Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 13, Number 12 (2017), pp. 8225-8246

http://www.ripublication.com

Supply Chain Deteriorating Inventory System for Retailer Partial Trade Credit Policy for Trapezoidal Type Demand

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Abstract

For items like fashion goods, Mobile phones and others, it is observed that, rate of demand is trapezoidal type function of time. Retailer’s ordering policy is formulated for the items having trapezoidal type demand rate. It is assumed that the retailer is dominant player in supply chain. The supplier offers credit period to the retailer and the retailer offers partial trade credit to his customer. The objective is to minimize total cost of an inventory system per time unit from retailer’s point of view. An algorithm is presented to find optimal ordering policy by retailer. Formulation is numerically validated, sensitivity analysis is carried out and managerial issues are derived.

Keywords: Supply chain, Deterioration, Partial trade credit, Trapezoidal type demand.

1. INTRODUCTION:

In current global and competitive market, in order to encourage sales and attract more customers, the supplier allows to delay payments for some fixed time, known as credit period; to his customers. Trade credit is a short term financing offered by wholesaler or manufacturer to retailer. Trade credits have been studied broadly in two categories. First is mainly focused on default financial risks. Second approach is through Operation Research, where researches from management sciences derives formulations of inventory systems and provides set of management decisions. One of the earliest EOQ model, which investigates trade credit is given by Goyal (1985); thereafter Chung (1998) derived theorems to determine the EOQ under the condition of permissible delay in payments. Under the condition of trade credit, Shah (1993a, 1993b, 2004) and Aggarwal and Jaggi (1995) derived inventory models with constant rate of deterioration. Model was extended for varying rate of deterioration by Chang et. al.(2002). Jamal et. al. (1997) and Chang and Dye (2001) considered shortages. Hwang and Shinn (1997) gave retailer's ordering and pricing policy for exponentially deteriorating items under permissible trade credit. Teng (2002) assumed that the selling price and the purchase price are not equal to rectify Goyal's model (1985). Shinn and Huang (2003) determined the retailer's optimal sale price and order size simultaneously under the condition of order – size – dependent delay in payment. Chung and Huang (2003) explored this problem by considering finite production rate and gave an algorithm to determine the retailer's optimal ordering policy. Huang and Chung (2003) extended Goyel's model (1985) to cash discount policy against early payment. The related articles and their cited references are by Chung et al. (2003), Chung and Liao (2004), Chang (2004), Huang (2004), Chung et al. (2005), Chung and Liao (2006), Huang (2007). In above stated articles, it is assumed that the supplier offers a credit period to his retailer. During this credit period, the retailer will earn interest on the accumulated revenue i.e. customer have to pay for the purchased goods immediately. This means the retailer avails trade credit from his supplier but would not pass it to the customer. This situation is called one level of trade credit. Such situation is impractical in current competitive market. Huang (2003) assumed that the retailer will offer the trade credit to boost his customer's demand to develop supply chain inventory model. Huang and Hsu (2008) derived supply chain model under the assumption that the retailer receives credit period from the supplier and the retailer just offers the partial trade credit to his customer. Shah et al. (2011) extended the model by assuming time dependent demand.

In a market where demand of the product is of trapezoidal type, above models cannot be used. For the first time, Hill (1995) formulated an inventory model with ramp type demand rate. In case of ramp type demand rate, the rate of demand increases linearly at the beginning, then it becomes constant until the end of replenishment cycle. Such demand pattern is mostly observed in new brand consumer goods which are likely to be introduced in the market. The demand rate of such products is generally increasing function of time at some extent, and then it becomes constant. Many researchers have studied inventory models with ramp type demand. Cheng and Wang (2009) extended this idea from ramp type demand to trapezoidal type demand. Cheng et al. (2011) extended the model for deteriorating items and by allowing shortages, with partial backlogging. Shukla and Suthar (2016) discussed an inventory model for deteriorating items for trapezoidal type demand with partially backlogged shortages.

In this article, an idea is explored when demand of an item is of trapezoidal type and rate of deterioration is constant. The retailer's total cost per time unit is minimized with respect to ordering policy. The model analyzes the effect of retailer's down payment, change in unit selling price, change in demand rate etc. on the retailer's managerial decisions.

2. ASSUMPTIONS AND NOTATIONS

The following assumptions and notations are used in the formulation of mathematical form of the proposed model.

1. The inventory system under consideration deals with a single item. Replenishment rate is infinite and the lead time is zero or negligible. The length of planning horizon is infinite. Model does not possess shortages.

2. The function $I(t)$ represents level of an inventory at any instant of time t, $0 \le t \le T$ where T is cycle time.

3. The demand $R(t) = \begin{cases} a(1+b_1 t) & ; \quad 0 \le t \le \lambda_1 \\ a(1+b_1\lambda_1) & ; \quad \lambda_1 \le t \le \lambda_2 \\ a(1+b_1\lambda_1)e^{-b_2(t-\lambda_2)} & ; \quad \lambda_2 \le t \le T \end{cases}$, where $a > 0$ is scale of parameter of demand and $0 < b_1, b_2 < 1$ is rate of change of demand,

4. During the ordering cycle, the item deteriorates with a constant rate, say $\theta\,(0 < \theta < 1)$. Moreover, deteriorated items is neither repaired nor replaced during cycle time.

5. EOQ Q (a decision variable), is an initial level of stock to the inventory system.

6. We consider C is the purchase cost / unit; P is the selling price / unit; h is the holding cost / unit / year; I_e is the interest earned / \$ / year; I_c is the interest charged / \$ / year ($I_e < I_c$); A is an ordering cost per order; $K(T)$ is an average cost of an inventory system per time unit.

7. The supplier offers credit period of M years to the retailer. When $M \le T$, the account is to be settled at M, the retailer pays off all units sold and pays interest charges at the rate I_c for the unsold items in the stock. When $M > T$, the account is settled at $T = M$ and the retailer does not pay any interest charges.

