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Modeling and Pricing of Weather Derivative Market

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Abstract

In recent years, one of the factors that had a significant impact on the economic development was represented by climatic change. At international level, the weather risk management stands for a priority for Governments, insurance companies and companies within the industries affected by the weather variability. The main objective of this study is to modelling and Pricing of Weather Derivative Market. The main goal of this study is fourfold: 1) First, we begin our approach to construct the temperature model under Ornstein – Uhlenbeck process which is driven by a Levy process rather than a standard Brownian motion is investigated. 2) Then we extent our approach to briefly present the weather derivatives market model. 3) Next we construct the Modeling and pricing of weather derivatives. 4) Finally, how weather forecasting and seasonal forecasting can potentially improve our valuation of weather derivative contracts. In addition, this paper ends with conclusion.

KeyWords: Weather Derivatives, Temperature, Ornstein – Uhlenbeck Process Heating Degree Days (HDDs), Cooling Degree Days CDDs and Arbitrage Pricing Theory.

INTRODUCTION

Even in our technology-based technology, it continues to live largely at the mercy of time. It influences our lives and choices, and has a huge impact on corporate earnings and profits. Until recently, very few financial tools offered corporate protection against climate risks. However, the beginning of the derivative of time - making the sea a tradable good - has changed all this. Almost 20 percent of the US economy is estimated to be directly affected by climate and the income and incomes of virtually every industry - agriculture, energy, entertainment, construction, travel, and others: "Climate is not just an environmental problem, it is a major economic factor, and at least a trillion dollars of our economy is climate sensitive”.

Weather derivatives are financial instruments that can be used by organizations or individuals as part of a risk management strategy to reduce the risk associated with adverse or unexpected weather conditions. Weather derivatives are index-based instruments that usually use meteorological data observed in a weather station to create an index on which a payment may be based. This index could be the total precipitation during a relevant period - which may be relevant for a hydroelectric generation company - or the number where the minimum temperature falls below zero, which could be relevant for a farmer who protects against frost damage. Unlike insurance-based coverage of "compensation”, There is no need to prove that a loss has been suffered. The settlement is objective, based on the final value of the weather index chosen during the chosen period. If a payment is due, it is usually done in a matter of a few days with the settlement period that is defined in the contract: unlike insurance contracts, there is no "loss adjustment" process.

The risks that companies face due to the climate are unique. Weather conditions tend to affect volume and use more than they directly affect price. An exceptionally warm winter, for example, can leave utilities and energy companies with excess supplies of oil or natural gas (because people need less to heat their homes). Alternatively, an exceptionally cold summer can leave hotel and airline seats empty. Although prices may change somewhat due to unusually high or low demand, price adjustments do not necessarily compensate for lost revenue because of untimely temperatures. Finally, weather risk is also unique in that it is highly localized, can´t be controlled and despite great advances in meteorological science, still cannot be predicted precisely and consistently.

Until recently, insurance was the main tool used by companies for protection against unexpected climatic conditions. However, the insurance provides protection only against catastrophic damages. Insurance does nothing to protect against the reduced demand that companies experience due to a climate that is warmer or colder than expected. In the late 1990s, people began to realize that if they quantified and indexed the climate in terms of monthly or seasonal temperatures, and added a dollar amount to each index value, they could in a sense "pack" and Commercial climate. In fact, this type of trade would be comparable to trading the variable values of stock indices, currencies, interest rates and agricultural products. The concept of climate as a tradable good, therefore, began to take shape. "Unlike the various perspectives provided by the government and independent forecasts, trading in weather derivatives gave market participants a quantifiable view of those prospects," said Agbeli Ameko, managing partner of energy and forecast firm Enter Cast. In 1997, the first OTC derivatives trade took place, and the field of climate risk management was born. According to Valerie Cooper, former executive director of the Weather Risk Management Association (WRMA), an $ 8 billion climate derivatives industry was developed within a few years of its creation.

In general, weather derivatives cover events of low risk and high probability. On the other hand, weather insurance usually covers high risk and low probability events, as defined in a highly adapted or customized policy. In this case, the company knows that this type of climate will affect its revenues. However, the same company would probably buy an insurance policy to protect against damages caused by a flood or hurricane (high risk and low probability events).

In 1999, the Chicago Mercantile Exchange (CME) took the derivatives of time a step further and introduced futures and options on futures traded, the first products of its kind. OTC time derivatives are privately negotiated, individualized settlements made between two parties. However, CME futures and futures options are standardized publicly traded contracts in the open market in an electronic auction environment, with continuous price negotiation and full price transparency.

1. STOCHASTIC MODEL FOR TEMPERATURE INDEX

In this Section, we discuss different stochastic models for temperature variations. We suggest an Ornstein-Uhlenbeck process driven by L´evy noise to model temperature fluctuations, but also present in detail other models proposed in the literature. Or

The Ornstein-Uhlenbeck (throughout: OU) process was proposed by Uhlenbeck and Ornstein (1930) in a physical modelling context, as an alternative to Brownian Motion, where some kind of mean reverting tendency is called for in order to adequately describe the situation being modelled. A Brownian motion may be a good model for a particle movement. After a hit the particle does not stop after changing position, but it moves continuously with decreasing speed. The Brownian motion is not differentiable anywhere.

In mathematics, the Ornstein–Uhlenbeck process (OU) is a stochastic process that, roughly speaking, describes the velocity of a massive Brownian particle under the influence of friction. The process is stationary Gauss–Markov process (which means that it is both a Gaussian and Markovian process), and is the only nontrivial process that satisfies these three conditions, up to allowing linear transformations of the space and time variables. Over time, the process tends to drift towards its long-term mean: such a process is called mean reverting.

There are a number of mean-reverting models proposed in the literature for daily average temperature, varying primarily in their description of the random `noise' of temperature variations. It is this unpredictable component of the daily average temperature that constitutes weather risk. We can consider three different prescriptions of this random noise term: a fractional Brownian motion, a standard Brownian motion and a Levy Process.

