Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 13, Number 12 (2017), pp. 8157-8177

http://www.ripublication.com

Zero Division Method for Finding an Optimal of Generalized Fuzzy Transportation Problems

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Abstract

In the literature, several methods are proposed for solving the transportation problem in fuzzy environment but all the proposed methods transportation costs are represented by normal fuzzy numbers. In this paper, a proposed method, namely, Zero Division Method (ZDM) is used for solving fuzzy transportation problems by assuming that a decision maker is uncertain about the precise values of the transportation cost only. But there is no uncertainty about the supply and demand of the product. In the proposed method transportation costs are represented by generalized trapezoidal fuzzy numbers. To illustrate the proposed method a numerical example is solved and the obtained result is compared with the results of existing approaches. The proposed method is easy to understand and apply on real life transportation problems for the decision makers.

Mathematics Subject Classification: 90C08, 90C90, 90C70, 90B06, 90C29.

Keywords: Fuzzy Transportation Problem (FTP); Generalized Trapezoidal Fuzzy Number (GTrFN); Ranking function; Zero Division Method (ZDM).

1. INTRODUCTION

Nowadays highly competitive market, the pressure on organization to find better ways to create and deliver value to customers becomes stronger. The transportation models provide a powerful framework to meet this challenge.

Transportation problem is an important network structured in linear programming (LP) problem that arises in several contexts and has deservedly received a great deal of attention in the literature. The main objective in the problem is to find the minimum total transportation cost of a commodity in order to satisfy demands at destinations using available supplies at origins. Transportation problem can be used for a wide variety of situations such as scheduling, production, investment, plant location, inventory control, employment scheduling, and many others. In general, transportation problems are solved with the assumptions that the transportation costs and values of supplies and demands are specified in a precise way i.e., in crisp environment. However, in many cases the decision makers has no crisp information about the coefficients belonging to the transportation problem. In these cases, the corresponding coefficients or elements defining the problem can be formulated by means of fuzzy sets, and the fuzzy transportation problem (FTP) appears in natural way.

The basic transportation problem was originally developed by Hitchcock [14].The transportation problems can be modeled as a standard linear programming problem, which can then be solved by the simplex method. However, because of its very special mathematical structure, it was recognized early that the simplex method applied to the transportation problem can be made quite efficient in terms of how to evaluate the necessary simplex method information (Variable to enter the basis, variable to leave the basis and optimality conditions). Charnes and Cooper [4] developed a stepping stone method which provides an alternative way of determining the simplex method information. Dantzig and Thapa [8] used simplex method to the transportation problem as the primal simplex transportation method. An Initial Basic Feasible Solution (IBFS) for the transportation problem can be obtained by using the North- West Corner rule, Row Minima, Column Minima, Matrix Minima, or Vogel's Approximation Method (VAM) [21]. The Modified Distribution Method (MODI) [5] is useful for finding the optimal solution of the transportation problem. In general, the transportation problems are solved with the assumptions that the coefficients or cost parameters are specified in a precise way i.e., in crisp environment.

In real life, there are many diverse situations due to uncertainty in judgments, lack of evidence etc. Sometimes it is not possible to get relevant precise data for the cost parameter. This type of imprecise data is not always well represented by random variable selected from a probability distribution. Fuzzy number [23] may represent the data. Hence fuzzy decision making method is used here.

Zimmermann [24] showed that solutions obtained by fuzzy linear programming method and are always efficient. Subsequently, Zimmermann's fuzzy linear programming has developed into several fuzzy optimization methods for solving the transportation problems. Chanas et al. [2] presented a fuzzy linear programming model for solving transportation problems with crisp cost coefficient, fuzzy supply and demand values. Chanas and Kuchta [3] proposed the concept of the optimal solution for the transportation problem with fuzzy coefficients expressed as fuzzy numbers, and developed an algorithm for obtaining the optimal solution. Saad and Abbas [22] discussed the solution algorithm for solving the transportation problem in fuzzy environment.

Liu and Kao [18] described a method for solving fuzzy transportation problem based on extension principle. Gani and Razak [13] presented a two stage cost minimizing fuzzy transportation problem (FTP) in which supplies and demands are trapezoidal fuzzy numbers. A parametric approach is used to obtain a fuzzy solution and the aim is to minimize the sum of the transportation costs in two stages.

Lin [16] introduced a genetic algorithm to solve transportation problem with fuzzy objective functions. Dinagar and Palanivel [9] investigated FTP, with the aid of trapezoidal fuzzy numbers. Fuzzy modified distribution method is proposed to find the optimal solution in terms of fuzzy numbers. Pandian and Natarajan [20] proposed a new algorithm namely, fuzzy zero point method for finding a fuzzy optimal solution for a FTP, where the transportation cost, supply and demand are represented by trapezoidal fuzzy numbers. Edward Samuel [11] proposed a new procedure for solving generalized trapezoidal fuzzy transportation problem, where precise values of the transportation costs only, but there is no uncertain about the demand and supply of the product. Edward Samuel [10] proposed a dual based approach for solving unbalanced fuzzy transportation problem, where precise values of the transportation costs only, but there is no uncertain about the demand and supply of the product.

