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Domination parameters of $f_{n.r}$

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Abstract

The domination parameters of a graph G of order n has been already introduced. It is defined as $\mathrm{D} \subseteq \mathrm{V(G)}$ is a dominating set of G, if every vertex $\mathrm{v} \in \mathrm{V}-\mathrm{D}$ is adjacent to atleast one vertex in D. In this paper, we have established various domination parameters of k^{th} power of Path, and square of the Centipede graph, also we have studied the relation between this parameters and illustrated with an examples.

Keywords: Graph, Domination, flower graph, Cater pillar graph

INTRODUCTION: 1.0

A graph $G=(V,E)$, where V is a finite set of elements, called vertices and E is a set of unordered pairs of distinct vertices of G called edges. The degree of a vertex v in G is the number of edges incident on it. Every pair of its vertices are adjacent in G is said to be complete, the complete graph on$'n'$vertices is denoted by K_n.

Let u and v be the vertices of a graph G,$a\ u-v$ walk of G is an alternating sequences $u=u_o, e, u, e_2, u_2, \dots .u_{n-1}, e_n, v_n=v$ of vertices and edges beginning with vertex u and ending with vertex v such that $e_i=u_{i-1}u_i$ for all $i=1,2,\dots..,n$. The number of edges in a walk is called its length. A walk in which all the vertices are distance in called a path. A path on$'n'$ vertices is denoted by P_n. A closed path is called a cycle, a cycle on $'n'$ vertices is denoted by C_n. Let $G=(V,E)$ be a simple connected graph, for any vertex $v\in V$, the open neighborhood is the set $N(v)=\{u\in V/uv\in E\}$ and the closed neighborhood of v is the set $N[v]=N(v)\cup\{u\}$. For a set $S\subset V$, the open neighborhood of S is $N(S) = \bigcup N(v)\,, v \in s$ and the closed neighborhood of S is $N[s] = N(S) \cup S$.

Definition 1.1:

A set D$\subseteq$ V is a dominating set of G if every vertex v $\in$ V – D is adjacent to at least one vertex of D. We call a dominating set D is a minimal if there is no dominating set D' $\subseteq$ V (G) with D′ $\subset$ D and D′ $\neq$ D. Further we call a dominating set D is minimum if these is no dominating set D $\subseteq$ V(G) with | D' | < | D|. The cardinality of a minimum dominating set is called the domination number denoted by γ(G) and the minimum dominating set D of G is also called a γ- set.

Definition 1.2:

A dominating set D is said to be a total dominating set if every vertex in V is adjacent to some vertex in D. The total domination number of G denoted by γ_t (G) is the minimum cardinality of a total dominating set.

Definition 1.3:

A dominating set D of a graph G is an independent dominating set, if the induced sub graph <D> has no edges. The independent domination number γ_i (G) is the minimum cardinality of a independent dominating set.

Definition 1.4:

A dominating Set D is said to be connected dominating set, if the induced sub graph <D> is connected. The connected domination number γ_c(G) is the minimum cardinality of a connected dominating set.

Definition 1.5:

A dominating Set D of a graph G is said to be a paired dominating set if the induced sub graph <D> contains at least one perfect matching, paired domination number γ_p (G) is the minimum cardinality of a paired dominating set.

Definition 1.6:

A dominating Set D of G is a split dominating set if the induced sub graph <V – D> disconnected Split domination number γ_s (G) is the minimum cardinality of a split dominating set.

Definition 1.7:

A dominating Set D of G is a non split dominating set, if the induced sub graph <V – D> is connected. Non split domination number γ_{ns} (G) is the minimum cardinality of a non split dominating set.

Definition 1.8:

A dominating set D of a graph G is called a global dominating set, if D is also a dominating set of $\overline{G}$. The global domination number γ_g (G) in the minimum cardinality of a global dominating set.

Definition 1.9:

A dominating set D is called a perfect dominating set, if every vertex in V – D in adjacent to exactly one vertex in D. The perfect domination number γ_{pr}(G) is the minimum cardinality of a perfect dominating set.

Definition: 1.10:

A Graph G is called a m × n flower graph if it has m vertices which form a m cycle and m sets of n – 2 vertices which form n cycles around them m cycle so that each n cycle uniquely intersects with the m cycle on a single edge. This graph will be denoted by fm×n. It is clear that fm×n has m(n-1) vertices & mn edges. The n cycles are called the petals and the m cycles are called the center of fm×n. Then m vertices which form the center are all of degree 4 & all the other vertices have degree 2.

Definition: 1.11:

A caterpillar is a tree with the property that a path remains if all leaves are deleted.

The n-sunlet graph is the graph on $2n$ vertices obtained by attaching n pendant edges to a cycle graph C_n

Definition: 1.12:

The friendship graph F_n can be constructed by joining n copies of the cycle graph C_3 with a common vertex

n −Centipede graph is a tree on $2n$ vertices obtained by joining the bottom of n −copies of the path graph P_2 laid in a row with edges and is denoted by $\mathbb{C}_n$.

