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S* Control Chart in Screw Quality Assessment

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Abstract

Quality control chart is a powerful tool in assessing quality of products and services. The assembled of control chart involves two phases. The first phase function as the quality standard reference which consist of well behave samples that are within the control limit. The second phase is the real samples that need to be quality tested. In this study we constructed S* control chart based on the two phases using screws data. The advantage of this chart is fulfilling the requirement of high dimensional data set where the number of dimensions is more than the number of sample sizes ($p > n$) as displays in screw. The samples taken involved only three screws ($n = 3$) but seven variables ($p = 7$) measuring the quality of the screws. S* control chart was successfully constructed and able to act as warning signals in detecting defects screw.

Keywords : large dimension, multivariate control chart, screw production, small sample size, statistical process control

INTRODUCTION

Statistical process control (SPC) is known to be effective methods for acquiring the quality standard of products or services through monitoring and controlling the process (Djauhari, Lee and Ismail, 2014). Originally, the application area of SPC is focusing in the manufacturing sector. Later, the expansion involving other areas, such as environmental science (Zimmerman, Dardeau, Crozier & Wagstaff, 1996); engineering (Mason & Young, 2002) and health care (Myles, German, Wilson and Wu, 2011; Smith, 2013).

One of the SPC methods is control chart and has a strong point due to the quality controller is able to decide based on valid real time results instead of doing assumptions using "last month" performance (Myles et al, 2011). A control chart can be constructed when data occur sequentially in timely order to detect the changes in the process. The control chart limits represented by two horizontal lines name as lower control limit (LCL) and upper control limit (UCL). The values outside the LCL and UCL are indicators of out-of-control process or defects and quick corrective action will be needed in ensuring the quality of good products and services (Djauhari et al 2014).

Generally, control chart can be assembled based on two phases. The primary interest of Phase I is to assess the process stability (in-control). The historical data (also known as reference sample) which is free from outliers is used to estimate all parameters such as mean vector and covariance matrix. Phase II aim to control the process using current data by benchmarking with the results of reference sample of Phase I. The success of Phase II is highly related with enhancement of Phase I (Coelho, Chakraborti & Graham, 2015)

In this paper, Phase I and II are used in constructing S* control chart (Sharif, Ismail and Omar, 2016) which was developed for high dimensional data set to assess the quality of screws. The uniqueness of this chart is conducting quality testing for data set that has many measurements (or dimensions, *p*) but limited samples (*n*). This constraint is executed in order to minimize the cost of manufacturing since all samples cannot be used for sales after quality testing.

The remainder of this paper is outlined as follows. In section 2, the data preparation for analysis is defined. In section 3, an empirical results based on S* control chart is discussed. Finally, the concluding remarks are presented in the last section.

DATA PREPARATION

In this study, screws data were used which involved 8 variables ($p = 8$) in determining the quality of screw namely, Body Diameter, D (X_1), Head Diameter, A (X_2), Head Height, H (X_3), Head Side Height, S (X_4), Key Engagement, T (X_5), Wall Thickness, G (X_6), Chamfer or Radius, K (X_7), Hex Socket, J (X_8), see Figure 1. The most important variable is Hex Socket, J (X_8) where the measurement must be exactly equal to 0.156.