8. The retailer offers the partial trade credit to the customers, say N. Hence, his customer must make a cash down payment to the retailer at the time of placing an order. The remaining balance must be paid at the end of the trade credit offered by the retailer. Thus, the retailer can incur interest from his customer's payment at the rate I_e. Here, Fraction of the cash down payment offered by the retailer to customer is α, $0 \le \alpha \le 1$.

3. MATHEMATICAL MODEL:

Mathematical formulation of deteriorating inventory system for retailer is presented in this section. The retailer's inventory depletes due to demand and deterioration. The level of an inventory system at any instant of time t, during $[0,T]$ is formulated as follows. The differential equation governing inventory system is,

$$\frac{dI(t)}{dt} = -\theta I(t) - R(t), \qquad 0 \le t \le T \tag{1}$$

with initial condition $I(0) = Q$ (2)

and boundary condition $I(T) = 0$ (3)

Using condition (3), the solution of differential equation (1) is,

$$I(t) = \begin{cases} I_1(t) & ; \quad 0 \le t \le \lambda_1 \\ I_2(t) & ; \quad \lambda_1 \le t \le \lambda_2 \quad ; \qquad 0 \le t \le T \\ I_3(t) & ; \quad \lambda_2 \le t \le T \end{cases} \tag{4}$$

Where,

$$I_1(t) = \frac{a}{(b_2-\theta)\theta^2}\begin{pmatrix} e^{\theta(\lambda_2-t)}b_1b_2\lambda_1\theta - e^{-Tb_2+T\theta+b_2\lambda_2-t\theta}b_1\lambda_1\theta^2 - b_1b_2t\theta + b_1t\theta^2 \\ +e^{\theta(\lambda_2-t)}b_2\theta - e^{-Tb_2+T\theta+b_2\lambda_2-t\theta}\theta^2 - e^{\theta(\lambda_1-t)}b_1b_2 + e^{\theta(\lambda_1-t)}b_1\theta \\ +b_1b_2 - b_1\theta - b_2\theta + \theta^2 \end{pmatrix}$$

$$I_2(t) = -\frac{a(b_1\lambda_1+1)\left(e^{-Tb_2+T\theta+b_2\lambda_2-t\theta}\theta - e^{(\lambda_2-t)\theta}b_2 + b_2 - \theta\right)}{\theta(b_2-\theta)}$$

and

$$I_3(t) = -\frac{a(b_1\lambda_1+1)\left(-e^{-(\lambda_2+t)b_2} + e^{-Tb_2+T\theta+b_2\lambda_2-t\theta}\right)}{b_2-\theta}$$

Using (2) and (4),

$$Q = I_1(0) = \frac{a\begin{pmatrix} b_1b_2\lambda_1\theta e^{\lambda_2\theta} - b_1\lambda_1\theta^2 e^{-Tb_2+T\theta+b_2\lambda_2} + e^{\lambda_2\theta}b_2\theta - e^{-Tb_2+T\theta+b_2\lambda_2}\theta^2 \\ -e^{\lambda_1\theta}b_1b_2 + e^{\lambda_1\theta}b_1\theta + b_1b_2 - b_1\theta - b_2\theta + \theta^2 \end{pmatrix}}{(b_2-\theta)\theta^2} \tag{5}$$

The total cost per time unit consists of following cost components in each case:

1. Purchase Cost: $PC = \dfrac{CQ}{T}$ (6)

2. Ordering Cost: $OC = \dfrac{A}{T}$ (7)

3. Inventory holding cost excluding interest charges: $IHC = \dfrac{h}{T}\int_0^T I(t)dt$ (8)

4. Interest charged by the supplier: $\dfrac{CI_c}{T}\int_T^M I(t)\,dt$ (9)

5. Interest earned by the retailer: $\dfrac{PI_e}{T}\int_0^M R(t)\,tdt$ (10)

Now depending upon the values of offered credit periods M and N five different cases may arise: **Case 1**: $N \le M \le T$, **Case 2**: $N \le T \le M$, **Case 3**: $T \le N \le M$, **Case 4**: $M < N \le T$ and **Case 5:** $T \le M < N$. Moreover, demand function is assumed to trapezoidal type, and hence depending upon length of λ_1 and λ_2 different sub-cases may arise. Hence, we compute interest components (9) and (10) for each case.

Case 1: Suppose that $N \le M \le T$ (Figure 1)

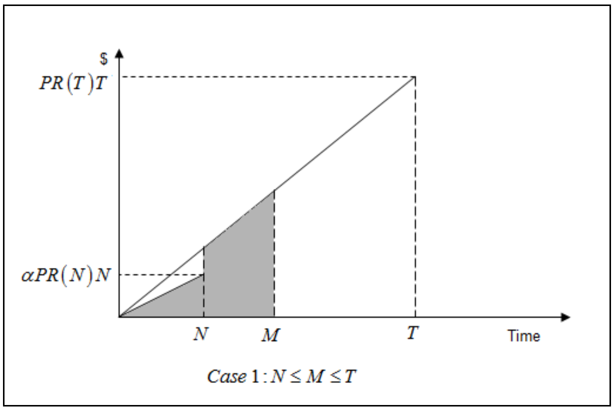

Figure 1: Interest earned by retailer in Case 1

Sub case A: $\lambda_1 \le \lambda_2 \le N \le M \le T$

Interest earned per time unit is,

$$IE_{1A} = \frac{PI_e}{T}\left[\alpha\left(\int_0^{\lambda_1} R_1(t)tdt + \int_{\lambda_1}^{\lambda_2} R_2(t)tdt + \int_{\lambda_2}^{N} R_3(t)tdt\right) + \int_N^M R_3(t)tdt\right] \tag{11}$$

and interest charged per time unit is, $IC_{1A} = \frac{CI_c}{T}\left[\int_M^T I_3(t)dt\right]$ (12)