Definition 1.13.: The Harary graph Hn,k is a graph on the n vertices $\{v_1, v_2, . . . , v_n\}$ defined by the following construction: • If k is even, then each vertex vi is adjacent to $v_{i\pm1}, v_{i\pm2},. . . , v_{i\pm k\ 2}$, where the indices are subjected to the wraparound convention that $v_i \equiv v_{i+n}$ (e.g. v_{n+3} represents v_3). • If k is odd and n is even, then $H_{n,k}$ is $H_{n,k-1}$ with additional adjacencies between each vi and $v_{i+\ n\ 2}$ for each i. • If k and n are both odd, then $H_{n,k}$ is $H_{n,k-1}$ with additional adjacencies

$\{ v_1, v_{1+ n-1\ 2} \}, \{v_1, v_{1+ n+1\ 2} \}, \{v_2, v_{2+ n+1\ 2} \}, \{v_3, v_{3+ n+1\ 2} \}, \cdots , \{v_{n-1\ 2} , v_n\}$

Definition 1.14:

Let x be any real value, then its upper sealing of x is denoted as x and is defined

$\lceil x \rceil = \begin{cases} x & \text{if } x \text{ is an integer} \\ k, & \text{where k is an integer lies in the interval } x < k < x + 1 \end{cases}$

the lower sealing of x is denoted as λxμ and is defined by

$\lfloor x \rfloor = \begin{cases} x & \text{if } x \text{ is an integer} \\ k, & \text{where k is an integer lies in the interval } x - 1 < k < x \end{cases}$

Definition 1.15:

The k^{th} power of a graph G is a graph with the same set of vertices of G and an edge between two vertices if there is a path of length almost k between them.G^2 is called the square of G, G^3 is called the cube of G etc.

Lemma 2.1

Let G be a connected graph with δ(G) ≥ 2, them γ(G) + γ'(G) = n if and only if G = P_4 or C_4.

Lemma 2.2

Let G be a connected graph with δ = 1 and Δ = n then γ (G) + γ' (G) = n + 1 if and only if G = $k_{1, n}$.

Lemma 2.3

For any tree with n ≥ 2 with more then two pendent vertices then there exists a vertex v ∈ V such that γ (T – v) = γ (T).

Theorem: 2.4

The domination number of the flower graph $G = f_{n.r}$ is defined as

$$\gamma(G) = \begin{cases} \lceil n/2 \rceil + (k-1)n & if\ r = 3k \\ kn & if\ r = 3k+1 \\ \left\lceil \frac{n}{3} \right\rceil + kn & if\ r = 3k+2 \end{cases}$$

Proof:

Flower graph $f_{n,r}$ is represented in figrme 1.1 as below

The vertices of G are denoted by $V = \{v_1, v_2, \dots, v_n;\ v_{11}, v_{12}, \dots, v_{1,r-2}; \dots, v_{n,1}, \dots, v_{n,r-2}\}$

That is $V = \{v_i, v_{ij} / i = 1, \dots, n; j = 1,2, \dots, r-2\}$

each petal p_i containing r

vertices such that

$\{v_i, v_{il}, v_{i2}, \ldots . v_{i,r-2}, v_{i+1}\}$

Case (i)

If n is even and $r = 3k$

then select the vertices

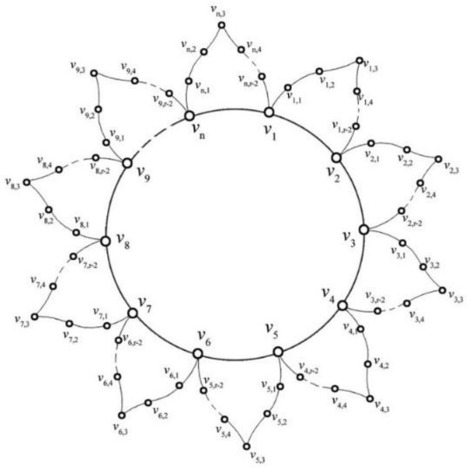

Figure 1. 1

$$D = \{v_1, v_3, \ldots, v_{n-1}, v_{1,3}, v_{1,6}, v_{1,9}, \ldots, v_{3,3}, v_{3,6}, v_{3,9}, \ldots, v_{5,3}, v_{5,6}, \ldots, v_{2,2}, v_{2,5}, v_{4,2}, v_{4,5} \ldots\}$$