In this paper, a proposed method, namely, Zero Division Method (ZDM) is used for solving a special type of fuzzy transportation problem by assuming that a decision maker is uncertain about the precise values of transportation cost only but there is no uncertainty about the supply and demand of the product. In the proposed method, transportation costs are represented by generalized trapezoidal fuzzy numbers. To illustrate the proposed method ZDM, a numerical example is solved and the obtained result is compared with the results of existing approaches. The proposed method is easy to understand and to apply in real life transportation problems for the decision makers.

This paper has eight sections. Section one is introduction which includes background of fuzzy transportation problem. In section two, we accommodate the fundamental concepts that include some useful definition, operation on fuzzy numbers and ranking function. In section three, we formulate generalized fuzzy transportation problem. In section four, we present ZDM to finding an fuzzy optimal solution. In section five, a numerical example is solved. In section six, result and discussion is presented. In section seven, to show the application of the proposed method, a real life fuzzy transportation problem is solved. And finally, the conclusions and future work are discussed in section eight.

2. PRELIMINARIES

In this section, some basic definitions, arithmetic operations and an existing method for comparing generalized fuzzy numbers are presented.

2.1. Definition [15]

A fuzzy set $\tilde{A}$, defined on the universal set of real numbers $\Re$ is said to be fuzzy number if it is membership function has the following characteristics:

(i) $\mu_{\tilde{A}}(x): \Re \to [0,1]$ is continuous.

(ii) $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a] \cup [d, \infty)$.

(iii) $\mu_{\tilde{A}}(x)$ Strictly increasing on [a, b] and strictly decreasing on [c, d]

(iv) $\mu_{\tilde{A}}(x)$ =1 for all $x \in [b,c], where\ \ a < b < c < d.$

2.2. Definition [15] a fuzzy number $\tilde{A} = (a,b,c,d)$ is said to be trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{(x-a)}{(b-a)}, & a \le x < b, \\ 1, & b \le x \le c, \\ \dfrac{(x-a)}{(b-a)}, & c < x \le d, \\ o, & otherwise. \end{cases}$$

2.3. Definition [6]

A fuzzy set $\tilde{A}$, defined on the universal set of real numbers $\Re$, is said to be generalized fuzzy number if its membership function has the following characteristics:

(i) $\mu_{\tilde{A}}(x): \Re \to [0, \omega]$ is continuous.

(ii) $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a] \cup [d, \infty)$.

(iii) $\mu_{\tilde{A}}(x)$ Strictly increasing on [a, b] and strictly decreasing on [c, d]

(iv) $\mu_{\tilde{A}}(x) = \omega$, for all $x \in [b,c], where\ \ 0 < \omega \le 1.$

2.4. Definition [6] A fuzzy number $\tilde{A} = (a,b,c,d;\omega)$ is said to be generalized trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \omega\dfrac{(x-a)}{(b-a)}, & a \le x < b, \\ \omega \quad , & b \le x \le c, \\ \omega\dfrac{(x-d)}{(c-d)}, & c < x \le d, \\ o \quad , & otherwise. \end{cases}$$

2.5. Arithmetic operations:

In this section, arithmetic operations between two generalized trapezoidal fuzzy numbers, defined on the universal set of real numbers $\Re$, are presented [6,7].

Let $\tilde{A}_1 = (a_1,b_1,c_1,d_1;\omega_1)$ and $\tilde{A}_2 = (a_2,b_2,c_2,d_2;\omega_2)$ are two generalized trapezoidal fuzzy numbers, then the following is obtained.

(i) $\tilde{A}_1 + \tilde{A}_2 = (a_1 + a_2,\ b_1 + b_2,\ c_1 + c_2,\ d_1 + d_2;\ \min(\omega_1,\omega_2)),$

(ii) $\tilde{A}_1 - \tilde{A}_2 = (a_1 - d_2,\ b_1 - c_2,\ c_1 - b_2,\ d_1 - a_2;\ \min(\omega_1,\omega_2)),$

(iii)
$$\begin{aligned} &\tilde{A}_1 \otimes \tilde{A}_2 \cong (a,b,c,d;\min(\omega_1,\omega_2)), \\ &where \\ &a = \min(a_1a_2, a_1d_2, a_2d_1, d_1d_2),\ b = \min(b_1b_2, b_1c_2, c_1b_2, c_1c_2), \\ &c = \max(b_1b_2, b_1c_2, c_1b_2, c_1c_2), \quad d = \max(a_1a_2, a_1d_2, a_2d_1, d_1d_2). \end{aligned}$$

(iv)
$$\lambda\tilde{A}_1 = \begin{cases} (\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1;\ \omega_1), & \lambda > 0, \\ (\lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1;\ \omega_1), & \lambda < 0. \end{cases}$$

(v)
$$\frac{\tilde{A}_1}{\tilde{A}_2} = (\frac{a_1}{d_2}, \frac{b_1}{c_2}, \frac{c_1}{b_2}, \frac{d_1}{a_2}; \min(w_1, w_2))$$
$$whre\, d_2 \ne 0, c_2 \ne 0, b_2 \ne 0, a_2 \ne 0.$$

2.6. Ranking function

An efficient approach for comparing the fuzzy numbers is by the use of ranking function[7,17,19], $\Re : F(\Re) \to \Re$, where F($\Re$) is a set of fuzzy numbers defined on the set of real numbers, which maps each fuzzy number into the real line, where a natural order exists, i.e.,

(i) $\tilde{A} >_{\Re} \tilde{B}$ if and only if $\Re(\tilde{A}) > \Re(\tilde{B})$