Sub case B: $\lambda_1 \le N \le \lambda_2 \le M \le T$

Interest earned per time unit is,

$$IE_{1B} = \frac{PI_e}{T}\left[\alpha\left(\int_0^{\lambda_1} R_1(t)tdt + \int_{\lambda_1}^{N} R_2(t)tdt\right) + \int_N^{\lambda_2} R_2(t)tdt + \int_{\lambda_2}^{M} R_3(t)tdt\right] \quad (13)$$

and interest charged per time unit is, $IC_{1B} = \frac{CI_c}{T}\left[\int_M^T I_3(t)dt\right]$ (14)

Sub case C: $\lambda_1 \le N \le M \le \lambda_2 \le T$

Interest earned per time unit is,

$$IE_{1C} = \frac{PI_e}{T}\left[\alpha\left(\int_0^{\lambda_1} R_1(t)tdt + \int_{\lambda_1}^{N} R_2(t)tdt\right) + \int_N^{M} R_2(t)tdt\right] \quad (15)$$

and interest charged per time unit is, $IC_{1C} = \frac{CI_c}{T}\left[\int_M^{\lambda_2} I_2(t)dt + \int_{\lambda_2}^{T} I_3(t)dt\right]$ (16)

Sub case D: $N \le \lambda_1 \le \lambda_2 \le M \le T$

Interest earned per time unit is,

$$IE_{1D} = \frac{PI_e}{T}\left[\alpha\left(\int_0^{N} R_1(t)tdt\right) + \int_N^{\lambda_1} R_1(t)tdt + \int_{\lambda_1}^{\lambda_2} R_2(t)tdt + \int_{\lambda_2}^{M} R_3(t)tdt\right] \quad (17)$$

and interest charged per time unit is, $IC_{1D} = \frac{CI_c}{T}\left[\int_M^T I_3(t)dt\right]$ (18)

Sub case E: $N \le \lambda_1 \le M \le \lambda_2 \le T$

Interest earned per time unit is,

$$IE_{1E} = \frac{PI_e}{T}\left[\alpha\int_0^{N} R_1(t)tdt + \int_N^{\lambda_1} R_1(t)tdt + \int_{\lambda_1}^{M} R_2(t)tdt\right] \quad (19)$$

and interest charged per time unit is,

$$IC_{1E} = \frac{CI_c}{T}\left[\int_M^{\lambda_2} I_2(t)dt + \int_{\lambda_2}^{T} I_3(t)dt\right] \tag{20}$$

Sub case F: $N \le M \le \lambda_1 \le \lambda_2 \le T$

Interest earned per time unit is, $IE_{1F} = \frac{PI_e}{T}\left[\alpha\int_0^N R_1(t)tdt + \int_N^M R_1(t)tdt\right]$ (21)

and interest charged per time unit is,

$$IC_{1F} = \frac{CI_c}{T}\left[\int_M^{\lambda_1} I_1(t)dt + \int_{\lambda_1}^{\lambda_2} I_2(t)dt + \int_{\lambda_2}^{T} I_3(t)dt\right] \tag{22}$$

Hence, the retailer's total cost per time unit is,

$$K_{1i}(T) = PC + OC + IHC + IC_{1i} - IE_{1i},\ i = A,B,C,D,E,F \tag{23}$$

Case 2: $N \le T \le M$ (Figure 2)

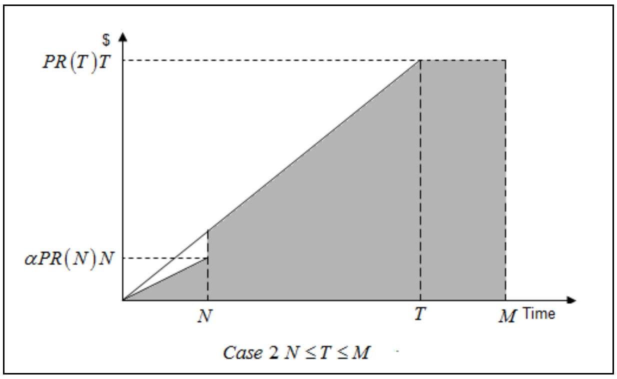

Figure 2: Interest earned by retailer in Case 2

Sub case A: $\lambda_1 \le \lambda_2 \le N \le T \le M$

Interest earned per time unit is,

$$IE_{2A} = \frac{PI_e}{T}\left[\alpha\left(\int_0^{\lambda_1} R_1(t)tdt + \int_{\lambda_1}^{\lambda_2} R_2(t)tdt + \int_{\lambda_2}^{N} R_3(t)tdt\right) + \int_N^T R_3(t)tdt + R_3(T)T(M-T)\right] \tag{24}$$

and interest charged per time unit is; $IC_{2A} = 0$ (25)

Sub case B: $\lambda_1 \le N \le \lambda_2 \le T \le M$

Interest earned per time unit is,

$$IE_{2B}=\frac{PI_e}{T}\left[\begin{array}{l}\alpha\left(\int_0^{\lambda_1}R_1(t)tdt+\int_{\lambda_1}^{N}R_2(t)tdt\right)+\int_N^{\lambda_2}R_2(t)tdt\\+\int_{\lambda_2}^{T}R_3(t)tdt+R_3(T)T(M-T)\end{array}\right] \tag{26}$$

and interest charged per time unit is; $IC_{2B}=0$ (27)

Sub case C: $N\le\lambda_1\le\lambda_2\le T\le M$

Interest earned per time unit is,

$$IE_{2C}=\frac{PI_e}{T}\left[\begin{array}{l}\alpha\int_0^{N}R_1(t)tdt+\int_{N}^{\lambda_1}R_1(t)tdt+\int_{\lambda_1}^{\lambda_2}R_2(t)tdt\\+\int_{\lambda_2}^{T}R_3(t)tdt+R_3(T)T(M-T)\end{array}\right] \tag{28}$$

and interest charged per time unit is; $IC_{2C}=0$ (29)

Therefore, the retailer's total inventory cost per time unit is,

$$K_{2i}(T)=PC+OC+IHC+IC_{2i}-IE_{2i},\ i=A,B,C \tag{30}$$

Case 3: $T\le N\le M$ i.e. $\lambda_1\le\lambda_2\le T\le N\le M$ (Figure 3)