The objective of this Ornstein–Uhlenbeck is related stochastic processes to a wide mathematical audience with a modest preparation in stochastic analysis.

Benth's paper [1] proposes a non-Gaussian Ornstein-Uhlenbeck model for the evolution of temperature, with seasonal mean and volatility, and with residuals generated by a Levy process.

For a particular location, let

i. $T(t)$ denote the daily average temperature on day *t*.

ii. $s(t)$ be a deterministic function, describing the seasonal variation. The temperature reverts on day *t* to the mean level.

iii. $\alpha(t)$ denote the rate of mean-reversion on day *t*.

iv. $\sigma(t)$ denote the volatility of daily temperature fluctuations. It is a seasonal function, taking into account the variation in volatility throughout the year.

For formal definition of Levy process, we refer the reader to the introduction paper [2]. This model was first introduced by [3] with Brownian motion.

We focus on models of the form

$$dT(t) = d\Lambda(t) + \alpha(t)(T(t) - \Lambda(t)dt + \sigma(t)dX(t)) \tag{1}$$

Where *X(t)* is a stochastic process on the probability space (Ω, F, P). Thus daily average temperature follows an Ornstein-Uhlenbeck process, reverting to mean level $\Lambda(t)$,our seasonal component. The speed of mean reversion, α, is constant in this model.

From Eq (1) we can directly use Ito formula for semimartingales to get the explicit solution

$$T(t) = s(t) + (T(0) - s(0))e^{\alpha t} + \int_0^t \sigma(u)e^{\alpha(t-u)}dX(u) \tag{2}$$

For the purpose of fitting this model to our daily average temperature, we reformulate Eq (2) by subtracting $T(t)$ from $T(t+1)$,

$$\Delta T(t) = \Delta s(t) - (1 - e^{\alpha})(T(t) - s(t)) + e^{\alpha}\int_0^{t+1} \sigma(u)e^{k(t-u)}dX(u) \tag{3}$$

With the notation $\Delta T(t) := X(t+1) - X(t)$. The stochastic integral could be approximated by

$$\Delta T(t) = \Delta s(t) - \left(1 - e^{\alpha}\right)\left(T(t) - s(t)\right) + e^{\alpha}\sigma(t)\Delta X(t) \tag{4}$$

Adding $T(t) - s(t) :\overset{\approx}{=} \overline{T}(t)$to both sides

$$\overline{T}(t+1) = e^{\alpha}\overline{T}(t) + e^{\alpha}\sigma(t)\varepsilon(t) \tag{5}$$

If we define $\overline{\varepsilon} := e^{\alpha}\overline{T}(t)\varepsilon(t)$, we will have the following three parts for modelling $T(t)$ step by step:

$$T(t) \Leftrightarrow s(t) + c(t) + \overline{\varepsilon}(t), \tag{6}$$

Where $s(t)$is the seasonal component, $c(t)$ is the cyclical component derived from Eq (5), and $\overline{\varepsilon}(t)$ is the stochastic part.

Now, we will build the OU process driven the general Levy processes.

Let $L = (L_t)_{t \geq 0}$ is a time homogeneous Levy process, for $\alpha > 0$, Ornstein – Unlenbeck (OU) type process has

$$\begin{aligned} X_t &= e^{-\alpha t} X_0 + \int_0^t e^{-\alpha(t-s)} dL_s \\ &= e^{-\alpha t} X_0 + e^{-\alpha t} \int_0^t e^{\alpha s} dL_s \end{aligned} \tag{7}$$

It is unique strong solution below SDE,

$$dX_t = -\alpha X_t dt + \sigma dL_t, \quad X_0 = x_0 \tag{8}$$

Where α denotes the arte of decay. The α enters the stationary solution of OU process. This leads difficulties solution of SDE. We can remove these difficulties by a simple change of time in the stochastic integrals [4].

We can rewrite OU process as follows

$$X_t = e^{\alpha t} X_0 + \int_0^t e^{-\alpha(t-s)} dL_{\alpha s} \tag{9}$$

If $Y = (Y_t)$is an OU process with marginal law D, then we say that Y is a D – OU process. When given a one dimensional marginal law is D if and only if D is self – decomposable [5]. We have the result that [4],

$$X_t = e^{-\alpha t} X_0 + e^{-\alpha t} \int_0^{\alpha t} e^s dL_s \tag{10}$$

An Ornstein–Uhlenbeck process, x_t, satisfies the following stochastic differential equation:

$$dx_i = \theta(\mu - x_t)dt + \sigma dW_t \tag{11}$$

Where $\theta > 0$, μ *and* $\sigma > 0$ are parameters and W_t denotes the Wiener process.

The above representation can be taken as the primary definition of an Ornstein–Uhlenbeck process [6] or sometimes also mentioned as the Vasicek model. [7].

The probability density function $f(x,t)$ of the Ornstein–Uhlenbeck process satisfies the Fokker–Planck equation

$$\frac{\partial f}{\partial t} = \theta \frac{\partial}{\partial x}[(x-\mu)f] + \frac{\sigma^2}{2}\frac{\partial^2 f}{\partial x^2} \tag{12}$$

The Green function of this linear parabolic partial differential equation, where $D = \frac{\sigma^2}{2}$, and the initial condition consisting of a unit point mass at location *y* is

$$f(x,t) = \sqrt{\frac{\theta}{2\pi D\left(1-e^{-2\theta t}\right)}} \exp\left\{\frac{-\theta}{2D}\left[\frac{\left(x-\mu-(y-\mu)e^{-\theta t}\right)^2}{1-e^{-2\theta t}}\right]\right\} \tag{13}$$

Which is Gaussian distribution with mean $\mu + (y-\mu)e^{-\theta t}$ and variance $\frac{\sigma^2}{2\theta}\left(1-e^{-2\theta t}\right)$. The stationary solution of this equation is the limit for time tending to infinity which is a Gaussian distribution with mean μ and variance $\frac{\sigma^2}{(2\theta)}$.

$$f_s(x) = \sqrt{\frac{\theta}{\pi\sigma^2}} e^{-\theta(x-\mu)^2/\sigma^2} \tag{14}$$

