Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 13, Number 11 (2017), pp. 8075–8084

http://www.ripublication.com/gjpam.htm

New nano generalized classes of $\tau_R(X)$

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Abstract

In this paper, another generalized class of $\tau_R(X)$ called mildly nano g-closed sets is introduced and the notion of mildly nano g-open sets in nano topological spaces is introduced and studied. The relationships of mildly nano g-closed sets with various other sets are investigated.

AMS subject classification: 54A05, 54C10, 54B05.
Keywords: mildly nano g-closed set, nano g-closed set, nano pre-closed set, nano pre-open set.

1. Introduction

Lellis Thivagar et al [3] introduced a nano topological space with respect to a subset X of an universe which is defined in terms of lower approximation and upper approximation and boundary region. The classical nano topological space is based on an equivalence relation on a set, but in some situation, equivalence relations are nor suitable for coping with granularity, instead the classical nano topology is extend to general binary relation based covering nano topological space

The main aim of this paper is to introduce another nano generalized class called mildly nano g-open sets in nano topological spaces. Moreover, this nano generalized class of $\tau_R(X)$, generalize nano g-open sets and mildly nano g-open sets. The relationships of mildly nano g-closed sets with various other sets are discussed.

2. Preliminaries

Throughout this paper $(U, \tau_R(X))$ (or X) represent nano topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset H of a space $(U, \tau_R(X))$, $Ncl(H)$ and $Nint(H)$ denote the nano closure of H and the nano interior of H respectively. We recall the following definitions which are useful in the sequel.

Definition 2.1. [5] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.

2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \cap X \neq \phi\}$.

3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Property 2.2. [3] If (U, R) is an approximation space and $X, Y \subseteq U$; then

1. $L_R(X) \subseteq X \subseteq U_R(X)$;

2. $L_R(\phi) = U_R(\phi) = \phi$ and $L_R(U) = U_R(U) = U$;

3. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;
4. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$;
5. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$;
6. $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$;
7. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$;
8. $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$;
9. $U_R U_R(X) = L_R U_R(X) = U_R(X)$;
10. $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

Definition 2.3. **[3]** Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by the Property 2.2, R(X) satisfies the following axioms:

1. U and $\phi \in \tau_R(X)$,
2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,
3. The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets and $[\tau_R(X)]^c$ is called as the dual nano topology of $[\tau_R(X)]$.

Remark 2.4. **[3]** If $[\tau_R(X)]$ is the nano topology on U with respect to X, then the set $B = \{U, \phi, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.

Definition 2.5. **[3]** If $(U, \tau_R(X))$ is a nano topological space with respect to U and if $H \subseteq U$, then the nano interior of H is defined as the union of all nano open subsets of A and it is denoted by $Nint(H)$.

That is, $Nint(H)$ is the largest nano open subset of H. The nano closure of H is defined as the intersection of all nano closed sets containing H and it is denoted by $Ncl(H)$.

That is, $Ncl(H)$ is the smallest nano closed set containing H.

Definition 2.6. **[3]** A subset H of a nano topological space $(U, \tau_R(X))$ is called;

1. nano pre-open set if $H \subseteq Nint(Ncl(H))$.

   The complement of nano pre-open set is called nano pre-closed.
2. nano regular open set if $H = Nint(Ncl(H))$.

   The complement of nano regular-open set is called nano regular-closed.

Definition 2.7. [4] Let $(U, \tau_R(X))$ be a nano topological space and let $H \subseteq U$, then H is called nano nowhere dense if $Nint(Ncl(H)) = \phi$.

Definition 2.8. [1] A subset H of a nano topological space $(U, \tau_R(X))$ is called nano g-closed if $Ncl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano open.

Theorem 2.9. [2] In any nano topological space, every nano regular-closed set is nano closed but not conversely.

3. New nano generalized classes of $\tau_R(X)$

Definition 3.1. In a nano topological space $(U, \tau_R(X))$, a subset H of U is said to be

1. weakly nano g-closed if $Ncl(Nint(H)) \subseteq G$ whenever $H \subseteq G$ and G is nano open in U;
2. mildly nano g-closed if $Ncl(Nint(H)) \subseteq G$ whenever $H \subseteq G$ and G is nano g-open in U;
3. strongly nano g-closed if $Ncl(H) \subseteq G$ whenever $H \subseteq G$ and G is nano g-open in U.