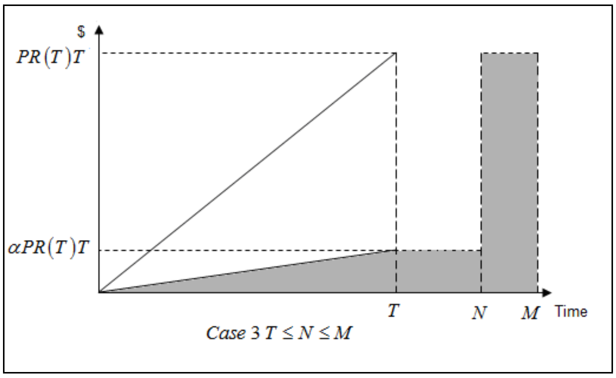

Figure 3: Interest earned by retailer in Case 3

Interest earned per time unit is,

$$IE_3 = \frac{PI_e}{T}\left[\begin{array}{l} \alpha\left(\int_0^{\lambda_1} R_1(t)tdt + \int_{\lambda_1}^{\lambda_2} R_2(t)tdt + \int_{\lambda_2}^{T} R_3(t)tdt\right) \\ +\alpha R_3(T)T(N-T) + R_3(T)T(M-N) \end{array}\right] \tag{31}$$

and interest charged per time unit is; $IC_3 = 0$ (32)

Therefore, the retailer's total inventory cost per time unit is,

$$K_3(T) = PC + OC + IHC + IC_3 - IE_3 \tag{33}$$

Thus, the retailer's total inventory cost per time unit for the case M ≤ N is,

$$K(T) = \begin{bmatrix} K_{1i}(T) & ,M \le T \\ K_{2i}(T) & ,N \le T \le M \\ K_{3i}(T) & ,T \le N \end{bmatrix} \tag{34}$$

It can be verified that $K_1(M) = K_2(M)$ and $K_2(N) = K_3(N)$. Hence, $K(T)$ is well – defined and continuous function of time T .

Case 4: $M < N \le T$

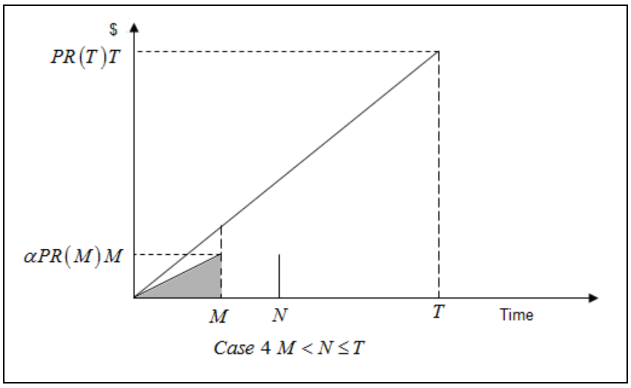

Figure 4: Interest earned by retailer in Case 4

Sub case A: $\lambda_1 \le \lambda_2 \le M < N \le T$

Interest earned per time unit is,

$$IE_{4A} = \frac{\alpha PI_e}{T}\left[\int_0^{\lambda_1} R_1(t)tdt + \int_{\lambda_1}^{\lambda_2} R_2(t)tdt + \int_{\lambda_2}^{M} R_3(t)tdt\right] \tag{35}$$

and interest charged per time unit is, $IC_{4A} = \frac{CI_c}{T}\left[\int_M^T I_3(t)dt\right]$ (36)

Sub case B: $\lambda_1 \le M \le \lambda_2 < N \le T$

Interest earned per time unit is, $IE_{4B} = \dfrac{\alpha P I_e}{T}\left[\int_0^{\lambda_1} R_1(t)tdt + \int_{\lambda_1}^{M} R_2(t)tdt\right]$ (37)

and interest charged per time unit is, $IC_4 = \dfrac{CI_c}{T}\left[\int_M^{\lambda_2} I_2(t)dt + \int_{\lambda_2}^{T} I_3(t)dt\right]$ (38)

Sub case C: $\lambda_1 \le M < N \le \lambda_2 \le T$

Interest earned per time unit is, $IE_{4C} = \dfrac{\alpha P I_e}{T}\left[\int_0^{\lambda_1} R_1(t)tdt + \int_{\lambda_1}^{M} R_2(t)tdt\right]$ (39)

and interest charged per time unit is, $IC_{4C} = \dfrac{CI_c}{T}\left[\int_M^{\lambda_2} I_2(t)dt + \int_{\lambda_2}^{T} I_3(t)dt\right]$ (40)

Sub case D: $M \le \lambda_1 \le \lambda_2 < N \le T$

Interest earned per time unit is, $IE_{4D} = \dfrac{\alpha P I_e}{T}\left[\int_0^{M} R_1(t)tdt\right]$ (41)

and interest charged per time unit is,

$$IC_{4D} = \frac{CI_c}{T}\left[\int_M^{\lambda_1} I_1(t)dt + \int_{\lambda_1}^{\lambda_2} I_2(t)dt + \int_{\lambda_2}^{T} I_3(t)dt\right] \tag{42}$$

Sub case E: $M \le \lambda_1 < N \le \lambda_2 \le T$

Interest earned per time unit is, $IE_{4E} = \dfrac{\alpha P I_e}{T}\left[\int_0^{M} R_1(t)tdt\right]$ (43)

and interest charged per time unit is,

$$IC_{4E} = \frac{CI_c}{T}\left[\int_M^{\lambda_1} I_1(t)dt + \int_{\lambda_1}^{\lambda_2} I_2(t)dt + \int_{\lambda_2}^{T} I_3(t)dt\right] \tag{44}$$

Sub case F: $M < N \le \lambda_1 \le \lambda_2 \le T$

Interest earned per time unit is, $IE_{4F} = \dfrac{\alpha P I_e}{T}\left[\int_0^{M} R_1(t)tdt\right]$ (45)

and interest charged per time unit is,

$$IC_{4F} = \frac{CI_c}{T}\left[\int_M^{\lambda_1} I_1(t)dt + \int_{\lambda_1}^{\lambda_2} I_2(t)dt + \int_{\lambda_2}^{T} I_3(t)dt\right] \tag{46}$$