The Ornstein–Uhlenbeck process is a prototype of a noisy relaxation process. Consider for example a Hookean spring with spring constant *k* whose dynamics is highly overdamped with friction coefficient γ. In the presence of thermal fluctuations with temperature *T*, the length *x(t)* of the spring will fluctuate stochastically around the spring rest length x_0; its stochastic dynamic is described by an Ornstein–Uhlenbeck process with:

$$\theta = k/\gamma, \quad \mu = x_0, \quad \sigma = \sqrt{2k_B T/\gamma}\ .$$

Where σ is derived from the Stokes – Einstein equation $D = \sigma^2/2 = k_B T/\gamma$ for the effective diffusion constant. In physical sciences, the stochastic differential equation of an Ornstein–Uhlenbeck process is rewritten as a Langevin equation

$$\dot{x}(t) = -\frac{k}{\gamma}(x(t) - x_0) + \xi(t) \tag{15}$$

Where $\xi(t)$ is white Gaussian noise $\langle \xi(t_1)\xi(t_2) \rangle = \frac{2k_B T}{\gamma}\delta(t_1 - t_2)$. Fluctuation are correlated as

$$\langle (x(t_0) - x_0)(x(t_0 + t) - x_0) \rangle = \frac{k_B T}{k}\exp\left(-|t|/\tau\right) \tag{16}$$

With correlation time $\tau = \gamma/k$.

At equilibrium, the spring stores an average energy $\langle E \rangle = \frac{k\langle (x - x_0)^2 \rangle}{2} = \frac{k_B T}{2}$ in accordance with the equipartition theorem.

The Ornstein–Uhlenbeck process is one of several approaches used to model (with modifications) interest rates, currency exchange rates, and commodity prices stochastically. The parameter μ represents the equilibrium or mean value supported by fundamentals; σ the degree of volatility around it caused by shocks, and θ the rate by which these shocks dissipate and the variable reverts towards the mean. One application of the process is a trading strategy known as pair's trade [8].

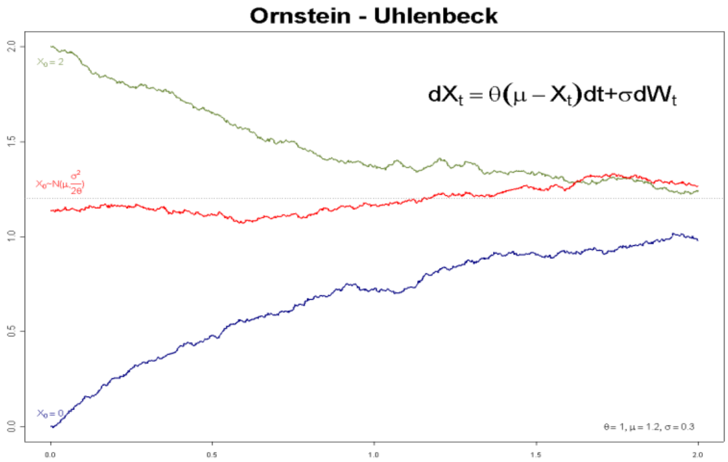

Figure 1

From fig 1, three sample paths of different OU – process with $\theta = 1, \mu = 1.2$ *and* $\sigma = 0.3$: the Bottom: Initial value a=0 (a.s)

Top: Initial value a = 2 (a.s)

Centre: Initial value normally distribution so that the process has invariant measure.

The Ornstein–Uhlenbeck process is an example of a Gaussian process that has a bounded variance and admits a stationary probability distribution, in contrast to the Wiener process; the difference between the two is in their "drift" term. For the Wiener process the drift term is constant, whereas for the Ornstein–Uhlenbeck process it is dependent on the current value of the process: if the current value of the process is less than the (long-term) mean, the drift will be positive; if the current value of the process is greater than the (long-term) mean, the drift will be negative. In other words, the mean acts as an equilibrium level for the process. This gives the process its informative name, "mean-reverting." The stationary (long-term) variance is given by

$$\operatorname{var}(x_t) = \frac{\sigma^2}{2\theta} \tag{17}$$

The Ornstein–Uhlenbeck process is the continuous-time analogue of the discrete-time AR(1) process. Asymptotic distribution of the MLE for the Ornstein-Uhlenbeck process:

$$\sqrt{n}\left(\begin{pmatrix}\hat{\theta}_n\\ \hat{\mu}_n\\ \hat{\sigma}_n\end{pmatrix}-\begin{pmatrix}\theta\\ \mu\\ \sigma\end{pmatrix}\right)\xrightarrow{d} N\left(\begin{pmatrix}0\\0\\0\end{pmatrix},\begin{pmatrix}\frac{e^{2t\theta}-1}{2} & 0 & \frac{\sigma}{2}\frac{e^{2t\theta}-1-2t\theta}{t^2\theta}\\ 0 & \frac{\sigma^2\left(e^{t\theta}+1\right)}{2\left(e^{t\theta}-1\right)\theta} & 0\\ \frac{\sigma}{2}\frac{e^{2t\theta}-1-2t\theta}{t^2\theta} & & \frac{\sigma^2}{2}\frac{\left(e^{2t\theta}-1\right)^2+2t^2\theta^2\left(e^{2t\theta}+1\right)-4t\theta\left(e^{2t\theta}-1\right)}{t^2\left(e^{2t\theta}-1\right)\theta^2}\end{pmatrix}\right) \tag{18}$$

2. WEATHER DERIVATIVES MARKET MODEL

Weather derivatives are generally structured as options, forward, futures and swaps based on different underlying of weather indexes. Here, introduce some indexes frequently used on the weather derivatives market, which is the underlying of the temperature.

Given a weather station, let us note by $T_i^{\max}$ and $T_i^{\min}$, respectively, the maximum and the minimum temperature (generally in degree Celsius) measured in one day i.

$$T_i = \frac{T_i^{\max} + T_i^{\min}}{2} \tag{19}$$

As above mentioned, one important underlying variables for weather derivatives is the degree – day. For a given location, the degree – day is the temperature value of the different between the temperature of given day and a temperature threshold.