(ii) $\tilde{A} <_{\Re} \tilde{B}$ if and only if $\Re(\tilde{A}) < \Re(\tilde{B})$

(iii) $\tilde{A} =_{\Re} \tilde{B}$ if and only if $\Re(\tilde{A}) = \Re(\tilde{B})$

Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1; \omega_1)$ *and* $\tilde{A}_2 = (a_2, b_2, c_2, d_2; \omega_2)$ *be two generalized trapezoidal fuzzy numbers and* $\omega = \min(\omega_1, \omega_2)$. *Then* $\Re(\tilde{A}_1) = \frac{\omega(a_1 + b_1 + c_1 + d_1)}{4}$ *and* $\Re(\tilde{A}_2) = \frac{\omega(a_2 + b_2 + c_2 + d_2)}{4}$.

3. FORMULATION OF FUZZY TRANSPORTATION PROBLEM

The fuzzy transportation problems, in which a decision maker is uncertain about the precise values of transportation cost from ith source to the jth destination, but sure about the supply and demand of the product, can be formulated as follows[12]:

Minimize $\tilde{x}_0$

Subject to

$$\sum_{j=1}^{n} x_{ij} \le a_i, \qquad i = 1,2,3.......m \tag{1}$$

$$\sum_{i=1}^{m} x_{ij} \ge b_j, \qquad j = 1,2,3......n$$

$$x_{ij} \ge 0, \qquad \forall\, i, j$$

Where a_i = the total availability of the product at ith source.

b_j = the total demand of the product at jth destination.

$\tilde{c}_{ij}$ = the approximate cost for transporting one unit quantity of the product from the ith source to the jth destination.

x_{ij} = the number of units of the product that should be transported from the ith source to the jth destination or decision variables.

$$\tilde{x}_0 = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} x_{ij} \text{ ; Total fuzzy transportation cost} \tag{2}$$

If $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ then the FTP is said to be balanced FTP, otherwise it is called an unbalanced FTP.

4. PROPOSED METHOD

In this section, a proposed method, namely, Zero division method (ZDM) for finding a fuzzy optimal solution using ranking function, in which transportation costs are represented as generalized trapezoidal fuzzy numbers .

Zero division method (ZDM) methodology is presented as follows:

Step 1. Initialization

It is well known that an unbalanced fuzzy transportation problem is equivalent to an ordinary balanced fuzzy transportation problem with one dummy column or one dummy row with generalized trapezoidal fuzzy zero costs added.

Step 2. Develop the cost table

If the total supply is not equal to the total demand, then a dummy row or dummy column must be added. The fuzzy transportation costs for dummy cells are always generalized trapezoidal fuzzy zero.

Step 3. Find the opportunity cost table

(a) Row Reduction:

Select the smallest fuzzy cost in each row of the given cost table and then subtract that from each fuzzy cost of that row.

(b) Column Reduction:

In the reduced matrix obtained from 3(a), select the smallest fuzzy cost in each column and then subtract that from each fuzzy cost of that column.

Step 4. Optimality criterion

(a) Verify each supply is less than or equal to the total demand, whose reduced fuzzy costs are zero.

(b) Now verify each demand is less than or equal to the total sum supply, whose reduced fuzzy costs are zero.

(c) If step 4(a) and step 4(b) are satisfied then go to step7 else go to step5.

Step 5. Revise the opportunity cost table

Draw the minimum number of horizontal and vertical lines to cover all fuzzy zeros in the revised cost table obtained from step3.

Step 6. Develop the new revised opportunity cost table

(a) From the cells uncovered by any line, choose the smallest fuzzy cost. Call this fuzzy cost value $\tilde{k}$.

(b) Subtract $\tilde{k}$ from every fuzzy cost in the cell not covered by a line.

(c) Add $\tilde{k}$ to every fuzzy cost in the cell covered by the two lines.

(d) Fuzzy costs in cells covered by one line remain unchanged.

(e) Then go to step4.

Step 7. **Determination of cell for allocation**

(a) New revised opportunity cost table having one fuzzy zero cost in each row and column, then find the fuzzy zero division value of all the fuzzy zeros in the new revised opportunity cost matrix by the following simplification, the fuzzy zero division value is denoted by

$$\tilde{A} = \frac{Add\ the\ adajacent\ sides\ of\ non\ zero\ fuzzy \cos ts\ i^{th} row\ and\ j^{th}\ column}{number\ of\ fuzzy\ zeros\ in\ adajacent\ of\ i^{th} row\ and\ j^{th}\ column}$$

(b) Choose the maximum of $\tilde{A}$ and allocate the maximum possible to the cell, and cross off in the usual manner, if there is a tie in the maximum value of $\tilde{A}$ select arbitrary.

(c) Repeat step 7(a) to step 7(b) until the entire demand at various destinations or available supply at various sources are satisfied.

Step 8. Feasible allocation

Compute total fuzzy transportation cost for the feasible allocation from the original fuzzy cost table.

