharatnam phase ϕ_P (b) the field purity ξ_F, (c) the **SCQES** component $E(S_x)$, (d) the **SCQES** component $E(S_y)$ for weak coupling between the two charge qubits $J/g = 0.2$ and $\theta = \pi/4$.

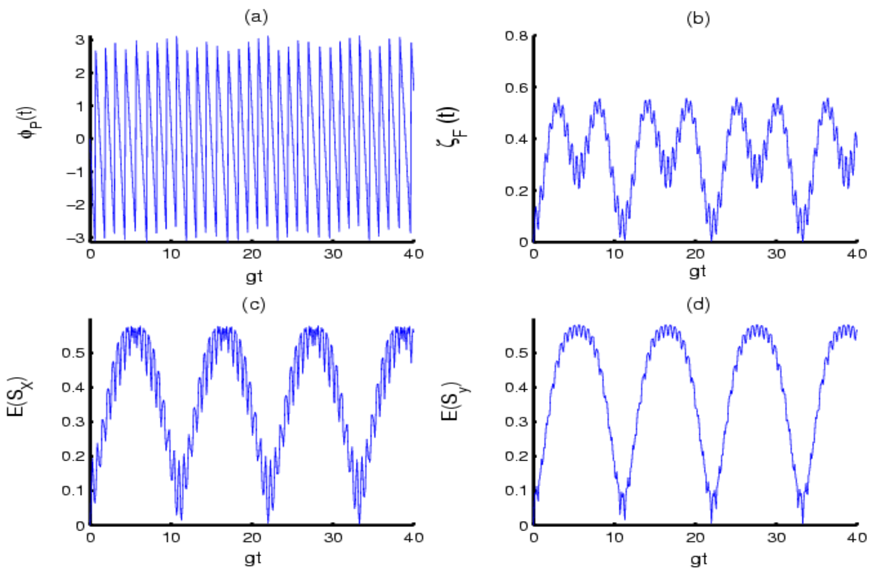

Figure-3: The evolution of: (a) Pancharatnam phase ϕ_P (b)field purity ξ_F, (c) The **SCQES** component $E(S_x)$, (d) The **SCQES** component $E(S_y)$ for weak coupling between the two charge qubits $J/g = 10$ and $\theta = 0$.

For further discussion the case of the strong coupling between the TSQs (i.e. $J/g = 10$), as comparing Fig. 3 with Fig. 2, one can see that the evolution of the correlation and squeezing quantifier has the same behavior with the increasing of the value of J/g, where, the evolution of the system is very interesting. It shows a tendency to form the doubly periodic structure is changed to be $gt = 11m, \ m = 1,2,3,...$, while, at this time the field purity and SCQES have zero values.

The Pancharatnam phase is clearly affected the strong coupling, where, it presents a new behavior. The distribution of the Pancharatnam phase oscillations is completely change with no disappearance during the evolution time.

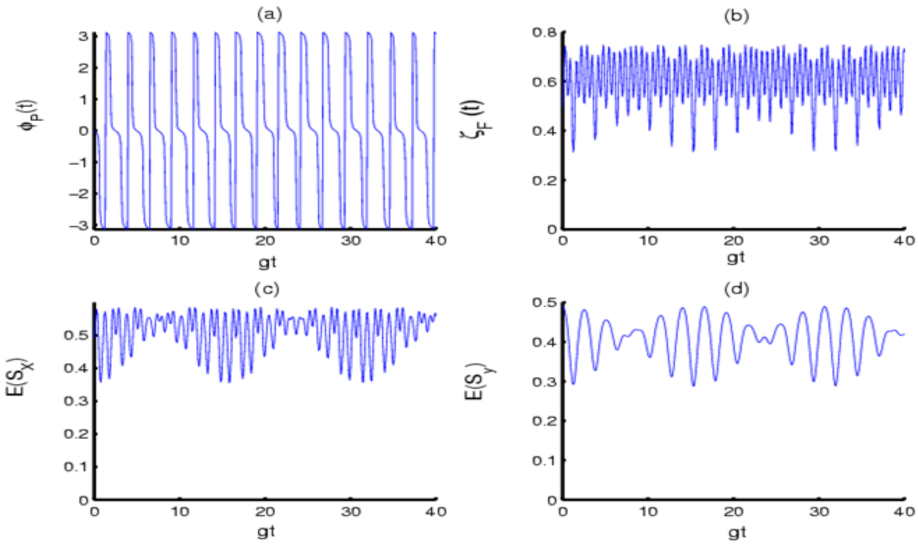

Figure-4: The evolution of: (a) Pancharatnam phase ϕ_P (b) field purity ξ_F, (c) The **SCQES** component $E(S_x)$, (d) The **SCQES** component $E(S_y)$ for weak coupling between the two charge qubits $J/g = 10$ and $\theta = \pi/4$.

From Figs (3 & 4) a reasonable comparison between the changing of the initial atomic state position from the excited state to the superposition state in the case strong regimes will enable us to understand the contribution of this effect of the kind of potential on the dynamical and physical properties of quantum quantifiers. The shape and intensity of the Pancharatnam phase oscillations is enhanced and being more periodic and regular. Also, no squeezing introduced by the SCQES components $E(S_x), E(S_y)$ and they have similar behavior with increasing the oscillations $\boldsymbol{E(S_y)}$ around the periodic time.

The promised results can be obtained from Fig (4-b), where, the purity of the field is enhanced and the field does not lost its purity during the time evolution.

6. CONCLUSION

It is well known that superconducting charge qubit devices can be used to prepare entangled states by adjusting some parameters. Due to their technical advantage and highly integrative characteristics, superconducting circuit quantum electrodynamic systems were considered to be very important in quantum information processing. In this paper we have discussed an artificial atomic system, which is consisting two superconducting charge qubits coupling with an LC-resonator. We focusing on the evolutionary properties of the field purity, Pancharatnam phase and single atom entropy squeezing of the system. There are showing that the evolutions of all physical quantities present an interesting structures and exhibit a regular and periodic structure. Also, the formation of the structure depends on the variation of the coupling strength of the two charge qubits and initial qubit state setting. Therefore, the results demonstrate that the double periodic structure may be a general property of two-qubit-cavity coupling systems. Also, the interesting and exciting results were one can obtained a high purity of the field in the case of the strong coupling regime, when, the two coupled superconducting qubits start the interaction from the superposition state. Our work shows how our proposed quantum system under a model that closely describes a realistic experimental scenario that can be implemented in the different tasks of quantum information and computational technologies with an optimal conditions.

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