This quantity is defined below.

Let define T_i as the mean temperature of a day i. We define Heating Degree Days (HDD_i - measure of cold in winder) and Cooling Degree Days (CDD_i - measure of heat in summer), generated on that day as

$$HDD_i = \max\left(T_{ref} - T_i, 0\right) \tag{20}$$

$$CDD_i = \max\left(T_i - T_{ref}, 0\right) \tag{21}$$

Where T_{ref} is a is the reference temperature (in general between 18^0 C and 20^0 predetermined temperature level and T_i is the average temperature calculated as in (19) for a given day i.

From the Eq. (10), we can write cumulative HDD (CHDD):

$$CHDD = \sum_{i=1}^{N} HDD_i \tag{22}$$

Where HDD_i is calculated as in (20) and N is the time horizon, which is generally a month or a season.

Based on above expression [9] of temperature model is an OU process driven by a Levy process that contains independent process as Brownian motions and two mean – reverting compound Poisson processes. The model is represented as follows:

$$dT_t = \left\{ \frac{dT_t^m}{dt} + b\left(T_t^m - T_t\right) \right\} dt + dL_t \tag{23}$$

where T_t^m is a cyclical process of temperature and represented in (24). Additionally, b is the mean-reversion parameter, and subscript t represents time.

$$T_t^m = A + Bt + C\sin\left(wt + \varphi\right) \tag{24}$$

Where $w = \frac{2\pi}{365}$, φ is the phase angle. The differential of the driving Levy process dL_t is defined as follows:

$$dL_t = \sigma_t dW_t + dY_t + dZ_t \tag{25}$$

The Brownian component of L_t will be approximated by the ARCH (1) model. To represent the different jump structures in temperature in the form of a single jump and a series of jumps, dY_t and dZ_t are defined as fast and slow mean-reverting OU processes driven by compound Poisson processes with intensities of λ_Y and λ_Z, and α and β being mean-reversion parameters, respectively [10].

$$dY_t = -\alpha Y_t dt + dQ_t$$
$$dZ_t = -\beta Z_t dt + dR_t \tag{26}$$

Where $Q_t = \sum_{i=1}^{N_i^Y} U_i, U_i$ are iid random variables, $U_i \sim N(\mu_Y, \delta_Y^2)$, and $R_t = \sum_{i=1}^{N_i^Z} V_i, V_i$ are iid random variables $V_i \sim N(\mu_Z, \delta_Y^2)$.

The solution to these non – Gaussian processes are [11] the following:

$$Y_t = y_0 e^{-\alpha t} + \int_0^t e^{\alpha(s-t)} dQ_s,$$
$$Z_t = z_0 e^{-\beta t} + \int_0^t e^{\beta(s-t)} dR_s \tag{27}$$

The solution to (23) is given as

$$T_t = T_t^m + e^{-bt}\left(T_0 - T_0^m\right) + e^{-bt}\int_0^t e^{bu} dL_u. \tag{28}$$

To find the value of a temperature-based derivative, one needs the distribution of the underlying temperature given in (28). However, this does not have a closed-form solution. One way to address this problem is to use a characteristic function of the temperature and apply inversion techniques to find the value of an HDD, an approximated distribution of CHDD, and an approximated distribution for temperature itself.

To find the [11] characteristic function given in (28). First, using L_1, the characteristic exponent of (25) will be determined, where characteristic exponent (u) is defined as $Ee^{iuL} = e^{\psi(u)}$. The solution to L_1 is

$$L_1 = \int_0^1 \sigma_u dW_u + y_0 e^{-\alpha} + \int_0^1 e^{\alpha(s-1)} dQ_s + z_0 e^{-\beta} + \int_0^1 e^{\alpha(s-1)} dR_s \tag{29}$$

Then, the characteristics exponents of the Levy components will be

$$\psi_{BM}(u) = -\frac{1}{2}u^2 C, \tag{30}$$

Where $C = E\left(\int_0^1 \sqrt{\sigma_t^2}\, dW_u\right)^2 = \int_0^1 \sigma_t^2 du$

And for jump process $\psi_Y(u) = iuy_0 e^{-\alpha} + \lambda_y \int_0^1 \left(e^{iu\mu_y e^{-\alpha(r-1)} - \left(1/2\right) u^2 \delta_Y^2 e^{2\alpha(r-1)}} - 1 \right) dr$ (31)

$\psi_Z(u)$ can be written similarly. It is not possible to evaluate the integral in (31). Then, by using linear approximation,

$$\psi_Y(u) = iuy_0 e^{-\alpha} + \lambda_y iu\mu_y \left(\frac{1-e^{-\alpha}}{\alpha} \right) - \lambda_y \frac{1}{2} u^2 \delta_Y^2 \left(\frac{1-e^{-2\alpha}}{2\alpha} \right). \quad (32)$$

Again, (u) can be written similarly. Finally, the characteristic function of the temperature model can be written explicitly.

$$E\left\{e^{iuT_i}\right\} = \exp\left\{ iu\left(T_i^m + e^{-bt}\left(T_0 - T_0^m\right)\right) + \int_0^t \psi_T\left(ue^{b(s-t)}\right) ds \right\}. \quad (33)$$

In explicit for,

$$E\left\{e^{iuT_t}\right\} = \theta_T(u) = \exp\left[\begin{array}{l} iu\left(T_t^m + e^{-bt}\left(T_0 - T_0^m\right)\right) + iu\left(\frac{1-e^{-bt}}{b}\right) \\ \left\{ y_0 e^{-\alpha} + \lambda_y \mu_y \left(\frac{1-e^{-\alpha}}{\alpha}\right) + z_0 e^{-\beta} + \lambda_z \mu_y \left(\frac{1-e^{-\beta}}{\beta}\right) \right\} \\ -\frac{1}{2} u^2 \left(\frac{1-e^{-2bt}}{2b}\right) . \left\{ C + \lambda_y \delta_y^2 \left(\frac{1-e^{-2\alpha}}{2\alpha}\right) + \lambda_z \delta_z^2 \left(\frac{1-e^{-2\beta}}{2\beta}\right) \right\} \end{array} \right] \quad (34)$$

Now, we focus on how to measure of HDDs and it is inversion technique will be used to find the value of an HDD and its distribution and hence the CHDD values. To find an approximating density function of temperature, inversion formula will be applied to the characteristic function of the temperature defined in (34).