That is $D = \left\{v_{2i-1}; v_{2i-1,3j}; v_{2i,3j-1} / i = 1,2, \ldots \lceil n/3 \rceil, j = 1,2, \ldots, \frac{r-2}{3}\right\}$

is the required dominating set of G

Therefore, $|D| = \lceil n/2 \rceil + (k-1)n \quad where\ k = \lceil r/3 \rceil$

$$= \lceil n/2 \rceil + (k-1)n$$

$[since\ \ n\ is\ even\ \lceil n/2 \rceil = \lceil n/2 \rceil]$

b) n is odd and $r = 3k$

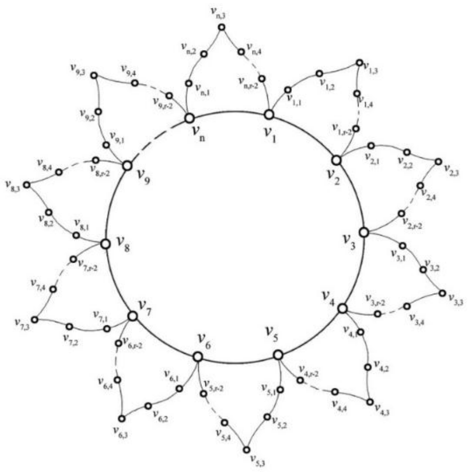

Figure 1. 1

Select the vertices of G

$$D = \left\{v_{2i-1}; v_n; v_{2i-1,3j}; v_{2i,3j-1} / i = 1,2 \dots, n;\ j = 1,2, \dots, \left[\left[\frac{r-2}{3}\right]\right]\right\}$$

is the required dominating set of G. and its cardinality

$|D| = [{}^{n}/_{2}] + 1 + (k-1)n$

$= [{}^{n}/_{2}] + (k-1)n$ $\quad [Since\ \ n\ is\ odd\ \ [{}^{n}/_{2}] = [{}^{n}/_{2}] + 1]$

Therefore, $\gamma(G) = [{}^{n}/_{2}] + (k-1)n.$

Case (ii)

If $r = 3k+1$, each petal contains the number of vertices in the form of $\{\ 3i + 1/i = 1,2,3 \dots\}$

$$D = \{v_{i,3j-2}/i = 1,2, \dots, n\ and\ j = 1,2, \dots, r-1\}$$

is the required domination set of G

Therefore, $|D| = n\left[\frac{r-1}{3}\right]$

$= n\left[\frac{3k+1-1}{3}\right]$ $\quad [\ since\ \ r = 3k+1]$

$= nk$ $\quad where\ k = [{}^{r}/_{3}]$

Case (iii)

If $r = 3k+2$

In this case each petal containing the number of vertices is of the form $3i + 2/i = 1,2,3 \dots$

(a) Suppose $n = 3m + 1$, the vertex set of G is

$V = \{v_i, v_{ij} / i = 1,2, \dots, n \; and \; j = 1,2, \dots, r-2\}$ Choose the vertices

$$D = \left\{v_{3i-2}; v_{n,r-1}; v_{3i-1,3j-1}; v_{3i,3j-2} / i = 1,2, \dots, n; j = 1,2, \dots, \left\lceil \frac{r-2}{3} \right\rceil \right\}$$

is the required dominating set of G and its cardinality is

$$|D| = \lceil n/3 \rceil + 1 + n\left(\frac{3k+2-2}{3}\right)$$
$$= \left\lceil \frac{n}{3} \right\rceil + nk$$

(b) Suppose $n \neq 3m + 1$ and $r = 3k + 2$ select the vertices

$$D = \left\{v_{3i-2}; v_{3i-2,3j}; v_{3i-1,3j-1}; v_{3i,3j-2} / i = 1,2, \dots, n; j = 1,2, \dots, \left\lceil \frac{r-2}{3} \right\rceil \right\}$$

is the required dominating set of G and its cardinality is,

$$|D| = \lceil n/3 \rceil + n\left(\frac{r-2}{3}\right)$$
$$= \left\lceil \frac{n}{3} \right\rceil + n\left\{\frac{3k+2-2}{3}\right\}$$
$$= \left\lceil \frac{n}{3} \right\rceil + nk$$

Therefore,

$$\gamma(G) = \begin{cases} \lceil n/2 \rceil + (k-1)n & if \;\; r = 3k \\ kn & if \;\; r = 3k+1 \\ \lceil n/3 \rceil + kn & if \;\; r = 3k+2 \end{cases}$$

Theorem: 2.5

The domination number γ $of \left(f_{n,3}\right)^2 \; is \lceil n/4 \rceil$

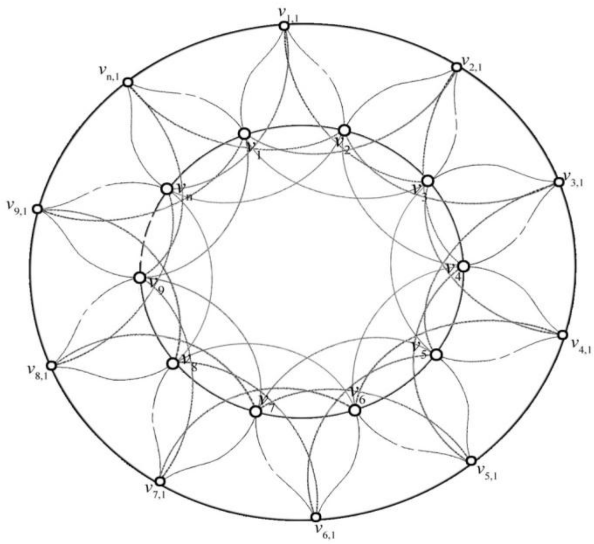

Figure 1. 3

Proof:

The graph $G = (f_{n,3})^2$

is graph in figure 1.3

as follows

The vertices of G are

$V = \{v_i; v_{i,1}/i = 1,\dots,n\}$ select the vertices

$$D(G) = \left\{\begin{matrix} v_{4i-2}, v_n & if\ n = 4n+1 \\ v_{4i-2} & if\ n \neq 4n+1 \end{matrix} \quad i = 1,2,\dots,n\right\}$$

Now the cardinality of D is

$$|D| = \left\{\begin{matrix} \lceil n/4 \rceil + 1 & if\ n = 4n+1 \\ \lceil n/4 \rceil & if\ n \neq 4n+1 \end{matrix}\right\}$$

$$\Rightarrow |D| = \left\{\left\lceil \frac{n}{4} \right\rceil for\ all\ n \qquad \left[since\ \left\lceil \frac{n}{4} \right\rceil + 1 = \lceil n/4 \rceil\ \right]\right.$$

Theorem: 2.6 The domination number of $G = (f_{n,4})^2\ is \lceil n/3 \rceil$

Proof:

Let the graph $G = (f_{n,4})^2$ and its

Vertices are $V = \{v_i; v_{i,1};\ v_{i,2}/\ i = 1,\dots,n\}$

Case (i) If $n = 3k + 1$, then select the vertices

$D = \{v_{3i-1,} v_n \,/ i = 1, \dots, n\}$ $\quad if\ n = 3k + 1$ and select

$$D = \{v_{3i-1}/i = 1,2, \dots, n\} \quad if\ n \neq 3k + 1$$

that is $D = \begin{Bmatrix} v_{3i-1} & if\ \ n \neq 3k + 1 \\ v_{3i-1}; v_n & otherwise\ for\ all\ \ i = 1,2, \dots, n \end{Bmatrix}$ is the required dominating set of G

Therefore, $\quad |D| = \begin{cases} \lceil n/3 \rceil + 1 & if\ n = 3K + 1 \\ \lceil n/3 \rceil & if\ n \neq 3k + 1 \end{cases}$

$\Rightarrow |D| = \lceil n/3 \rceil \quad for\ all\ n, \qquad \{since\ n = 3k + 1,\ \lceil n/3 \rceil + 1 = \lceil n/3 \rceil\}$

$\Rightarrow \gamma(G) = \left\lceil \frac{n}{3} \right\rceil$

Theorem: 2.7

Let G he a caterpillar graph with $3n$ vertices

Then, $\gamma(G) = r_t(G) = \gamma_s(G) = r_{ct}(G) = n$ and

$$\gamma_{ns}(G) = \gamma_p(G) = r_i(G) = 2$$

Proof:

The caterpillar graph with $3n$ vertices are given as figure 1.4

The vertices of G are denoted by

$V = \{v_i; v_{i,1}; v_{i,2}/i = 1,2, \dots, n\}$ choose the vertices

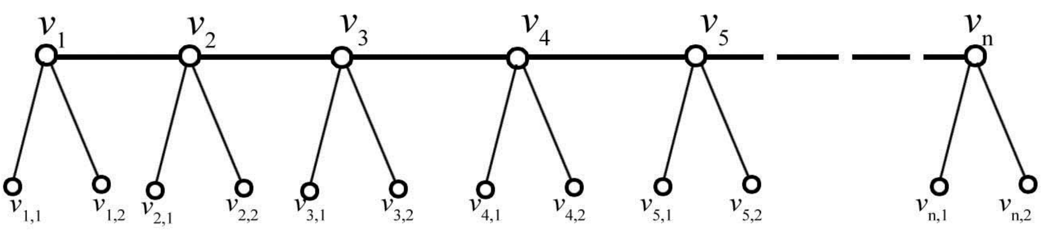

Figure 1. 4

$D = \{v_i/i = 1,2, \dots, n\}$ is the required dominating set of G. Also all the elements of G are adjacent with the element of D which satisfies the property of totally connected domination and the induced sub graph of G is connected in G.

$V(G) - D$ is a disconnected graph and $\quad |D| = n$

Therefore, $\quad D_s(G) = \{v_i/i = 1,2, \dots, n\}$ $\quad$ and $\quad \gamma_s(G) = n$

Hence, $\gamma(G) = \gamma_t(G) = \gamma_s(G) = \gamma_{ct}(G) = n$

Let, $D_1 = \{v_{i,1}; v_{i,2}/i = 1,2, \dots, n\}$ and $|D_1| = 2n$

Now the elements of D_1 are independent set in G, also removal of D_1 from $V(G)$ we get a connected graph, as P_n also each pair $(v_{i,1}, v_{i,2})$ from a matching of G.