Hence, the retailer's total cost per time unit is,

$$K_{4i}(T) = PC + OC + IHC + IC_{4i} - IE_{4i},\ i = A,B,C,D,E,F \tag{47}$$

Case 5: $T \le M < N$, i.e. $\lambda_1 \le \lambda_2 \le T \le M < N$ (Figure 5)

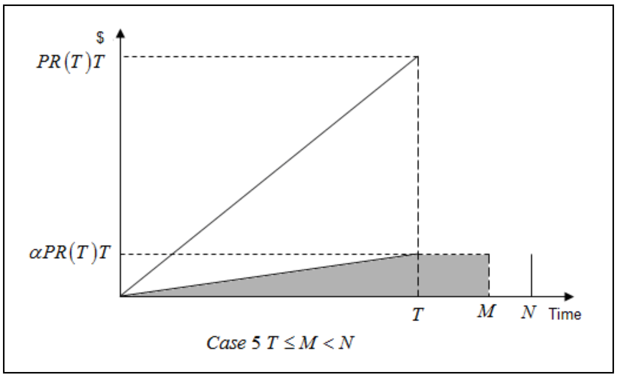

Figure 5: Interest earned by retailer in Case 5

Interest earned per time unit is,

$$IE_5 = \frac{PI_e}{T}\left[\int_0^{\lambda_1} \alpha R_1(t)tdt + \int_{\lambda_1}^{\lambda_2} \alpha R_2(t)tdt + \int_{\lambda_2}^{T} \alpha R_3(t)tdt + \alpha R_3(T)T(M-T)\right] \tag{48}$$

and interest charged per time unit is; $IC_5 = 0$ (49)

Hence, the retailer's total cost per time unit is,

$$K_5(T) = PC + OC + IHC + IC_5 - IE_5 \tag{50}$$

Therefore, the retailer's total cost per time unit for $M > N$ is,

$$K(T) = \begin{bmatrix} K_{4i}(T) & ,M \le T \\ K_5(T) & ,M \ge T \end{bmatrix} \tag{51}$$

At $T = M$, $K_4(M) = K_5(M)$.So, $K(T)$ is well–defined, continuous function of time.

4.: Computational Algorithm:

Step 1: Assign values to all the parameters.

Step 2: If $M \ge N$ then go to Step 3

Else go to Step 7.

Step 3: Compute T by solving $\frac{dK_{1i}(T)}{dT}=0, \frac{dK_{2i}(T)}{dT}=0, \frac{dK_{3}(T)}{dT}=0$

Step 4: If $M \le T$ then go to Step 5

Else if $N \le T \le M$ then go to Step 6

Else $K_3(T)$ is optimal

Step 5: If { $\lambda_1 \le \lambda_2 \le N \le M \le T$ then $K_{1A}(T)$ is optimal

Else if { $\lambda_1 \le N \le \lambda_2 \le M \le T$ then $K_{1B}(T)$ is optimal

Else if { $\lambda_1 \le N \le M \le \lambda_2 \le T$ then $K_{1C}(T)$ is optimal

Else if { $N \le \lambda_1 \le \lambda_2 \le M \le T$ then $K_{1D}(T)$ is optimal

Else if { $N \le \lambda_1 \le M \le \lambda_2 \le T$ then $K_{1E}(T)$ is optimal

Else { $K_{1F}(T)$ is optimal}}}}}}

Step 6: If { $\lambda_1 \le \lambda_2 \le N \le T \le M$ then $K_{2A}(T)$ is optimal

Else if { $\lambda_1 \le N \le \lambda_2 \le T \le M$ then $K_{2B}(T)$ is optimal

Else { $K_{2C}(T)$ is optimal}}}

Step 7: Compute T by solving $\frac{dK_{4i}(T)}{dT}=0, \frac{dK_{5}(T)}{dT}=0$

Step 8: If $M < N \le T$ then go to Step 9

Else $K_5(T)$ is optimal.

Step 9: If { $\lambda_1 \le \lambda_2 \le M < N \le T$ then $K_{4A}(T)$ is optimal

Else if { $\lambda_1 \le M \le \lambda_2 < N \le T$ then $K_{4B}(T)$ is optimal

Else if { $\lambda_1 \le M < N \le \lambda_2 \le T$ then $K_{4C}(T)$ is optimal

Else if { $M \le \lambda_1 \le \lambda_2 < N \le T$ then $K_{4D}(T)$ is optimal

Else if { $M \le \lambda_1 < N \le \lambda_2 \le T$ then $K_{4E}(T)$ is optimal

Else { $K_{4F}(T)$ is optimal}}}}}}

Step 10: Compute EOQ Q using equation (5)

5. Numerical Examples

Mathematical formulation is illustrated with numerical examples for five cases.

Example: 1 $N \le M \le T$: It is assumed that $M = 90/365$, $N = 60/365, \theta = 0.55$ and consider the following parametric values in proper units: $a = 900$, $b_1 = 0.05$, $b_2 = 0.05$, $A = 1000$, $\alpha = 0.1$, $C = 10$, $P = 20$, $h = 7$, $I_c = 0.2$, $I_e = 0.1$, $\lambda_1 = 15/365$, $\lambda_2 = 45/365$. Using sub case A, Optimal value of T is 0.3797 years i.e. 138.5 days approx. Using optimal T, EOQ Q=379.13 and total cost of an inventory system is K=13850.37 \$. The graph given in Figure 6 (Appendix A) shows that total cost of an inventory system per time unit is strictly convex.