Before applying the inversion formula, the following shortcuts are derived from (34). Let (x) and $\theta(z)$ be the density function and characteristic function of temperature, respectively.

$$T^* = \left(T_t^m + e^{-bt}\left(T_0 - T_0^m\right)\right) \quad (35)$$

$$M = \left(\frac{1-e^{-bt}}{b}\right) \left\{ y_0 e^{-\alpha} + \lambda_y \mu_y \left(\frac{1-e^{-\alpha}}{\alpha}\right) + z_0 e^{-\beta} + \lambda_z \mu_z \left(\frac{1-e^{-\beta}}{\beta}\right) \right\} \quad (36)$$

$$M^* = T^* + M \quad (37)$$

$$V = \left(\frac{1-e^{-2bt}}{2b}\right) \left\{ C + \lambda_y \delta_Y^2 \left(\frac{1-e^{-2\alpha}}{2\alpha}\right) + \lambda_z \delta_Y^2 \left(\frac{1-e^{-2\beta}}{2\beta}\right) \right\} \quad (38)$$

Then, by inversion formula $f(x) = \left(\frac{1}{2\pi}\right)\int_{-\infty}^{\infty} e^{-izx}\theta(z)dz$ the result is

$$f(x) = \frac{1}{\sqrt{2\pi V}} e^{-(x-M^*)^2 / 2V} \tag{39}$$

Because weather derivatives are defined on CHDDs, their distribution will be defined. Clearly, HDD values may show autocorrelation. In addition, due to the nature of the proposed temperature model in terms of the independence of the included processes and motivation to keep the process simple, the model assumes the independence of the HDDs. With this assumption, the approximated distribution of CHDD can be found using

$$CHDD \sim N\left(PB - PM - PT^*, PV\right) \tag{40}$$

HDDs are clearly contingent claims on how temperature deviates from a base temperature. As one way to find the expected value of an HDD, this study will first find its Fourier transform. Then, the inverse Fourier transform will be applied to both the HDD's Fourier transform and the characteristic function of temperature [12].

Let $x = T_t, w(x)$ is HDDs payoff function given in (20), base (B) and $\hat{w}(z) = F[w(x)], z \in C$ is its generalized Fourier transformation. Then $\hat{w}(z) = \int_{-\infty}^{\infty} \exp(izx)w(x)dx$. Then

$$\hat{w}(z) = -\frac{e^{izB}}{z^2}, \quad \operatorname{Im} z < 0. \tag{41}$$

Now, the inversion will be applied to $\hat{w}(z)\theta_T(-z)$, where $\hat{w}(z)$ is defined in (10) and θ_T is the characteristics function defined in (34). Let temperature in (28) be defined in shorthand notation as $T_t = T * + \Lambda_t$ where T^* is defined as in (35) and

$$\Lambda_t = e^{-bt}\int_0^t e^{bu} dL_u \tag{42}$$

The characteristics function of Λ_t can be obtained from (34) and written as

$$\theta_\Lambda(u) = \exp\left(iu\left(\frac{1-e^{-bt}}{b}\right)\left\{ \begin{array}{l} y_0 e^{-\alpha} + \lambda_y \mu_y \left(\frac{1-e^{-\alpha}}{\alpha}\right) + z_0 e^{-\beta} + \\ \lambda_z \mu_z \left(\frac{1-e^{-\beta}}{\beta}\right) - \frac{1}{2}u^2\left(\frac{1-e^{-2bt}}{2b}\right) \\ \left(C + \lambda_y \delta_Y^2 \left(\frac{1-e^{-2\alpha}}{2\alpha}\right) + \lambda_z \delta_z^2 \left(\frac{1-e^{-2\beta}}{2\beta}\right)\right) \end{array} \right\} \right) \tag{43}$$

Then, $E[HDD] = E\left[\left(\frac{1}{2\pi}\right)\int_{iv-\infty}^{iv-\infty} e^{izT_i}\hat{w}(z)dz\right]$, where *E* represents expectations

$$E[EDD] = \frac{e^{-T^*}}{2\pi}\int_R e^{-iuT^*}\hat{w}(u-i)\theta_\Lambda(-u+i)du \tag{44}$$

Finally, results indicated that the integral is equal to $B - M - T^*$; therefore,

$$E[HDD] = B - M - T^* \tag{45}$$

3. PRICING OF WEATHER DERIVATIVES

The market for weather derivatives is a typical example of an incomplete market, because temperature itself cannot be traded, and so one cannot form a parallel between temperate and equities. Therefore, we have to consider the market price of risk ¸ in order to obtain unique prices for such contracts. Since there is not yet a real market from which we can obtain prices, we assume for simplicity that the market price of risk is constant. Furthermore, we assume that we are given a risk free asset with constant interest rate *r* and a contract that <u>for</u> each degree Celsius pays one unit of currency. Thus, under a martingale measure Q, characterized by the market price of risk λ ¸, our price process also denoted by T_t satisfies the following dynamics:

$$dT_t = \left\{\frac{dT_t^m}{dt} + a\left(T_t^m - T_t\right) - \lambda\sigma_t\right\}dt + \sigma_t dV_t \tag{46}$$

where, $(V_t, t \geq 0)$ is a Q–Wiener process. Since the price of a derivative is expressed as a discounted expected value under martingale measure Q, we start by computing the expected value and the variance of T_t under the measure Q . Indeed, as a Girsanov transformation only changes the drift term, the variance of Q is the same under both measures. Therefore,

$$Var\left[T_t \middle| F_s\right] = \int_s^t \sigma_u^2 e^{-2a(t-u)} du \tag{47}$$

The stochastic process describing the temperature we are looking for should have a mean – reverting property. Putting all the assumption together, we model temperature by a stochastic process solution (1) of the following SDE

$$dT_t = a\left(T_t^m - T_t\right)dt + \sigma_t dW_t \tag{48}$$

Where $a \in R$ determine the speed of the mean – reversion. The problem with (48) is that it is actually not reverting to T_t^m in the long – run [3]. To obtain a process that really revert so the mean (24) and we have to differentiation with respect to *t*, then we can get $dT_t^m = B + wC\cos(wt + \varphi)$

(49)

To the drift term in (48). As the mean temperature T_t^m is not constant this term will adjusted the drift so that the solution of the SDE has the long run mean T_t^m.