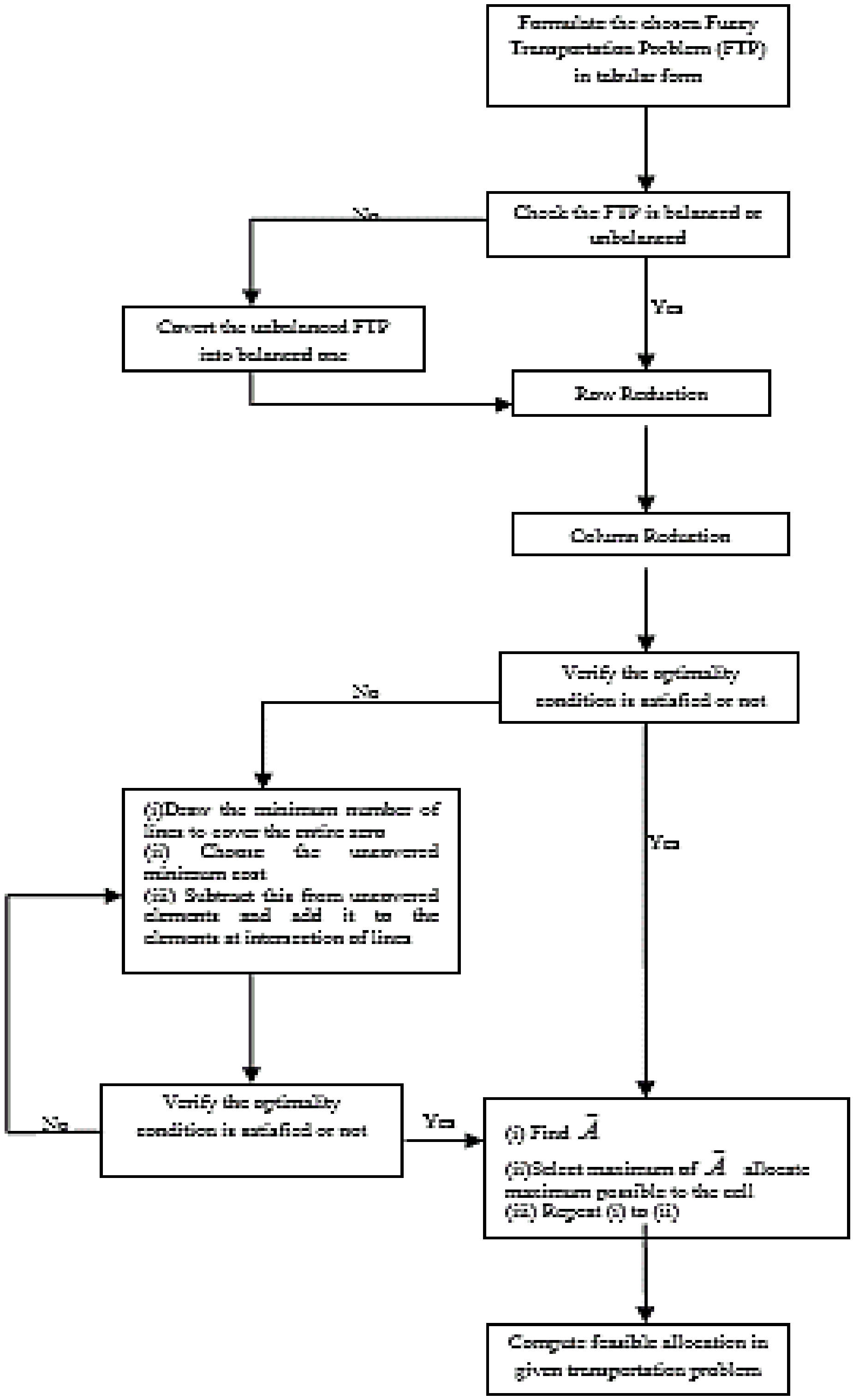


Figure 1: Flow chart of the ZDM

5. NUMERICAL EXAMPLES

To illustrate the proposed method, namely, Zero Division Method (ZDM), the following Fuzzy Transportation Problem is solved.

5.1 Problem.1

A company has four warehouses W_1, W_2, W_3, W_4. It is required to deliver a product from these four warehouses to three customers C_1, C_2, C_3. Amount in stock at these four warehouses W_1, W_2, W_3 and W_4 are 4, 8, 7 and 15 units respectively and the requirement of the three customers C_1, C_2 and C_3 of the product are 7, 9 and 18 units respectively. But due to frequently variation in the rates of fuel and several other reasons, the owner of the company is uncertain about the transportation cost. The transporting cost for one unit quantity of product from each warehouse to each customer is represented by generalized trapezoidal fuzzy number. Determine the fuzzy optimal transportation of the products such that the total transportation cost is minimum. Let the data be represented in the following table 1.

Table 1

	C_1	C_2	C_3	Supply (a_i)
W_1	(2,3,4,7;.3)	(3,7,9,14;.5)	(10,12,14,15;.6)	4
W_2	(2,3,4,8;.4)	(2,3,5,6;.2)	(0,1,2,3;.2)	8
W_3	(3,5,7,10;.5)	(2,4,6,9;.3)	(3,7,9,11;.5)	7
W_4	(0,1,3,4;.3)	(4,6,7,13;.7)	(2,3,4,5;.4)	15
Demand (b_j)	7	9	18	

Since $\sum_{i=1}^{4} a_i = 34 = \sum_{j=1}^{3} b_j = 34$, so the chosen problem is a balanced FTP.