Example: 2 $N \le T \le M$ It is assumed that $M = 65/365$, $N = 35/365, \theta = 0.55$ and consider the following parametric values in proper units: $a = 900$, $b_1 = 0.05$, $b_2 = 0.05$, $A = 100$, $\alpha = 0.1$, $C = 10$, $P = 20$, $h = 7$, $I_c = 0.2$, $I_e = 0.1$, $\lambda_1 = 15/365$, $\lambda_2 = 25/365$. Using sub case A, Optimal value of T is 0.1271 years i.e. 46.39 days approx. Using optimal T, EOQ Q=118.64 and total cost of an inventory system is K=10381.22 \$. The graph given in Figure 7 (Appendix A) shows that total cost of an inventory system per time unit is strictly convex.

Example: 3 $T \le N \le M$ It is assumed that $M = 70/365$, $N = 65/365, \theta = 0.55$ and consider the following parametric values in proper units: $a = 900$, $b_1 = 0.05$, $b_2 = 0.05$, $A = 100$, $\alpha = 0.1$, $C = 10$, $P = 20$, $h = 7$, $I_c = 0.2$, $I_e = 0.1$, $\lambda_1 = 30/365$, $\lambda_2 = 40/365$. Optimal value of T is 0.1287 years i.e. 46.97 days approx. Using optimal T, EOQ Q=120.36 and total cost of an inventory system is K=10500.12 \$. The graph given in Figure 8 (Appendix A) shows that total cost of an inventory system per time unit is strictly convex.

Example: 4 $M < N \le T$ It is assumed that $M = 50/365$, $N = 70/365, \theta = 0.55$ and consider the following parametric values in proper units: $a = 900$, $b_1 = 0.05$, $b_2 = 0.05$, $A = 1000$, $\alpha = 0.1$, $C = 10$, $P = 20$, $h = 7$, $I_c = 0.2$, $I_e = 0.1$, $\lambda_1 = 25/365$, $\lambda_2 = 35/365$. Using Sub case A, Optimal value of T is 0.3777 years i.e. 137.86 days approx. Using optimal T, EOQ Q=376.99 and total cost of an inventory system is K=14036.46 \$. The graph given in Figure 9 (Appendix A) shows that total cost of an inventory system per time unit is strictly convex.

Example: 5 $T \le M < N$ It is assumed that $M = 60/365$, $N = 70/365, \theta = 0.55$ and consider the following parametric values in proper units: $a = 900$, $b_1 = 0.05$, $b_2 = 0.05$, $A = 100$, $\alpha = 0.1$, $C = 10$, $P = 20$, $h = 7$, $I_c = 0.2$, $I_e = 0.1$, $\lambda_1 = 20/365$, $\lambda_2 = 40/365$. Optimal value of T is 0.1293 years i.e. 47.19 days approx. Using optimal T, EOQ Q=120.95 and total cost of an inventory system is K=10520.83 \$. The graph given in Figure 10 (Appendix A) shows that total cost of an inventory system per time unit is strictly convex.

Sensitivity analysis:

Sensitivity analysis is carried out, by assuming following parametric values in proper units: $a = 1000$, $b_1 = 0.05$, $b_2 = 0.05$, $\theta = 0.55$ $A = 1000$, $\alpha = 0.1$, $C = 10$, $P = 20$, $h = 5$, $I_c = 0.2$, $I_e = 0.1$, $\lambda_1 = 15/365$, $\lambda_2 = 30/365$.

Table 1: Sensitivity with respect to α

	$M = 60/365$, $N = 40/365$, $M \geq N$				$M = 35/365$, $N = 45/365$, $M < N$			
α	T	Q	K	Case	T	Q	K	Case
0.1	0.3658	403.61	14713.42	Case 1-A	0.3720	411.20	15041.11	Case 4-A
0.3	0.3638	401.26	14683.74	Case 1-A	0.3706	409.44	15018.79	Case 4-A
0.7	0.3599	396.52	14623.91	Case 1-A	0.3677	405.91	14973.86	Case 4-A

Table 2: Sensitivity with respect to b_1

	$M = 60/365$, $N = 40/365$, $M \geq N$				$M = 35/365$, $N = 45/365$, $M < N$			
b_1	T	Q	K	Case	T	Q	K	Case
0.05	0.3658	403.61	14713.42	Case 1-A	0.3720	411.20	15041.11	Case 4-A
0.1	0.3653	403.86	14736.81	Case 1-A	0.3716	411.48	15065.29	Case 4-A
0.15	0.3649	404.11	14760.19	Case 1-A	0.3712	411.75	15089.47	Case 4-A

Table 3: Sensitivity with respect to a

	$M = 60/365$, $N = 40/365$, $M \geq N$				$M = 35/365$, $N = 45/365$, $M < N$			
a	T	Q	K	Case	T	Q	K	Case
1000	0.3658	403.61	14713.42	Case 1-A	0.3720	411.20	15041.11	Case 4-A
1050	0.3571	412.84	15310.74	Case 1-A	0.3636	421.08	15657.24	Case 4-A
1100	0.3490	421.82	15904.97	Case 1-A	0.3558	430.73	16270.44	Case 4-A

Table 4: Sensitivity with respect to b_2

	$M = 60/365$, $N = 40/365$, $M \geq N$				$M = 35/365$, $N = 45/365$, $M < N$			
b_2	T	Q	K	Case	T	Q	K	Case
0.05	0.3658	403.61	14713.42	Case 1-A	0.3720	411.20	15041.11	Case 4-A
0.1	0.3750	412.38	14639.37	Case 1-A	0.3816	420.21	14962.94	Case 4-A
0.15	0.3851	421.86	14562.50	Case 1-A	0.3919	429.96	14881.81	Case 4-A

Table 5: Sensitivity with respect to P

	$M = 60/365,\ N = 40/365,\ M \geq N$				$M = 35/365,\ N = 45/365,\ M < N$			
P	T	Q	K	Case	T	Q	K	Case
30	0.3658	403.61	14713.42	Case 1-A	0.3720	411.20	15041.11	Case 4-A
40	0.3614	398.32	14646.56	Case 1-A	0.3718	410.91	15037.40	Case 4-A
50	0.3570	392.96	14578.90	Case 1-A	0.3716	410.61	15033.68	Case 4-A