Starting at $T_s = x$, now we got the following temperature model

$$dT_t = \left\{ \frac{dT_t^m}{dt} + a\left(T_t^m - T_{tt}\right) \right\} dt + \sigma_t dW_t, \quad t > s \tag{50}$$

Whose solution is

$$T_t = \left(x - T_s^m\right)e^{-a(t-s)} + T_t^m + \int_s^t e^{-(t-\tau)} \sigma_\tau dW_\tau \tag{51}$$

Where $T_t^m = A + Bt + C\sin(wt + \varphi)$

Moreover, it follows from (51) that

$$E^P\left[T_t \middle| F_s\right] = \left(T_s - T_s^m\right)e^{-a(t-s)} + T_t^m \tag{52}$$

Hence, in view of Eq (50) we must have

$$E^Q\left[T_t \middle| F_s\right] = E^P\left[T_t \middle| F_s\right] - \int_s^t \lambda \sigma_u e^{-a(t-u)} du \tag{53}$$

Evaluating the integrals in one of the intervals where σ is constant, we get that

$$E^Q\left[T_t \middle| F_s\right] = E^P\left[T_t \middle| F_s\right] - \frac{\lambda \sigma_i}{a}\left(1 - e^{-a(t-s)}\right) \tag{54}$$

And the variance is

$$Var\left[T_t \middle| F_s\right] = \frac{\sigma_i^2}{2a}\left(1 - e^{-2a(t-s)}\right) \tag{55}$$

Now, we need to compute the covariance of the temperature between two different days. Indeed, for $0 \le s \le t \le u$

$$Cov\left[T_t, T_u \middle| F_s\right] = e^{-a(u-t)} Var\left[T_t \middle| F_s\right] \tag{56}$$

Suppose now that t_1 and t_n denote the first and last day of a month and start the process at some time s from the month before $[t_1, t_n]$. To compute the expected value and variance of T_t in this case, we split the integrals in (53) and (47) into two integrals where σ is constant in each one of them. We then get

$$E^Q\left[T_t \middle| F_s\right] = E^P\left[T_t \middle| F_s\right] - \frac{\lambda}{a}\left(\sigma_i - \sigma_j\right)e^{-a(t-t_1)} + \frac{\lambda\sigma_i}{a}e^{-a(t-s)} - \frac{\lambda\sigma_j}{a} \tag{57}$$

And the variance is

$$Var\left[T_t \middle| F_s\right] = \frac{1}{2a}\left(\sigma_i^2 - \sigma_j^2\right)e^{-2a(t-t_1)} - \frac{\sigma_i^2}{2a}e^{-2a(t-s)} + \frac{\sigma_j^2}{2a} \tag{58}$$

The generalization to larger time intervals becomes now obvious.

Next, as mentioned before, most weather derivatives involving the temperature are based on heating or cooling degree-days. We will construct how to price a standard heating degree-day option. We aware of that option contract and the buyer of a HDD call, for example, pays the seller a premium at the beginning of the contract. In return, if the number of HDDs for the contract period is greater than the predetermined strike level the buyer will receive a payout. The size of the payout is determined by the strike and the tick size. The tick size is the amount of money that the holder of the call receives for each degree-day above the strike level for the period. Often the option has a cap on the maximum payout unlike, for example, traditional options on stocks. The parameters of a typical weather option are: The contract type (call or put), The contract period (e.g. January 2001), The underlying index (HDD or CDD), An official weather station from which the temperature data are obtained, the strike level, the tick size and The maximum payout (if there is any).

To find a formula for the payout of an option, let K denote the strike level and α the tick size. Let the contract period consist of n days. Then the number of HDDs and CDDs for that period are

$$HDD_n = \sum_{i=1}^{n} HDD_i, \quad and \quad C_n = \sum_{i=1}^{n} CDD_i \tag{59}$$

Now we can write the payment of an uncapped HDD call as

$$\chi = \alpha \max\{H_n - K, 0\} \quad (60)$$

The payouts for similar contracts like HDD puts and CDD calls/puts are defined in the same way.

where, for simplicity $\alpha = 1$ unit of currency/HDD and

$$H_n = \sum_{i=1}^{n} \max\{18 - T_t, 0\} \quad (61)$$

The contract (60) is a type of an arithmetic average Asian option. In the case of a log-normally distributed underlying process, no exact analytic formula for the price of such an option is known. Here we have an underlying process, which is normally distributed, but the maximum function complicates the task to find a pricing formula. We therefore try to make some sort of approximation. We know that, under *Q*, under given information at time *s*,

$$T_t \sim N(\mu_t, \upsilon_t) \quad (62)$$

where μ_t is given by (57) and υ_t by (58). Now suppose that we want to find the price of a contract whose payout depends on the accumulation of HDDs during some period in the winter.

4. FORECASTING OF WEATHER PRICING

In this chapter, how weather forecasting and seasonal forecasting can potentially improve our valuation of weather derivative contracts. Dynamical models of the atmosphere known as atmospheric general circulation models (AGCMs) produce most weather forecasts. These models are based on discrete numerical methods for attempting to solve the continuous equations that are believed to govern large-scale atmospheric flows.