Therefore, $\gamma_i(G) = \gamma_{ns}(G) = \gamma_p(G) = 2n$

Theorem: 2.8

G be any caterpillar graph win $3n$ vertices its base elements are $\{v_i/i = 1,2 \dots, n\}$ and $\{v_i'/i = 1,2 \dots, n\}$ is the duplication of v_i and G_k is the duplication of the graph G then

$$\gamma(G_k) = \gamma_s(G_k) = \gamma_p(G_k) = \gamma_i(G_k) = \gamma_{ct}(G_k) = \gamma_t(G_k) = n$$

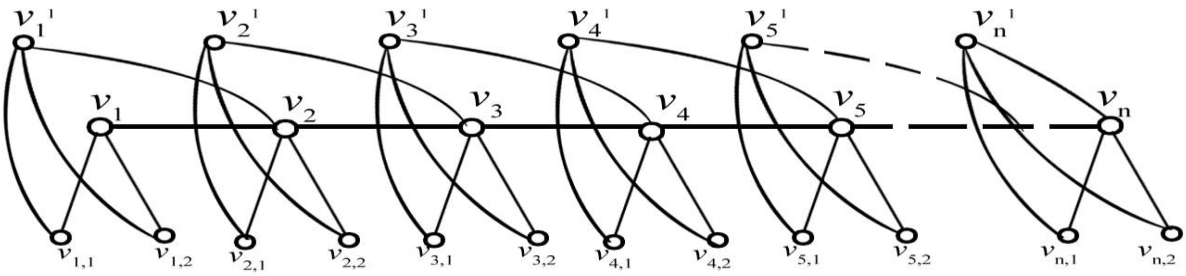

Figure 1. 5

Proof:

The base vertices of G, $V = \{v_i/i = 1, \dots, n\}$ and the duplication of V is denoted by $V' = \{v_i'/i = 1,2, \dots, n\}$ now the duplication V' *of* V forms a dominating set of G_k.

Therefore, $\gamma(G_k) = n$, also the element of V' are independent and $V(G_k) - V'$ is connected.

Therefore, $\gamma(G_n) = \gamma_i(G_k) = \gamma_{ns}(G_n) = n$

the vertices $V = \{v_i/i = 1, \dots, n\}$ is also a dominating set of G_k, the elements of G_k are connected, also it forms a matching, removal of $V(G_k) - V$ is disconnected and its cardinality is n . Therefore, $\gamma_s(G_k) = \gamma_p(G_k) = \gamma_{ct}(G_k) = n.$

Result: 2.9

G be any there regular graph with $4n$ vertices and $\gamma(G) = n$ then G has a perfact dominating set.

Proof:

G be a three regular graph with $4n$ vertices and $\gamma(G) = n$, that is each vertex $v_i \in G$

dominates its neighbors, $D = \{v_i / \ i = 1, \dots, n\}$ and $|D| = n$, which implies $N(v_i) \cap N(v_j) = \varphi$ for all $v_i, v_j \in D$, which implies D is a perfect dominating set of G.

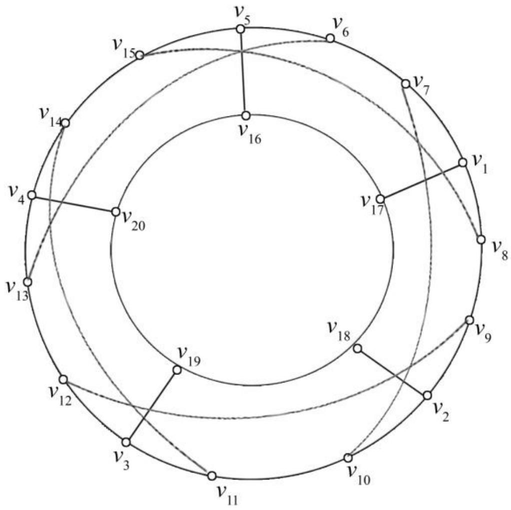

Figure 1. 6

Example

The graph represented in figure 1.6

$$\bigcap_{i=1}^{5} N(v_i) = \varphi, \qquad i = 1, \dots, 5$$

Therefore, $D = \{v_1, v_{2,} v_3, v_4, v_5\}$

is the perfect dominating set of G.

also D is the independent and

split domination of the graph.