The complements of the above mentioned sets are called their respective nano open sets.

Example 3.2. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\}$. Then the nano topology $\tau_R(X) = \{\phi, \{a\}, \{b, d\}, \{a, b, d\}, U\}$. Then $\{c\}$ is weakly nano g-closed, mildly nano g-closed and strongly nano g-closed.

Theorem 3.3. In a nano topological space $(U, \tau_R(X))$, a subset H of U is mildly nano g-closed $\Leftrightarrow$ $Ncl(Nint(H)) \subseteq H$.

Proof. $\Rightarrow$ If $Ncl(Nint(H)) \nsubseteq H$, there exists $x \in U$ such that $x \in Ncl(Nint(H)) - H$. Then $x \in Ncl(Nint(H)) - H \subseteq U - H$ and so $H \subseteq U - \{x\}$ where $U - \{x\}$ is nano g-open being nano open. Thus $H \subseteq U - \{x\}$ where $U - \{x\}$ is nano g-open. But $Ncl(Nint(H)) \nsubseteq U - \{x\}$ since $x \in Ncl(Nint(H))$. This implies that H is not mildly nano g-closed which proves the necessary part.

$\Leftarrow$ Let $Ncl(Nint(H)) \subseteq H$ and G be any nano g-open subset such that $H \subseteq G$. Then $Ncl(Nint(H)) \subseteq H \subseteq G$. This implies that H is mildly nano g-closed which proves the sufficiency part. ■

Theorem 3.4. In a nano topological space $(U, \tau_R(X))$, a subset H of U is weakly nano g-closed $\Leftrightarrow$ $Ncl(Nint(H)) \subseteq H$.

Proof. $\Rightarrow$ If $Ncl(Nint(H)) \nsubseteq H$, there exists $x \in U$ such that $x \in Ncl(Nint(H)) - H$. Then $x \in Ncl(Nint(H)) - H \subseteq U - H$ and so $H \subseteq U - \{x\}$ where $U - \{x\}$ is nano open. Thus $H \subseteq U - \{x\}$ where $U - \{x\}$ is nano open. But $Ncl(Nint(H)) \nsubseteq U - \{x\}$

since $x \in Ncl(Nint(H))$. This implies that H is not weakly nano g-closed which proves the necessary part.

$\Leftarrow$ Let $Ncl(Nint(H)) \subseteq H$ and G be any nano open set such that $H \subseteq G$. Then $Ncl(Nint(H)) \subseteq H \subseteq G$. This implies that H is weakly nano g-closed which proves the sufficiency part. ■

Theorem 3.5. In a nano topological space $(U, \tau_R(X))$, a subset H of U is mildly nano g-closed $\Leftrightarrow$ H is weakly nano g-closed.

Proof. Proof follows by Theorem 3.3 and Theorem 3.4. ■

Theorem 3.6. In a nano topological space $(U, \tau_R(X))$, a subset H of U is strongly nano g-closed $\Leftrightarrow$ $Ncl(H) \subseteq H$.

Proof. $\Rightarrow$ If $Ncl(H) \nsubseteq H$, there exists $x \in U$ such that $x \in Ncl(H) - H$. Then $x \in Ncl(H) - H \subseteq U - H$ and so $H \subseteq U - \{x\}$ where $U - \{x\}$ is nano g-open being nano open. Thus $H \subseteq U - \{x\}$ where $U - \{x\}$ is nano g-open. But $Ncl(H) \nsubseteq U - \{x\}$ since $x \in Ncl(H)$. This implies that H is not strongly nano g-closed which proves the necessary part.

$\Leftarrow$ Let $Ncl(H) \subseteq H$ and G be any nano g-open set such that $H \subseteq G$. Then $Ncl(H) \subseteq H \subseteq G$. This implies that H is strongly nano g-closed which proves the sufficient part. ■

Theorem 3.7. In a nano topological space $(U, \tau_R(X))$, a subset H of U is nano g-closed $\Leftrightarrow$ $Ncl(H) \subseteq H$.