We will start by considering the simplest case of using weather forecasts in weather derivative pricing, which is the calculation of the fair price of a linear swap contract on a separable and linear index (e.g. a CAT index). Now we will discuss that the, only forecasts of the expected temperature need to be used and probabilistic forecasts are not necessary. In the first case, we consider is the calculation of the fair price of a linear swap contract on a separable non-linear index (e.g. HDDs). Probabilistic weather forecasts must now be used, but there is no particular difficulty in merging historical data and the probabilistic forecast. Then, we consider the general case, which includes the calculation of the fair price for all other contracts (non-linear swaps and options) and the calculation of the distribution of outcomes for all contracts. This is the most difficult case, and we will present three techniques for solving this problem.

We now consider the estimation of the fair strike of a linear swap on a separable linear index based on daily temperatures. This includes linear swaps on CAT indices and linear swaps on HDDs and CDDs in those cases where there is no chance of the temperature crossing the baseline. By definition, the fair strike of a linear swap is given by the expectation of the index distribution

$$\text{Fair strike} = E(x) \tag{63}$$

Since we are considering a separable index, we can write the aggregate contract index x in terms of daily indices z:

$$x = \sum_{i=1}^{N_d} z_i \tag{64}$$

and hence the expected index is the sum of the mean daily indices:

$$E(x) = \sum_{i=1}^{N_d} E(z_i) \tag{65}$$

For a CAT index

$$z_i = T_i \tag{66}$$

If we are part way through the contract then evaluating $E(x)$ involves using measured temperature, forecasts, and expectations from historical data. If we are using a forecast with N_f values in it, then on day N_0 of the contract

$$E(x) = \sum_{i=1}^{N_d} T_i \tag{67}$$

$$= \sum_{i=1}^{N_0-1} T_i^{hist} + \sum_{i=N_0}^{N_0+N_f-1} m_i^{fc} + \sum_{i=N_0+N_f}^{N_d} m_i^{c\lim} \tag{68}$$

where T_i^{hist} are the known historical temperatures, m_i^{fc} are single forecasts giving the expected temperature over the forecast period, and $m_i^{c\lim}$ are climatological mean temperatures from historical data.

Now, we include HDD and CDD indices for which there is some chance that the temperature will cross the baseline. We can no longer express the mean of the daily index in terms of the mean temperature, but rather it becomes a function of the whole distribution of daily temperature, $f(T)$:

$$E(z_i) = \int_{-\infty}^{\infty} f_i(T) z_i(T) dT \tag{69}$$

For normally distributed temperatures, this integral can usually be evaluated in terms of the mean and the standard deviation of temperature. For HDDs

$$\begin{aligned} E(z_i) &= \int_{-\infty}^{\infty} \phi_i(T) z_i(T) dT \\ &= \int_{-\infty}^{T_0} \phi_i(T) T dT \\ &= (T_0 - m_i)\Phi(T_0) + s_i \phi_i(T_0) \end{aligned} \tag{70}$$

where Φ_I is the cumulative normal distribution of temperatures on day $i, \frac{1}{s_i}\phi_i$ is the density of temperatures on day *i*, and mi and s_i are the mean and standard deviation of temperatures on day *i*.

Fair value for the swap contract for an arbitrary distribution of temperature is now given by

$$E(x) = \sum_{i=1}^{N_0 - 1} z_i\left(T_i^{hist}\right) + \sum_{i=N_0}^{N_0 + N_f - 1} \int f_i(T) z_i(T) dT + \sum_{i=N_0+N_f}^{N_d} \int f_i(T) z_i(T) dT \tag{71}$$

The first term is the accumulated index due to historical temperatures. The second term is the expected contribution due to forecasts, and the third term is the expected contribution due to temperature beyond the end of the forecast. The distribution of temperature in the second term can be taken from a probabilistic forecast and in the third term from historical data. The days in the third term could be treated together as one block, and the mean of the aggregate index estimated from historical values of the aggregate index (i.e. index modelling, but for part of the contract only).

For normally distributed temperatures and for HDDs this equation becomes

$$\begin{aligned} E(x) = &\sum_{i=1}^{N_0-1} \max\left(T_0 - T_i^{hist}, 0\right) + \sum_{i=N_0}^{N_0+N_f-1} \left(T_0 - m_i^{fc}\right)\Phi_i\left(T_0'\right) + s_i^{fc}\phi\left(T_0'\right) + \\ &\sum_{i=N_0+N_f}^{N_d} \left(T_0 - m_i^{clim}\right)\Phi_i\left(T_i''\right) + s_i^{clim}\phi\left(T_0''\right) \end{aligned} \tag{72}$$

Where $T_0' = \frac{T_0 - m_i^{fc}}{s_i^{fc}}$ and $T_0'' = \frac{T_0 - m_i^{clim}}{s_i^{clim}}$

The probabilistic forecast comes in via the mean and the standard deviations of temperature on each day during the forecast (m_i^{fc} *and* s_i^{fc}), and the historical data used in the third term comes in via the mean and standard deviation of historical temperatures m_i^{clim} *and* s_i^{clim}.

The standard deviation of the index is the square root of the variance of the index. For a separable index, the variance is the sum of the terms in the covariance matrix of daily index values during the contract period. For a CAT index (the example we will use for illustration) it is the sum of the terms in the covariance matrix of the daily temperatures. For a contract covered partly by forecast and partly by historical data we can split the terms in the covariance matrix into those that involve the forecast only, those that involve historical data only, and those that involve a mix of historical data and forecast.