Hence, $\gamma_p(G) = \gamma_i(G) = 5$

Result: 2.10

Let G be the n-sunlet graph then,

$$\gamma(G) = \gamma_i(G) = \gamma_{ct}(G) = \gamma_p(G) = \gamma_t(G) = \gamma_s(G) = \gamma_{ns}(G) = n$$

Theorem:2.11

Let G be any H_n graph ($H_n$from the path P_n by the clique cover contribution) then $\gamma(G) = \gamma_i(G) = \gamma_s(G) = \lceil n/2 \rceil$

Proof:

H_n g raph is given in figure (1.7)

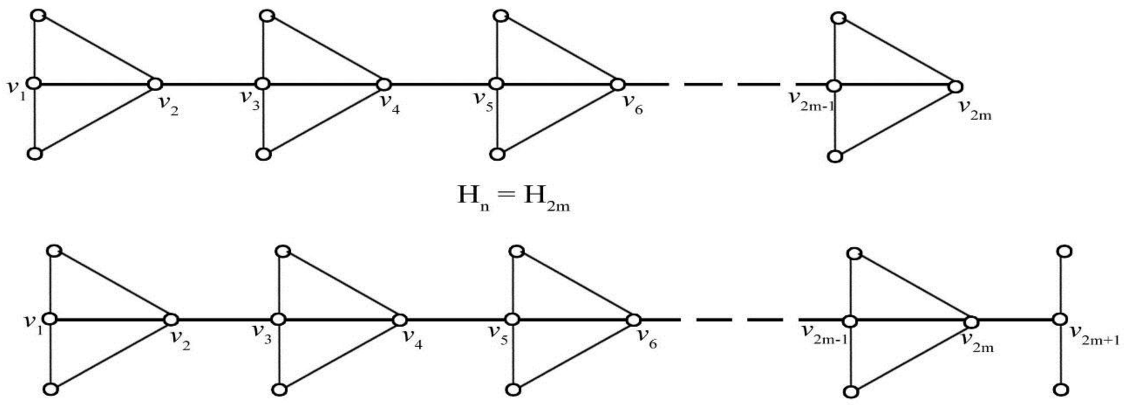

Figure 1.7

The base vertices of $G = H_n$ are denoted by $V = \{v_i / i = 1, \ldots, n\}$

If $n = 2m$ choose

$D = \{v_{2i-1} / i = 1, \ldots, m\}$ and

If $n = 2m + 1$, $D = \{v_{2i-1}, v_n / i = 1, \ldots, m\}$ is the required dominating set of G, the elements of D are independent in G , the induced sub graph $V(G) - D$ is a disconnected in G.

Therefore, $|D|(G) = |D_i(G)| = |D_s(G)| = \left\lceil \frac{n}{2} \right\rceil = \lceil n/2 \rceil$ $if\ n\ is\ even$

$|D(G)| = |D_i(G)| = |D_s(G)| = \lceil n/2 \rceil + 1$ if n is odd

$= \left\lceil \frac{n}{2} \right\rceil$

Hence, $\gamma(G) = \gamma_i(G) = \gamma_s(G) = \left\lceil \frac{n}{2} \right\rceil$ for all n.

Result: 2.12

Let G be a Fan graph F_n with $n + 1$ vertices then $\gamma(G) = \gamma_i(G) = \gamma_{ns}(G) = 1$ and

$$\gamma_t(G) = \gamma_p(G) = \gamma_s(G) = \gamma_{ct}(G) = 2$$

Result: 2.13

Let F_n be a fan graph and G be a new graph after the duplication of the vertex v' by v and $\deg(v) = n$ then

$\gamma(G) = \gamma_i(G) = \gamma_p(G) = \gamma_{ns}(G) = \gamma_{ct}(G) = 2$

Theorem : 2.14

Let G_1 and G_2 be any two graphs with domination numbers respectively on γ_1 and γ_2 then $\gamma\ (G) =$ min $\{\gamma_1 , \gamma_2\}$ when $G = G_1 + G_2$

Proof:

Let $\gamma\ (G_1) = \gamma_1$ and $\gamma\ (G_2) = \gamma_2$ without lose of generality we take $\gamma_1 \leq \gamma_2$ now γ_1 is the domination number and D_1 is the minimum dominating set of G_1 that is all the elements of $V(G_1) - D_1(G_1)$ is adjacent with all the element of D_1 in G_1

Since $G = G_1 + G_2$ then all the elements of G_2 is adjacent with every element of G_1 that is every element of G_2 is adjacent with the element of D_1

$\Rightarrow$ all the elements of $V(G_1) - D_1$ and $V(G_2)$ is adjacent with the element of D_1

$\Rightarrow D_1$ is a dominating set of $G_1 + G_2$

$\Longrightarrow \gamma\ (G) = \gamma_1$

Theorem:2.15

Let G be any path containing n vertices $n \geq 3$ then $H = G - v$ be any sub graph of G where v is any cut vertex of G and $n \neq 3m + 1$ then $\gamma(G) \leq \gamma\ (H)$.