Proof. $\Rightarrow$ If $Ncl(H) \nsubseteq H$, there exists $x \in U$ such that $x \in Ncl(H) - H$. Then $x \in Ncl(H) - H \subseteq U - H$ and so $H \subseteq U - \{x\}$ where $U - \{x\}$ is nano open. Thus $H \subseteq U - \{x\}$ where $U - \{x\}$ is nano open. But $Ncl(H) \nsubseteq U - \{x\}$ since $x \in Ncl(H)$. This implies that H is not nano g-closed which proves the necessary part.

$\Leftarrow$ Let $Ncl(H) \subseteq H$ and G be any nano open set such that $H \subseteq G$. Then $Ncl(H) \subseteq H \subseteq G$. This implies that H is nano g-closed which proves the sufficient part. ■

Theorem 3.8. In a nano topological space $(U, \tau_R(X))$, a subset H of U is strongly nano g-closed $\Leftrightarrow$ H is nano g-closed.

Proof. Proof follows from Theorem 3.6 and Theorem 3.7. ■

Proposition 3.9. In a nano topological space $(U, \tau_R(X))$,

1. Every nano g-closed set is weakly nano g-closed.

2. Every strongly nano g-closed set is mildly nano g-closed.

Proof. Obvious. ■

Remark 3.10. The converses of Proposition 3.9 are not true in general as shown in the following Example.

Example 3.11. In Example 3.2,

1. then $\{b\}$ is weakly nano g-closed set but not nano g-closed.
2. then $\{d\}$ is mildly nano g-closed set but not strongly nano g-closed.

Remark 3.12. In a nano topological space $(U, \tau_R(X))$, the following relations hold for a subset H of U.

$$\begin{array}{ccc} \text{strongly nano } g\text{-closed} & \longleftrightarrow & \text{nano } g\text{-closed} \\ \downarrow & & \downarrow \\ \text{mildly nano } g\text{-closed} & \longleftrightarrow & \text{weakly nano } g\text{-closed} \end{array}$$

Where $A \longleftrightarrow B$ means A implies and is implied by B and $A \to B$ means A implies B but not conversely.

Theorem 3.13. In a nano topological space $(U, \tau_R(X))$, for a subset H of U, the following properties are equivalent.

1. H is mildly nano g-closed;
2. $Ncl(Nint(H))\backslash H = \phi$;
3. $Ncl(Nint(H)) \subseteq H$;
4. H is nano pre-closed.

Proof. (1) $\Leftrightarrow$ (2) H is mildly nano g-closed $\Leftrightarrow$ $Ncl(Nint(H)) \subseteq H$ by Theorem 3.3 $\Leftrightarrow$ $Ncl(Nint(H))\backslash H = \phi$.

(2) $\Leftrightarrow$ (3) $Ncl(Nint(H))\backslash H = \phi \Leftrightarrow Ncl(Nint(H)) \subseteq H$.

(3) $\Leftrightarrow$ (4) $Ncl(Nint(H)) \subseteq H \Leftrightarrow H$ is nano pre-closed by (1) of Definition 3.1. ■

Theorem 3.14. In a nano topological space $(U, \tau_R(X))$, if H is mildly nano g-closed, then $H \cup (U - Ncl(Nint(H)))$ is mildly nano g-closed.

Proof. Since H is mildly nano g-closed, $Ncl(Nint(H)) \subseteq H$ by Theorem 3.3. Then $U - H \subseteq U - Ncl(Nint(H))$ and $H \cup (U - H) \subseteq H \cup (U - Ncl(Nint(H)))$. Thus $U \subseteq H \cup (U - Ncl(Nint(H)))$ and so $H \cup (U - Ncl(Nint(H))) = U$. Hence $H \cup (U - Ncl(Nint(H)))$ is mildly nano g-closed. ■

Theorem 3.15. In a nano topological space $(U, \tau_R(X))$, the following properties are equivalent:

1. H is a nano closed set and an nano open set,

2. H is a nano regular-closed set and an nano open set,

3. H is a mildly nano g-closed set and an nano open set.

Proof. (1) $\Rightarrow$ (2): Since H is nano closed and nano open, $H = Ncl(H)$ and $H = Nint(H)$ implies $H = Ncl(Nint(H))$ and $H = Nint(H)$. Hence H is nano regular-closed and nano open.