$$\sigma_x^2 = \sum_{i=1}^{N_d}\sum_{j=1}^{N} E\left(T_i'T_j'\right)$$

$$= \sum_{i=N_0}^{N_0+N_f-1}\sum_{j=N_0}^{N_0+N_f-1} E\left(T_i'T_j'\right) + \sum_{i=N_0+N_f}^{N_d}\sum_{j=N_0+N_f}^{N_d} E\left(T_i'T_j'\right) + 2\sum_{i=N_0}^{N_0+N_f-1}\sum_{j=N_0+N_f}^{N_d} E\left(T_i'T_j'\right)$$

$$= \sum_{i=N_0}^{N_0+N_f-1}\sum_{j=N_0}^{N_0+N_f-1} c_{ij} + \sum_{i=N_0+N_f}^{N_d}\sum_{j=N_0+N_f}^{N_d} c_{ij} + 2\sum_{i=N_0}^{N_0+N_f-1}\sum_{j=N_0+N_f-1}^{N_d} c_{ij}$$

$$= \sum_{i=N_0}^{N_0+N_f-1}\sum_{j=N_0}^{N_0+N_f-1} s_i^{fc} s_j^{fc}\rho_{ij} + \sum_{i=N_0+N_f-1}^{N_d}\sum_{j=N_0+N_f-1}^{N_d} s_i^{clim} s_i^{clim}\rho_{ij} + 2\sum_{i=N_0}^{N_0+N_f-1}\sum_{j=N_0+N_f-1}^{N_d} s_i^{fc} s_i^{clim}\rho_{ij}$$

$$= \sigma_{fc}^2 + \sigma_{pfc}^2 + \sigma_{\mathrm{cov}}^2 \tag{73}$$

σ_{fc}^2 is depends on forecast variances and correlations between temperatures during the forecast period. Then, σ_{pfc}^2 represents climatological temperature variances and correlations. And, finally, σ_{cov}^2 represents forecast variances, climatological temperature variances, and correlations between temperature during the forecast and the post-forecast period.

How weather forecasts can be combined with methods based on daily modelling. Combining forecasts with daily models is perhaps the most elegant way to incorporate forecasts into weather pricing because of the natural way that daily models cope with the issue of time dependence and the evaluation of the σ_{cov}^2 term in equation (63). However, as we will see the methods – although elegant – are reasonably complex. Implementing these methods is probably only justifiable economically if an organisation is trading options very frequently based on forecasts. Now describe long contract based method based on daily modelling and introduce Pruning method [13 - 18].

First, we use a daily temperature model of a large number of temperature tracks, which cover the completely remaining period of the contract. These tracks should be initialised from the most recent historical data. We also calculate the probability density associated with each track from the daily temperature model. Second, we calculate another probability density for each track using a probabilistic forecast. These probabilities contain the forecast information. Third, we convert each track into an index value. The index values are weighted using a weight that is proportional to the second (forecast) density divided by the first (historical) density. The weighted index values then define the index distribution. In statistics, the weighting of simulations in this way is called 'importance sampling' [19].

The mathematical basis for the pruning method is the following.

Let $p(\mathbf{T})$ be the pay-off due to temperature track $\mathbf{T}$, $f(\mathbf{T})$ be the climatological probability of $\mathbf{T}$, and $g(\mathbf{T})$ be the forecast probability of $\mathbf{T}$. Then the climatological expected pay-off $\mu_p^{c\lim}$ is given by

$$\mu_p^{c\lim} = \int p(\mathbf{T}) f(\mathbf{T}) d\mathbf{T} \tag{74}$$

where the integral is over all possible tracks for $\mathbf{T}$. The forecast expected pay-off μ_p^{fc} is given by

$$\mu_p^{fc} = \int p(\mathbf{T}) g(\mathbf{T}) d\mathbf{T} \tag{75}$$

To evaluate $\mu_p^{c\lim}$ we choose a set of tracks that are equally spaced along the climatological CDF, $F(\mathbf{T})$. In other words, all the values of $dF(\mathbf{T}) = f(\mathbf{T}) d\mathbf{T}$ are equal, so $dF(\mathbf{T}) = dF = \frac{1}{N}$

where N is the number of tracks. The integral becomes

$$\mu_p^{c\lim} = \int p(\mathbf{T}) dF \approx \frac{1}{N} \sum p(\mathbf{T}) \tag{76}$$

where the sum is over all the tracks in a discrete set of possible tracks.

If we evaluate μ_p^{fc} using the same set of tracks,

$$\begin{aligned} \mu_p^{fc} &= \int p(\mathbf{T}) g(\mathbf{T}) d\mathbf{T} = \int p(\mathbf{T}) g(\mathbf{T}) \frac{dF}{f(\mathbf{T})} = \int p(\mathbf{T}) \frac{g(\mathbf{T})}{f(\mathbf{T})} dF \\ &= \int p(\mathbf{T}) w(\mathbf{T}) dF \approx \frac{1}{N} \sum p(\mathbf{T}) w(\mathbf{T}) \end{aligned} \tag{77}$$

where the weights $w(\mathbf{T})$ are given by

$$w(\mathbf{T}) = \frac{g(\mathbf{T})}{f(\mathbf{T})} \tag{78}$$

In other words, we sum the pay-offs for all possible tracks, but with weights. The weighting to be used is the forecast probability density of a certain track divided by the climatological probability density of that track.

5. CONCLUSION

Weather risk has some specificities compared to other sources of economic risk: in particular, it is a local geographical risk, which cannot be controlled. The impact of weather is also very predictable: the same causes will always lead to the same effects. Moreover, weather risk is often referred to as a volumetric risk, its potential impacts being mainly on the volume and not (at least directly) on the price. This explains why hedging of weather risk via the trading of commodities futures is di¢ cult and imperfect. Weather derivatives are financial instruments whose value and/or cash flows depend on the occurrence of some meteorological events, which are easily measurable, independently authentifiable, and sufficiently transparent to act as triggering underlying for financial contracts. Typically, location is clearly identified and measurement is provided by independent and reliable sources.

The underlying meteorological events can be considered as no catastrophic. Use of weather derivatives generalizes standard financial risk management practice based on hedging foreign exchange and interest rate risks via financial derivatives. The main objective of this study is to modelling and Pricing of Weather Derivative Market. The main goal of this study was discussed four objective. We begin our approach to construct the temperature model under Ornstein – Uhlenbeck process that is driven by a Levy process rather than a standard Brownian motion is investigated. Then, we extent our approach to briefly presented the weather derivatives market model. In addition, we followed to constructed the Modeling and pricing of weather derivatives. Finally, we discussed how weather forecasting and seasonal forecasting could potentially improve our valuation of weather derivative contracts. In addition, this paper ended with conclusion.

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