Table 6: Sensitivity with respect to λ_1

	$M = 60/365,\ N = 40/365,\ M \geq N$				$M = 35/365,\ N = 45/365,\ M < N$			
λ_1	T	Q	K	Case	T	Q	K	Case
15/365	0.3658	403.61	14713.42	Case 1-A	0.3720	411.20	15041.11	Case 4-A
20/365	0.3656	403.63	14720.68	Case 1-A	0.3719	411.23	15048.654	Case 4-A
25/365	0.3654	403.62	14727.68	Case 1-A	0.3717	411.23	15055.928	Case 4-A

Table 7: Sensitivity with respect to λ_2

	$M = 60/365,\ N = 40/365,\ M \geq N$				$M = 35/365,\ N = 45/365,\ M < N$			
λ_2	T	Q	K	Case	T	Q	K	Case
30/365	0.3658	403.61	14713.42	Case 1-A	0.3720	411.20	15041.11	Case 4-A
40/365	0.3653	403.43	14726.07	Case 1-A	0.3716	411.05	15054.34	Case 4-B
50/365	0.3647	403.13	14737.57	Case 1-B	0.3710	410.76	15066.43	Case 4-C

Table 8: Sensitivity with respect to N

	$M = 60/365,\ M \geq N$				$M = 35/365,\ M < N$				
N	T	Q	K	Case	N	T	Q	K	Case
40/365	0.3658	403.61	14713.42	Case 1-A	45/365	0.3720	411.20	15041.11	Case 4-A
45/365	0.3681	406.40	14748.64	Case 1-A	55/365	0.3720	411.20	15041.11	Case 4-A
50/365	0.3706	409.49	14787.72	Case 1-A	65/365	0.3720	411.20	15041.11	Case 4-A

Table 9: Sensitivity with respect to θ

	$M = 60/365,\ N = 40/365,\ M \geq N$				$M = 35/365,\ N = 45/365,\ M < N$			
θ	T	Q	K	Case	T	Q	K	Case
0.55	0.3658	403.61	14713.42	Case 1-A	0.3720	411.20	15041.11	Case 4-A
0.65	0.3494	390.94	14931.43	Case 1-A	0.3553	398.21	15265.27	Case 4-A
0.75	0.3349	379.49	15143.00	Case 1-A	0.3404	386.48	15482.66	Case 4-A

Convexity for sensitivity analysis is presented in Figure 11 – 17 (Appendix A).

6. CONCLUSION:

In this article an ordering policy is discussed under two level trade credit policy; for items having trapezoidal type demand rate, and deteriorates with a constant rate of deterioration. Here, the retailer is the dominant decision maker. The following managerial issues are observed.

1. Increase in customer's cash down fraction at the time of placing an order will decrease retailer's EOQ and order frequency too. This will help retailer to earn more interest.
2. When demand of item increases, total cost per time unit for the retailer will decrease. Moreover, Increase in demand parameter b_1 and b_2 will increase retailer's EOQ and as a result total cost of an inventory system per time unit will increase.
3. Increase in selling price will decrease retailer's order size. Hence, retailer may procure less to avail benefits of trade credit more frequently.
4. In case $M \geq N$, when N increases the retailer will procure more to balance the loss of interest earned as longer credit period is offered to the customer by the retailer.
5. Increase in rate of deterioration reduces cycle time and retailer's EQO and results in to increased total cost of an inventory system per time unit.

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Appendix A

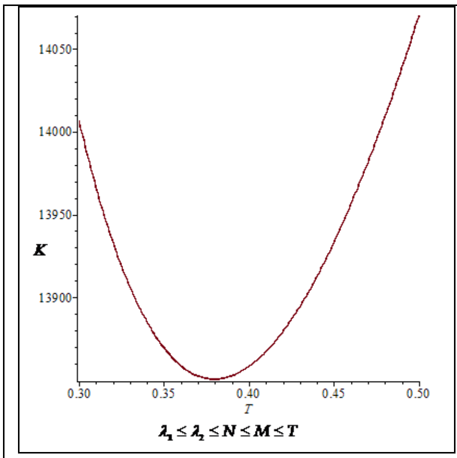


Figure 6 (Example 1)

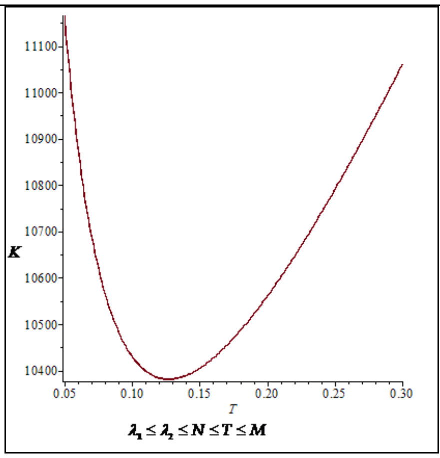
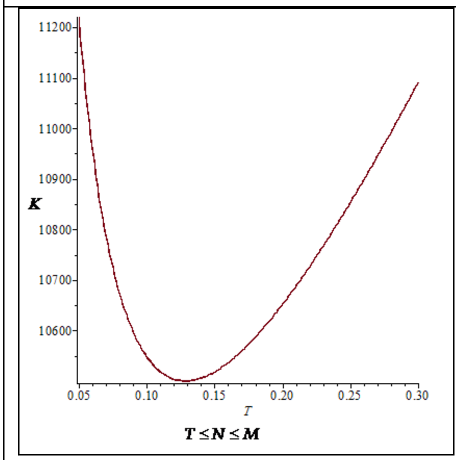


Figure 7 : (Example 2)



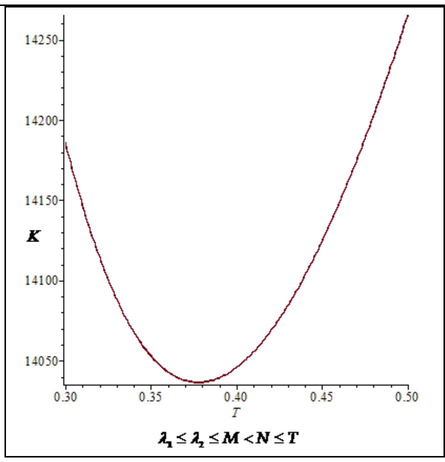


Figure 8: (Example 3)