Iteration 1. Using step 3, we get

	C_1	C_2	C_3	Supply (a_i)
W_1	**(-9,-3,3,9;.3)**	(-11,1,8,19;.3)	(0,7,12,16;.2)	**4**
W_2	(-6,0,4,13;.2)	(-8,-1,6,13;.2)	**(-6,-2,2,6;.2)**	**8**
W_3	(-11,-2,4,13;.2)	**(-14,-2,2,14;.3)**	(-9,0,6,12;.2)	**7**
W_4	**(-9,-3,3,9;.3)**	(-7,1,8,20;.3)	(-5,-1,4,8;.2)	15
Demand (b_j)	**7**	9	18	

Using step 4**, row 4 and column 2 and 3** are not satisfied.

Improvement is required.

Iteration 2. Using step 5 and step 6, we get

	C_1	C_2	C_3	Supply (a_i)
W_1	**(-9,-3,3,9;.3)**	(-19,-3,9,24;.2)	(-8,3,13,21;.2)	**4**
W_2	(-11,-1,8,21;.2)	(-8,-1,6,13;.2)	**(-6,-2,2,6;.2)**	**8**
W_3	(-16,-3,8,21;.2)	**(-14,-2,2,14;.3)**	(-9,0,6,12;.2)	**7**
W_4	**(-9,-3,3,9;.3)**	(-15,-3,9,25;.2)	**(-13,-5,5,13;.2)**	**15**
Demand (b_j)	**7**	9	**18**	

Using step 4, **here column 2 is satisfied**. Repeat the process.

Iteration 3. Again apply step 5 and step 6, we get

	C_1	C_2	C_3	Supply (a_i)
W_1	**(-9,-3,3,9;.3)**	(-32,-9,10,32;.2)	(-8,3,13,21;.2)	**4**
W_2	(-11,-1,8,21;.2)	**(-21,-7,7,21;.2)**	**(-6,-2,2,6;.2)**	**8**
W_3	(-24,-4,14,34;.2)	**(-14,-2,2,14;.3)**	(-17,-1,12,25;.2)	**7**
W_4	**(-9,-3,3,9;.3)**	(-28,-9,10,33;.2)	**(-13,-5,5,13;.2)**	**15**
Demand (b_j)	**7**	**9**	**18**	

Using step 4, here **optimal table** is attains.

Using step 7, allocate the values:

	C_1	C_2	C_3	Supply (a_i)
W_1	(-9,-3,3,9;.3) **4**	(-32,-9,10,32;.2)	(-8,3,13,21;.2)	4
W_2	(-11,-1,8,21;.2)	(-21,-7,7,21;.2) 2	(-6,-2,2,6;.2) **6**	8
W_3	(-24,-4,14,34;.2)	(-14,-2,2,14;.3) **7**	(-17,-1,12,25;.2)	7
W_4	(-9,-3,3,9;.3) **3**	(-28,-9,10,33;.2)	(-13,-5,5,13;.2) **12**	15
Demand (b_j)	7	9	18	

Using step 8, we get.

	C_1	C_2	C_3	Supply (a_i)
W_1	(2,3,4,7;.3) **4**	(3,7,9,14;.5)	(10,12,14,15;.6)	4
W_2	(2,3,4,8;.4)	(2,3,5,6;.2) **2**	(0,1,2,3;.2) **6**	8
W_3	(3,5,7,10;.5)	(2,4,6,9;.3) **7**	(3,7,9,11;.5)	7
W_4	(0,1,3,4;.3) **3**	(4,6,7,13;.7)	(2,3,4,5;.4) **12**	15
Demand (b_j)	7	9	18	

The minimum fuzzy transportation cost is

= **4** (2,3,4,7;.3) + **2** (2,3,5,6;.2) + **6** (0,1,2,3;.2) + **7** (2,4,6,9;.3)

+ **3** (0, 1, 3, 4; .3) + **12** (2, 3, 4, 5; .4)

= (50, 91, 137, 193; .2)

The ranking function R (A) = 23.6

The conventional solution obtained by **NWCR [1]**

	C_1	C_2	C_3	Supply (a_i)
W_1	(2,3,4,7;.3) **4**	(3,7,9,14;.5)	(10,12,14,15;.6)	4
W_2	(2,3,4,8;.4) **3**	(2,3,5,6;.2) **5**	(0,1,2,3;.2)	8
W_3	(3,5,7,10;.5)	(2,4,6,9;.3) **4**	(3,7,9,11;.5) **3**	7
W_4	(0,1,3,4;.3)	(4,6,7,13;.7)	(2,3,4,5;.4) **15**	15
Demand (b_j)	7	9	18	

The minimum fuzzy transportation cost is

= **4** (2,3,4,7;.3) + **3** (2,3,4,8;.4) + **5** (2,3,5,6;.2) + **3** (3,7,9,11;.5)

+ **4** (2, 4, 6, 9; .3) + **15** (2, 3, 4, 5; .4)

= (71, 118, 164, 226; .2)

The ranking function R (A) = 29.0

The conventional solution obtained by **MMM [1]**

	C_1	C_2	C_3	Supply (a_i)
W_1	(2,3,4,7;.3)	(3,7,9,14;.5) **2**	(10,12,14,15;.6) **2**	4
W_2	(2,3,4,8;.4)	(2,3,5,6;.2)	(0,1,2,3;.2) **8**	8
W_3	(3,5,7,10;.5)	(2,4,6,9;.3) **7**	(3,7,9,11;.5)	7
W_4	(0,1,3,4;.3) **7**	(4,6,7,13;.7)	(2,3,4,5;.4) **8**	15
Demand (b_j)	7	9	18	