Proof:

Let G be a path containing $'n'$ vertices and the vertices of G are $V = \{v_1, v_2, v_3, \ldots, v_n\}$ with $d(v_1) = d(v_n) = 1$ and $d(v_i) = 2$ for all $i = 2, \ldots, n-1$

Case (i) $H = G - v_1$ *or* $H = G - v_n \Rightarrow H = P_{n-1}$ *and* $n \neq 3m + 1$

we know that, $\gamma(P_n) = \lceil n/3 \rceil \Rightarrow \gamma(H) = \gamma(P_{n-1})$

$$= \left\lceil \frac{n-1}{3} \right\rceil$$

$= \lceil n/3 \rceil$ $[\because n \neq 3m + 1]$

$$= \gamma(P_n)$$

$$= \gamma(G)$$

Case (ii)

$H = G - v_i, i = 2,3, \ldots, n-1$

Let v_i be the i^{th} vertex in the path P_n, then H containing two paths P_1 and P_2 such that

P_1 contains $i-1$ vertices from v_1 to v_{i-1} and P_2 containing $n-i$ vertices from v_{i+1} to v_n

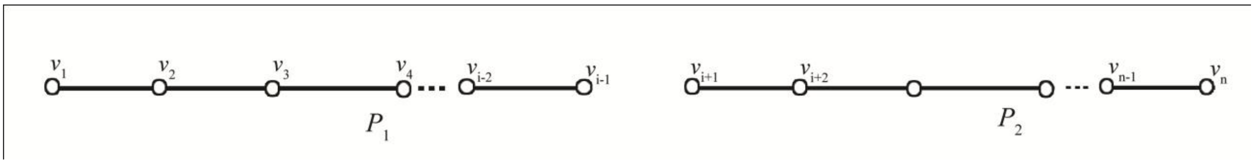

Figure 1.8

since $n \neq 3m+1$, $\quad L(P_1)+L(P_2) \neq 3m$

Therefore, $L(P_1)+L(P_2)=3m-1$ *or* $L(P_1)+L(P_2)=3m-2$

Case (i)

$L(P_1)+L(P_2)=3m-1$ and $L(P_n)=3m$

$$i-1+n-i=3m-1 \qquad ------- \ (A)$$

From (A) , P_1 or P_2 containing any one of the following form

(i) either $i-1 \equiv 0(mod\ 3)$ and $n-i \equiv 2\ (mod\ 3)$ or

$$i-1 \equiv 1(mod\ 3) \quad and \quad n-i \equiv 1(mod\ 3)$$

Let $i-1=r_1$ *and* $n-i=r_2$ then

$$\left\lceil\frac{i-1}{3}\right\rceil = l_1 \ \ and \ \ \left\lceil\frac{n-i}{3}\right\rceil = l_2$$

$\Rightarrow \left\lceil\frac{i-1}{3}\right\rceil = l_1$ and $\left\lceil\frac{n-i}{3}\right\rceil = l_2+1$

$\Rightarrow l_1+l_2+1=m$

$\Rightarrow \gamma(P_1)+\gamma(P_2)=\gamma(G)$

$\Rightarrow \gamma(H)=\gamma(G) \qquad ------- (i)$

Suppose , $i-1=1(mod\ 3)$ *and* $n-i=1(mod\ 3)$

Let $\left\lceil\frac{i-1}{3}\right\rceil = l_1$ *and* $\left\lceil\frac{n-i}{3}\right\rceil = l_2$

$\Rightarrow \left\lceil\frac{i-1}{3}\right\rceil = l_1+1$ *and* $\left\lceil\frac{n-i}{3}\right\rceil = l_2+1$

$\Rightarrow l_1+l_2+2>m \qquad [\because l_1+l_2+1=m]$

$\Rightarrow \gamma(P_1)+\gamma(P_2) \geq \gamma(G) \qquad ------- (ii)$

Therefore, $\gamma(G) \leq \gamma(H)$ when $n=3m$

case(ii)

$L(P_1) + L(P_2) = 3m - 2$

then $P1$ or P_2 containing the number of vertices in any one of the following form

either $i - 1 \equiv 0\ (mod\ 3)$ and $n - i \equiv 1\ (mod\ 3)$ or vice verse

or $i - 1 \equiv 2(mod\ 3\ )$ and $n - i \equiv 2(mod\ 3)$

Suppose, $i - 1 \equiv 0(mod\ 3\ )$ and $n - i \equiv 1(mod\ 3)$

Let $\left\lfloor\frac{i-1}{3}\right\rfloor = l_1$ and $\left\lfloor\frac{n-i}{3}\right\rfloor = l_2$

Then $\left\lceil\frac{i-1}{3}\right\rceil = l_1$ and $\left\lceil\frac{n-i}{3}\right\rceil = l_2 + 1$

$\Rightarrow$ $l_1 + l_2 + 1 = m$

Therefore, $\gamma(P_1) + \gamma(P_2) = \gamma(G)$

$\gamma(H) = \gamma(G)$ $-----$ (iii)

suppose, $i - 1 \equiv 2(mod\ 3)$ and $n - i \equiv 2(mod\ 3)$

Let $\left\lfloor\frac{i-1}{3}\right\rfloor = l_1$ and $\left\lfloor\frac{n-i}{3}\right\rfloor = l_2$

$\Rightarrow \left\lceil\frac{i-1}{3}\right\rceil = l_1 + 1$ and $\left\lceil\frac{n-i}{3}\right\rceil = l_2 + 1$