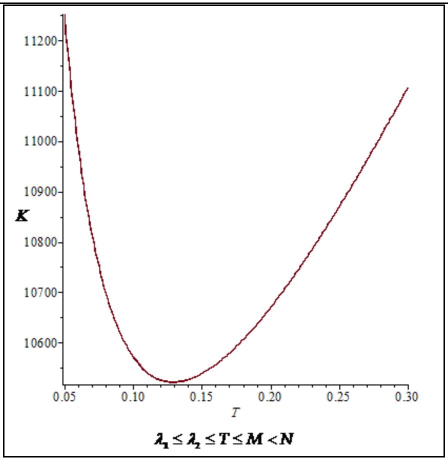


Figure 9: (Example 4)

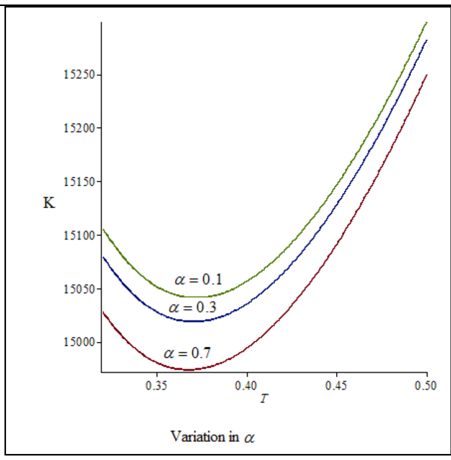
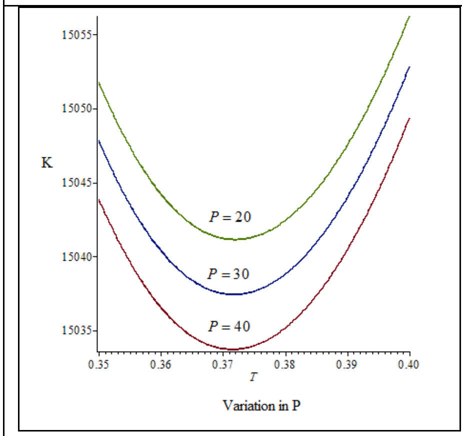


Figure 10: (Example 5)

Figure 11: Variation in α



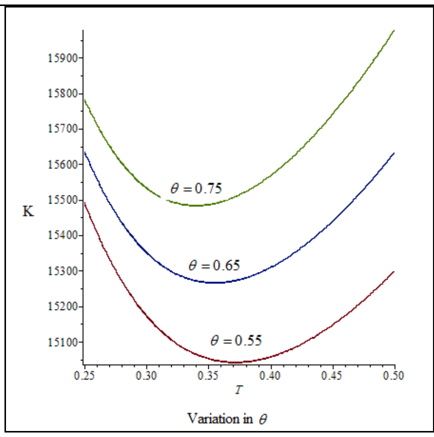
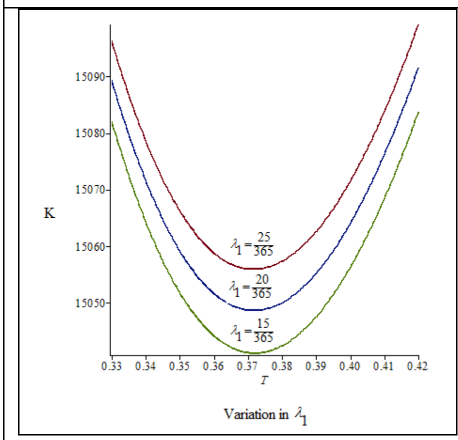


Figure 12: Variation in P

Figure 13: Variation in θ



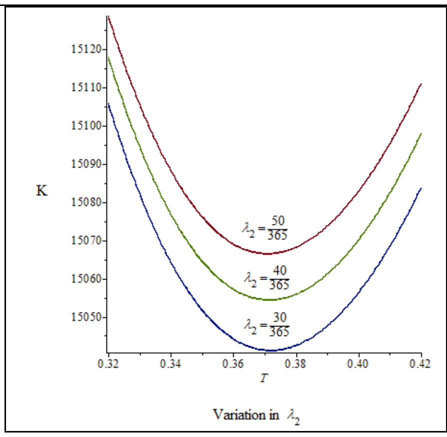


Figure 14: Variation in λ_1

Figure 15: Variation in λ_2

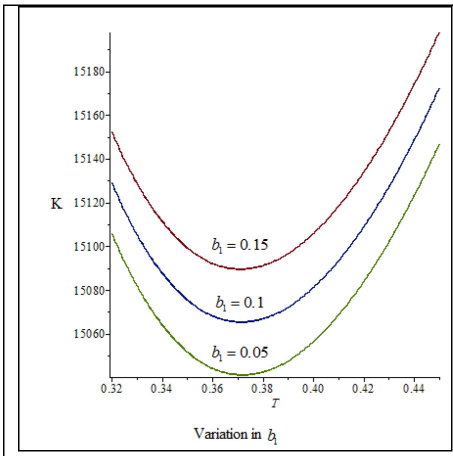


Figure 16: Variation in b_1

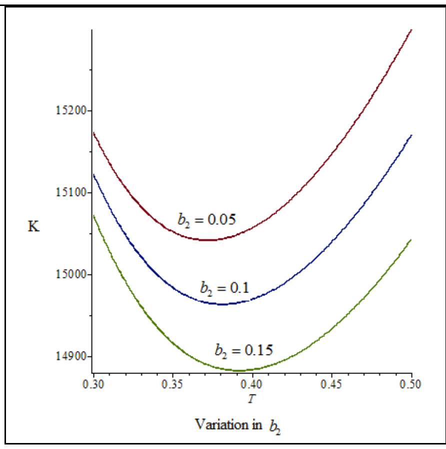


Figure 17: Variation in b_2