The minimum fuzzy transportation cost is

= **2**(3,7,9,14;.5) + **2** (10,12,14,15;.6) + **8** (0,1,2,3;.2) + **7** (2,4,6,9;.3)

+ **7** (0, 1, 3, 4; .3) + **8** (2, 3, 4, 5; .4)

= (56, 105, 157, 213; .2)

The ranking function R (A) = 26.6

The conventional solution obtained by **VAM [21]**

	C_1	C_2	C_3	Supply (a_i)
W_1	(2,3,4,7;.3) **4**	(3,7,9,14;.5)	(10,12,14,15;.6)	4
W_2	(2,3,4,8;.4)	(2,3,5,6;.2)	(0,1,2,3;.2) **8**	8
W_3	(3,5,7,10;.5)	(2,4,6,9;.3) **7**	(3,7,9,11;.5)	7
W_4	(0,1,3,4;.3) **3**	(4,6,7,13;.7) **2**	(2,3,4,5;.4) **10**	15
Demand (b_j)	7	9	18	

The minimum fuzzy transportation cost is

= **4** (2,3,4,7;.3) + **8** (0,1,2,3;.2) + **7** (2,4,6,9;.3) + **3** (0, 1, 3, 4; .3) + **2** (4,6,7,13;.7)

+ **10** (2, 3, 4, 5; .4)

= (50, 93, 137, 203; .2)

The ranking function R (A) = 24.2

The conventional solution obtained by **MODI [5]**

	C_1	C_2	C_3	Supply (a_i)
W_1	(2,3,4,7;.3) **4**	(3,7,9,14;.5)	(10,12,14,15;.6)	4
W_2	(2,3,4,8;.4)	(2,3,5,6;.2) **2**	(0,1,2,3;.2) **6**	8
W_3	(3,5,7,10;.5)	(2,4,6,9;.3) **7**	(3,7,9,11;.5)	7
W_4	(0,1,3,4;.3) **3**	(4,6,7,13;.7)	(2,3,4,5;.4) **12**	15
Demand (b_j)	7	9	18	

The minimum fuzzy transportation cost is

= **4** (2,3,4,7;.3) + **2** (2,3,5,6;.2) + **6** (0,1,2,3;.2) + **7** (2,4,6,9;.3)

+ **3** (0, 1, 3, 4; .3) + **12** (2, 3, 4, 5; .4)

= (50, 91, 137, 193; .2)

The ranking function R (A) = 23.6

5.2. Results with normalization process

If all the values of the parameters used in problem.1 are first normalized and then the Problem is solved by using the ZDM, then the fuzzy optimal value is $\tilde{x}_0 = (50,\ 91,\ 137,\ 193;\ 1)$.

5.3. Results without normalization process

If all the values of the parameters of the same problem.1 are not normalized and then the Problem is solved by using the ZDM, then the fuzzy optimal value is $\tilde{x}_0 = (50,\ 91,\ 137,\ 193;.2)$.

5.4. Remark

Results with normalization process represent the overall level of satisfaction of decision maker about the statement that minimum transportation cost will lie between 91and 137 units as 100% while without normalization process, the overall level of satisfaction of the decision maker for the same range is 20%. Hence, it is better to use generalized fuzzy numbers instead of normal fuzzy numbers, obtained by using normalization process.

5.5. Problem. 2

	D_1	D_2	D_3	D_4	Supply (a_i)
S_1	(18,19,20,22;.6)	(20,21,22,24;.6)	(15,17,20,21;.5)	(3,4,5,7;.2)	120
S_2	(22,24,26,28;.5)	(35,36,37,40;.7)	(8,9,10,11;.2)	(5,7,9,11;.3)	70
S_3	(30,31,32,33;.4)	(35,36,37,41;.7)	(18,19,20,22;.6)	(13,15,16,17;.5)	50
Demand (b_j)	60	40	30	110	

5.6. Problem. 3

	D_1	D_2	D_3	D_4	Supply (a_i)
S_1	(1,2,3,5;.2)	(1,3,4,7;.5)	(9,11,12,14;.3)	(5,7,8,11;.6)	6
S_2	(1,2,3,6;.4)	(0,2,6,13;.5)	(5,6,7,9;.5)	(1,2,3,4;.2)	1
S_3	(3,5,6,8;.2)	(5,8,9,12;.6)	(12,15,16,19;.4)	(7,9,10,15;.6)	10
Demand (b_j)	7	5	3	2	

5.7. Problem. 4 [12]

	D_1	D_2	D_3	Supply (a_i)
S_1	(1,4,9,19;.5)	(1,2,5,9;.4)	(2,5,8,18;.5)	10
S_2	(8,9,12,26;.5)	(3,5,8,12;.2)	(7,9,13,28;.4)	14
S_3	(11,12,20,27;.5)	(0,5,10,15;.8)	(4,5,8,11;.6)	15
Demand (b_j)	15	14	10	

5.8. Problem. 5 [12]

	D_1	D_2	D_3	D_4	Supply (a_i)
S_1	(1,4,8,11;.6)	(0,1,2,3;.4)	(3,7,10,16;.5)	(1,2,4,5;.5)	70
S_2	(4,10,12,18;.3)	(1,3,6,10;.2)	(0,1,3,4;.5)	(3,5,9,15;.2)	55
S_3	(4,9,12,15;.2)	(6,10,14,18;.6)	(2,3,5,6;.2)	(4,6,8,10;.2)	90
Demand (b_j)	85	35	50	45	