$\Rightarrow l_2 + l_2 + 2 > m$

$\Rightarrow \gamma(P_1) + \gamma(P_2) > m$ $------$ (iv)

In all cases, $\gamma(G) \leq \gamma(H)$

Corollary : 2.16

suppose $G = P_n$ having n vertices if $n = 3m + 1$ then,

$\gamma(H) \leq \gamma(G)$ $if\ H = G - v_i/i = 3k + 1,$ $k = 0,1,2\ldots$

Proof:

Let $G = P_n$, then the vertices of P_n are $V = \{v_i/i = 1, \ldots, n\}$

$$H = G - v_i/i = 3k + 1, k = 0,1,2\ldots$$

Case (i) $H = G - v_1$ or $H = G - v_n$

H is a path containing $3m$ vertices then

$$\gamma(H) = \left\lceil\frac{3m}{3}\right\rceil = m \quad and \quad \gamma(G) = \left\lceil\frac{n}{3}\right\rceil = \left\lceil\frac{m+1}{3}\right\rceil = m + 1$$

Therefore, $\gamma(H) < \gamma(G)$.

Case (ii)

$H = G - v_i/i = 3k+1, k = 1,2,\ldots$ then H is a graph containing two paths P_1 and P_2.

$L(P_1) = i-1;\ \ L(P_2) = n-i$ with $i-1 \equiv 0(mod\ 3)$ and $n-i \equiv 0(mod\ 3)$

[since $i = 3k+1; k = 1,2,\ldots$]

$$\Rightarrow \left\lceil \frac{i-1}{3} \right\rceil = l_1 \quad and \quad \left\lceil \frac{n-i}{3} \right\rceil = l_2$$

$$\Rightarrow l_1 + l_2 < m+1 \qquad \left[\because \left\lceil \frac{n}{3} \right\rceil = m+1\right]$$

$$\Rightarrow \gamma(P_1) + \gamma(P_2) \le \gamma(G)$$

Hence the proof

Theorem:2.17

Let G be a path having n vertices $v_k{}' \in G_k$ be the duplication of $v_k \in G$ then

$\gamma(G) \le \gamma(G_k)$

Proof:

Let $V = \{v_1, v_2, \ldots, v_n\}$ be the vertex set of $G = P_n$

If $n = 3m$ and $v_k = v_{3i-1}/i = 1,2,3,\ldots$ then

$D(G_k) = \{v_{3i-1}, v_{k-1}/i = 1,2,3 \ldots\}$ is the dominating set of G_k and

$$|D(G_k)| = \left[{}^{n}/_{3}\right] + 1$$

$$= \left\lceil {}^{n}/_{3} \right\rceil + 1 \qquad \left[\because n = 3m \Rightarrow \left\lceil {}^{n}/_{3} \right\rceil = \left[{}^{n}/_{3}\right]\right]$$

If $v_k \ne v_{3i-1}/i = 1,2,\ldots$ then

$D(G_k) = \{v_{3i-1}/i = 1,2,\ldots,\}$ is the required dominating set of G and $|D(G_k)| = \left[{}^{n}/_{3}\right] = \left\lceil {}^{n}/_{3} \right\rceil$

If, $n = 3k+1$ and $v_k = v_{3i-1}/i = 1,2,\ldots$

$D(G_k) = \{v_{3i-2}, v_n/i = 1,2,\ldots\}$ is the required dominating set of G and its cardinality $|D(G_k)| = \left[\ {}^{n}/_{3}\right] + 1 = \left\lceil {}^{n}/_{3} \right\rceil$

Suppose, $v_k \ne v_{3i-1}/i = 1,2,\ldots$ then the required dominating set of G_k is

$D(G_k) = \left\{v_{3i-1,} v_{n-1}/i = 1,2,\ldots\right\}$ and its cardinality is

$$|D(G_k)| = \left[{}^{n}/_{3}\right] + 1 = \left\lceil{}^{n}/_{3}\right\rceil$$

$$n = 3m + 2 \ and \ v_k = v_{3i-1}/i = 1,2, \dots$$

The required dominating set of G_k is

$D(G_k) = \{v_{3i-1}, v_{n-1}/i = 1,2, \dots\}$ and its cardinality is

$|D(G_k)| = \left[{}^{n}/_{3}\right] + 1 = \left\lceil{}^{n}/_{3}\right\rceil$

Suppose, $n = 3m + 2$ and $v_k \neq v_{3i-1}$ then

$D(G_k) = \{v_{3i-2}, v_{n-1}/i = 1,2, \dots\}$ is the required dominating set and its cardinality is

$|D(G_k)| = \left[{}^{n}/_{3}\right] + 1 = \left\lceil{}^{n}/_{3}\right\rceil$

Therefore, in all cases $\left[{}^{n}/_{3}\right] \leq \gamma(G_k) \leq \left[{}^{n}/_{3}\right] + 1$

Hence, $\gamma(G) \leq \gamma(G_k)$

Hence the proof.

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