(2) $\Rightarrow$ (3): Since H is nano regular-closed and nano open, $H = Ncl(Nint(H))$ and $H = Nint(H)$. Since $H = Ncl(Nint(H))$, $Ncl(Nint(H)) \subseteq H$. By Theorem 3.3, H is mildly nano g-closed and nano open.

(3) $\Rightarrow$ (1): Since H is mildly nano g-closed, $Ncl(Nint(H)) \subseteq H$ by Theorem 3.3. Again H is nano open implies $Ncl(H) \subseteq H$. Thus H is nano closed and nano open. ■

Proposition 3.16. In a nano topological space $(U, \tau_R(X))$, every nano closed set is mildly nano g-closed.

Proof. Let G be any nano g-open subset of U such that $H \subseteq G$. Then $cl(int(H)) \subseteq cl(H) = H \subseteq G$ and hence H is mildly nano g-closed. This shows that H is nano closed $\Rightarrow$ mildly nano g-closed. ■

Remark 3.17. The converse of Proposition 3.16 is not true in general as shown in the following example.

Example 3.18. In Example 3.2, then $\{b\}$ is mildly nano g-closed set but not nano closed.

Theorem 3.19. In a nano topological space $(U, \tau_R(X))$, if H is mildly nano g-closed and G is a subset such that $H \subseteq G \subseteq Ncl(Nint(H))$, then G is mildly nano g-closed.

Proof. Since H is mildly nano g-closed, $Ncl(Nint(H)) \subseteq H$ by (3) of Theorem 3.13. Thus by assumption, $H \subseteq G \subseteq Ncl(Nint(H)) \subseteq H$. Then $H = G$ and so G is mildly nano g-closed. ■

Corollary 3.20. In a nano topological space $(U, \tau_R(X))$, if H is a mildly nano g-closed set and an nano open set, then $Ncl(H)$ is mildly nano g-closed.

Proof. Since H is nano open in U, $H \subseteq Ncl(H) = Ncl(Nint(H))$. H is mildly nano g-closed implies $Ncl(H)$ is mildly nano g-closed by Theorem 3.19. ■

Theorem 3.21. In a nano topological space $(U, \tau_R(X))$, a nano nowhere dense subset is mildly nano g-closed.

Proof. If H is a nano nowhere dense subset in U then $Nint(Ncl(H)) = \phi$. Since $Nint(H) \subseteq Nint(Ncl(H))$, $Nint(H) = \phi$. Hence $Ncl(Nint(H)) = Ncl(\phi) = \phi \subseteq H$. Thus, H is mildly nano g-closed in $(U, \tau_R(X))$ by Theorem 3.3. ■

Remark 3.22. The converse of Theorem 3.21 is not true in general as shown in the following Example.

Example 3.23. In Example 3.2, then $\{b\}$ is mildly nano g-closed set but not nano nowhere dense.

Remark 3.24. In a nano topological space $(U, \tau_R(X))$, the intersection of two mildly nano g-closed subsets is mildly nano g-closed.

Proof. Let P and Q be mildly nano g-closed subsets in $(U, \tau_R(X))$. Then $Ncl(Nint(P)) \subseteq P$ and $Ncl(Nint(Q)) \subseteq Q$ by Theorem 3.3. Also $Ncl[Nint(P \cap Q)] = Ncl[Nint(P) \cap Nint(Q)] \subseteq Ncl(Nint(P)) \cap Ncl(Nint(Q)) \subseteq P \cap Q$. This implies that $P \cap Q$ is mildly nano g-closed by Theorem 3.3. ■

Remark 3.25. In a nano topological space $(U, \tau_R(X))$, the union of two mildly nano g-closed subsets need not be mildly nano g-closed.

Example 3.26. Let $U = \{a, b, c\}$ with $U/R = \{\{a\}, \{b, c\}\}$ and $X = \{b, c\}$. Then the nano topology $\tau_R(X) = \{\phi, \{b, c\}, U\}$. Then $\{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, U\}$ is mildly nano g-closed set. Then $P = \{b\}$ and $Q = \{c\}$ is mildly nano g-closed sets. Hence $P \cup Q = \{b, c\}$ is not mildly nano g-closed.

Theorem 3.27. In a nano topological space $(U, \tau_R(X))$, a subset H is mildly nano g-open if and only if $H \subseteq Nint(Ncl(H))$.