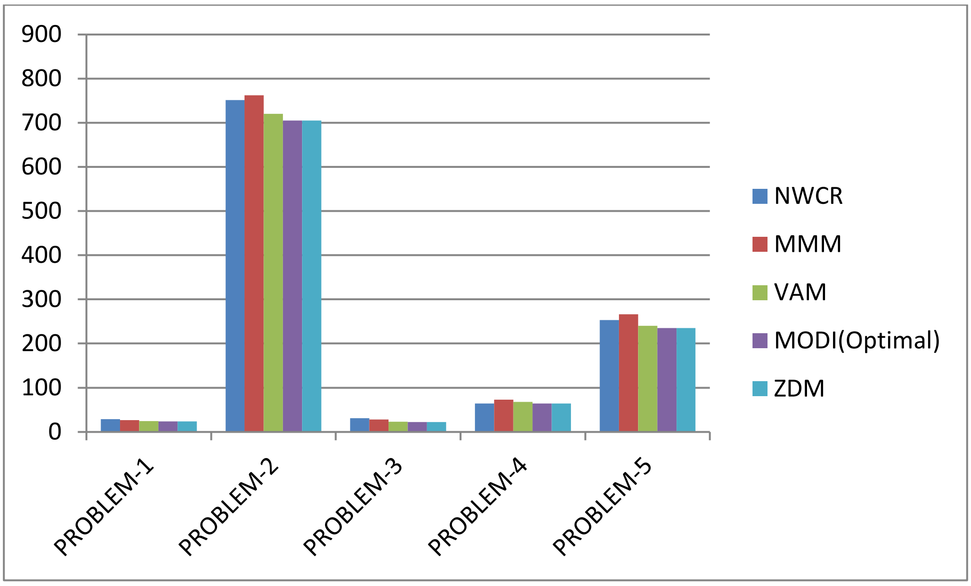
6. COMPARATIVE STUDY AND RESULT ANALYSIS

From the investigations and the results given in Table 2 it clear that ZDM is better than **NWCR [1]**, **MMM [1]** and **VAM [21]** for solving fuzzy transportation problem and also, the solution of the fuzzy transportation problem is given by ZDM is an optimal solution.

Table 2

S.NO	ROW	COLUMN	NWCR	MMM	VAM	MODI	ZDM
1.	4	3	29	26.6	24.2	**23.6**	**23.6**
2.	3	4	751.5	762	720	**705**	**705**
3.	3	4	30.80	28.25	22.7	**22.3**	**22.3**
4.	3	3	64.4	73.1	67.6	**64.4**	**64.4**
5.	3	4	253.0	266.5	240.0	**235.0**	**235.0**

Table 2 represents the solution obtained by **NWCR [1]**, **MMM [1], VAM [21], MODI [5]** and **ZDM.** This data speaks the better performance of the proposed method. The graphical representation of solution obtained by varies methods of this performance, displayed in graph 1.



Graph -1

	PROBLEM1	PROBLEM2	PROBLEM3	PROBLEM4	PROBLEM5
NNCR	29	751.5	30.8	64.4	253
MMM	26.6	762	28.3	73.1	266.5
VAM	24.2	720	22.7	67.6	240
MODI	23.6	705	22.3	64.4	235
ZDM	23.6	705	22.3	64.4	235

Graph 1 – Graphical representation of the obtained results.

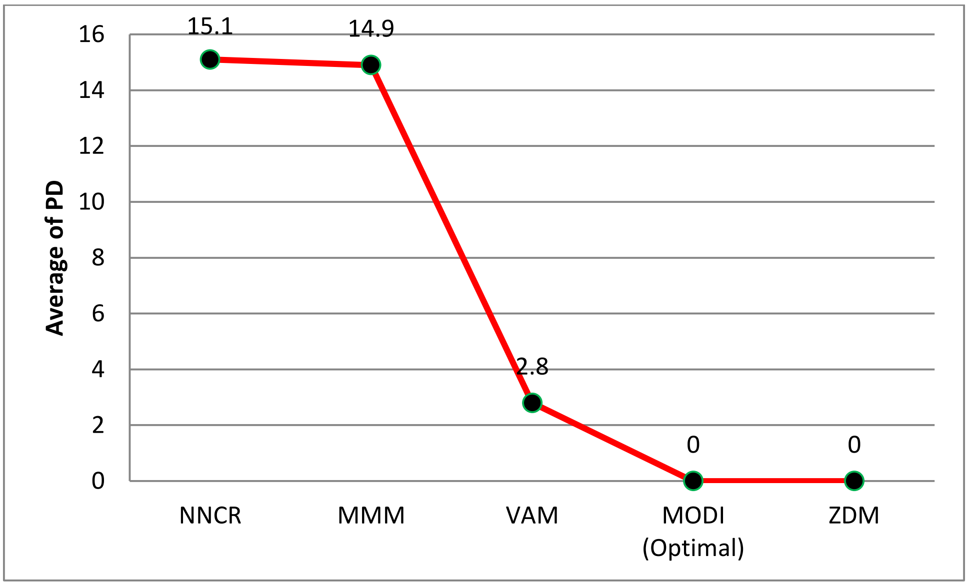
Table 3, represents the difference between the optimal results and obtain results. Percentage of difference (PD) and average of PD are shown in table 4. During this process, it is observed that the performance of the proposed method is superior to other methods discussed in this table, which is shown in Graph 2.