Proof. H is mildly nano g-open $\Leftrightarrow$ $U \backslash H$ is mildly nano g-closed $\Leftrightarrow$ $U \backslash H$ is nano pre-closed by (4) of Theorem 3.13 $\Leftrightarrow$ H is nano pre-open $\Leftrightarrow$ $G \subseteq Nint(Ncl(H))$. ■

Theorem 3.28. In a nano topological space $(U, \tau_R(X))$, if the subset H is mildly nano g-closed, then $Ncl(Nint(H)) \backslash H$ is mildly nano g-open in $(U, \tau_R(X))$.

Proof. Since H is mildly nano g-closed, $Ncl(Nint(H)) \backslash H = \phi$ by (2) of Theorem 3.13. Thus $Ncl(Nint(H)) \backslash H$ is mildly nano g-open in $(U, \tau_R(X))$. ■

Theorem 3.29. In a nano topological space $(U, \tau_R(X))$, if H is mildly nano g-open, then $Nint(Ncl(H)) \cup (U - H) = U$.

Proof. Since H is mildly nano g-open, $H \subseteq Nint(Ncl(H))$ by Theorem 3.27. So $(U - H) \cup H \subseteq (U - H) \cup Nint(Ncl(H))$ which implies $U = (U - H) \cup Nint(Ncl(H))$. ■

Theorem 3.30. In a nano topological space $(U, \tau_R(X))$, if H is mildly nano g-open and $Nint(Ncl(H)) \subseteq G \subseteq H$, then G is mildly nano g-open.

Proof. Since H is mildly nano g-open, $H \subseteq Nint(Ncl(H))$ by Theorem 3.27. By assumption $Nint(Ncl(H)) \subseteq G \subseteq H$. This implies $H \subseteq Nint(Ncl(H)) \subseteq G \subseteq H$. Thus $H = G$ and so G is mildly nano g-open. ■

Corollary 3.31. In a nano topological space $(U, \tau_R(X))$, if H is a mildly nano g-open set and a nano closed set, then $Nint(H)$ is mildly nano g-open.

Proof. If H is a mildly nano g-open set and a nano closed set in $(U, \tau_R(X))$, then $Nint(Ncl(H)) = Nint(H) \subseteq Nint(H) \subseteq H$. Thus, by Theorem 3.30, $Nint(H)$ is mildly nano g-open in $(U, \tau_R(X))$. ■

4. Nano $\mathcal{O}$-set

Definition 4.1. A subset H of a nano topological space $(U, \tau_R(X))$ is called a nano $\mathcal{O}$-set if $H = P \cup Q$ where P is nano g-closed and Q is nano pre-open.

Example 4.2. In Example 3.2, then $\{\phi, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}, U\}$ is nano $\mathcal{O}$-set.

Proposition 4.3. Every nano pre-open (resp. nano g-closed) set is a nano $\mathcal{O}$-set.

Remark 4.4. The separate converses of Proposition 4.3 are not true in general as shown in the following Example.

Example 4.5. In Example 3.2, then $\{a, c\}$ is nano $\mathcal{O}$-set but not nano pre-open.

Remark 4.6. The following example shows that the concepts of nano pre-open and nano g-closed are independent of each other.

Example 4.7. In Example 3.2,

1. then $\{c\}$ is nano g-closed set but not nano pre-open.
2. then $\{b\}$ is nano pre-open set but not nano g-closed.

Theorem 4.8. Let $(U, \tau_R(X))$ be a nano topological space and $H \subseteq U$. Then H is mildly nano g-open if and only if $F \subseteq Nint(Ncl(H))$ whenever F is nano g-closed and $F \subseteq H$.

Proof. Suppose H is mildly nano g-open. If F is nano g-closed and $F \subseteq H$, then $U - H \subseteq U - F$ and so $Ncl(Nint(U - H)) \subseteq U - F$. Therefore $F \subseteq U - Ncl(Nint(U - H)) = Nint(Ncl(H))$.

Conversely, the condition holds. Let G be an nano g-open set such that $U - H \subseteq G$. Then $U - G \subseteq H$ and so $U - G \subseteq Nint(Ncl(H))$. Therefore $Ncl(Nint(U - H)) \subseteq G$. Thus $U - H$ is mildly nano g-closed and so H is mildly nano g-open. ■

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