Table 3

Difference between the optimal results and obtained results						
S.No	**NWCR**	**MMM**	**VAM**	**MODI**		**ZDM**
1.	5.4	3.0	0.6	**23.6**	**0.0**	**0.0**
2.	46.5	57.0	15.0	**705**	**0.0**	**0.0**
3.	8.5	6.0	0.4	**22.3**	**0.0**	**0.0**
4.	0.0	8.7	3.2	**64.4**	**0.0**	**0.0**
5.	18.0	31.5	5.0	**235.0**	**0.0**	**0.0**

Table 4

Percentage of Difference (PD)					
S.No	**NWCR**	**MMM**	**VAM**	**MODI**	**ZDM**
1.	22.9	12.7	2.5	**0.0**	**0.0**
2.	6.6	8.1	2.1	**0.0**	**0.0**
3.	38.1	26.9	1.8	**0.0**	**0.0**
4.	0	13.5	5	**0.0**	**0.0**
5.	7.7	13.4	2.1	**0.0**	**0.0**
Average of PD	**15.1**	**14.9**	**2.8**	**0.0**	**0.0**



Graph - 2

Graph 2 – Graphical representation of average PD

Above discussion ensures the feasibility of the proposed method in solving transportation problems. The ZDM solution procedure is also computationally very easier.

7. APPLICATION OF THE REAL LIFE FUZZY TRANSPORTATION PROBLEM [12].

Colliery	D_1	D_2	D_3	D_4	D_5	D_6	Supply (a_i) in ton
S_1	(20,27,35,41;.7)	(9,11,12,14;.6)	(10,15,18,20;.7)	(15,20,22,24;.7)	(10,15,18,20;.8)	(9,12,15,18;.7)	124
S_2	(20,25,35,41;.7)	(9,11,12,16;.8)	(9,11,12,14;.6)	(10,15,21,23;.8)	(10,15,18,23;6)	(9,12,15,18;.8)	120
S_3	(9,10,12,16;7)	(65,70,74,76;.8)	(20,25,35,41;.8)	(12,15,22,24;.7)	(10,12,14,18;.6)	(15,20,26,28;.8)	150
S_4	(9,11,12,14;.6)	(10,15,21,24;.7)	(20,25,35,41,.6)	(10,15,18,20;.6)	(8,10,12,14;.8)	(15,20,25,28;.7)	170
Demand (b_j) in ton	112	90	84	92	106	80	

In this section, to show that the application of the proposed method, the data collected from an owner of a regional coal company situated in Jharia is shown in table below:

Since, the owner of the company is certain about the supply and demands of the coal at different collieries and washeries, respectively, so in the above table, these parameters of diesel and several other reasons, the owner of the company is uncertain about the transportation cost (in rupees) from different collieries and washeries. So according to past experiences of the owner, the transportation costs are represented by generalized trapezoidal fuzzy number. The owner of the company needs to keep the transportation cost minimum.

7.1. We apply our proposed method, ZDM to compute optimal solution is

Colliery	D_1	D_2	D_3	D_4	D_5	D_6	Supply (a_i) in ton
S_1	(20,27,35,41;.7)	(9,11,12,14;.6) **80**	(10,15,18,20;.7)	(15,20,22,24;.7)	(10,15,18,20;.8)	(9,12,15,18;.7) **44**	**124**
S_2	(20,25,35,41;.7)	(9,11,12,16;.8)	(9,11,12,14;.6) **84**	(10,15,21,23;.8)	(10,15,18,23;6)	(9,12,15,18;.8) **36**	**120**
S_3	(9,10,12,16;7) **112**	(65,70,74,76;.8)	(20,25,35,41;.8)	(12,15,22,24;.7) **38**	(10,12,14,18;.6)	(15,20,26,28;.8)	**150**
S_4	(9,11,12,14;.6)	(10,15,21,24;.7) **10**	(20,25,35,41,.6)	(10,15,18,20;.6) **54**	(8,10,12,14;.8) **106**	(15,20,25,28;.7)	**170**
Demand (b_j) in ton	**112**	**90**	**84**	**92**	**106**	**80**	

The minimum fuzzy transportation cost is

= **80** (9,11,12,14;.6) + **44** (9,12,15,18;.3) + **84** (9,11,12,14;.6) + **36** (9,12,15,18;.8)

+ **112**(9,10,12,16;.7) + **38** (12, 15, 22, 24; .7) + **10** (10, 15, 21, 24; .7)+**54** (10,15,18,20;.6)

+**106** (8, 10, 12, 14; .8)

= (5414,6802,8280,9712;.6)

The ranking function R (A) = 4531.2

8. CONCLUSION AND FUTURE WORK

In this paper new method, namely, ZDM is finding an optimal solution of generalized fuzzy transportation problem. The values of transportation costs are represented by generalized fuzzy numbers. The advantage of the proposed method is discussed and a numerical example is solved. The ZDM is easy to understand and apply for solving the fuzzy transportation problems occurring in real life situations. In future using the proposed method, the existing software's for solving fuzzy transportation problems.

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