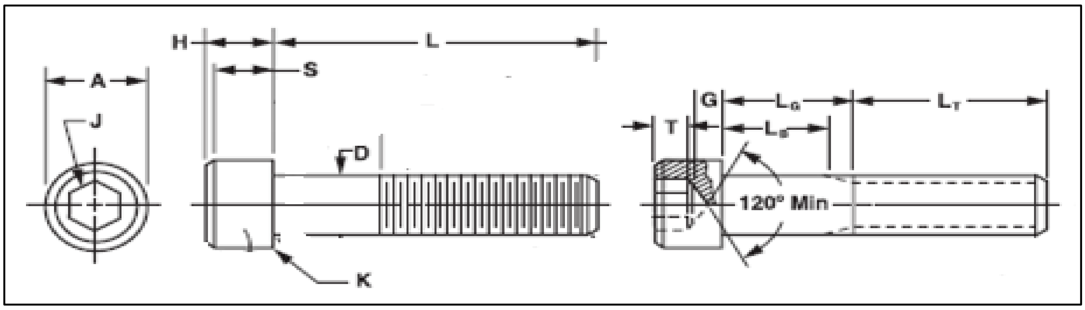

Figure. 1: Technical drawing of screw

The initial step to construct the S* control chart for screw is to check the measurement of X_8. If the value of X_8 is not exactly equal to 0.156, the sample is automatically considered as defect. If satisfied, the statistics is then computed based on seven dimensions only (X_1 to X_7).

RESULTS AND DISCUSSIONS

Phase I S control chart*

A Phase I control chart is constructed by using the first 7 subgroups (g = 7) of screws historical data (please refer to Appendix 1). For every subgroup, we selected only three samples of screws (n = 3) to minimize the quality testing cost. The mean vector 7 × 1 and covariance matrix 7 × 7 based on S* statistics are as follows;

$$\bar{X} = \begin{vmatrix} 0.027 \\ 0.000 \\ -0.005 \\ 0.002 \\ 0.004 \\ 0.001 \\ -0.005 \end{vmatrix}$$

$$S = \begin{vmatrix} 0.027 & & & & & & \\ 0.000 & 0.250 & & & & & \\ -0.005 & 0.063 & 0.027 & & & & \\ 0.002 & -0.025 & -0.011 & 0.004 & & & \\ 0.004 & -0.025 & -0.003 & 0.001 & 0.005 & & \\ 0.001 & 0.000 & 0.001 & 0.000 & 0.001 & 0.000 & \\ -0.005 & 0.038 & 0.013 & -0.006 & -0.003 & 0.000 & 0.017 \end{vmatrix}$$

Next, based on the multivariate control chart approach, we conducted a repeated test of $H_0: \Sigma_i = \Sigma_R$ versus $H_1: \Sigma_i \neq \Sigma_R$.

The above hypothesis, implies that we have to compare the second, third and the rest with the first covariance matrix using the statistical test defined as below

$$S^* = A^t\, S_p^{-1}\, A \qquad (1)$$

Where,

$$A = \left[vec\left(S_{i,L}\right) - vec\left(S_{R,L}\right)\right]$$

$$S_p = M^t\left(I_{p^2} + K\right)\left(S_{R,L} \otimes S_{R,L}\right)M$$

$$M = (m_{ij}) = \begin{cases} 1\,; & (i,j) = (C_2^a + b, b) \text{ for } b = 1,2,\ldots,a \\ 0; & \text{elsewhere} \end{cases}$$

To compute the equation (1), we begin with identifying M. In general, the duplication matrix M of size $(k \times p^2)$ can be presented in matrix form as a block matrix.

$$M = (M_1|M_2|\ldots|M_p)$$

where $k = \frac{1}{2}p(p+1)$ and this matrix can be partitioned into p blocks. M_1 is a matrix with the first element is equal to 1 and the other elements are zero.

After that, all the covariance elements are transformed into the vector space. Then, we choose only the lower element of that matrix for next computational, $vec\left(\boldsymbol{S}_{1,L}\right)$. Generally, covariance matrix is a symmetric matrix where the lower and the upper element of matrix consist of the same elements (duplication). By removing the redundant elements, singularity problem can be solved (Sharif, 2013). All the statistical result is then computed for the remainder of 7sub-groups. The LCL = 0 and UCL = 41.337 are based on Chi-Square with k degree of freedom (where $k = \frac{1}{2}7(7+1) = 28$) and probability of rejection 0.05.

Table 1 and Fig. 2 present the results. Based on Table 1, although the S* statistics values varies among sub groups but Figure 2 revealed that no observations lie outside the control limits which indicated good quality of screws.

Table 1. Statistical test using S* test

Subgroup	**S* statistics**
1	12.813
2	36.325
3	13.359
4	19.134
5	16.328
6	35.990
7	23.452

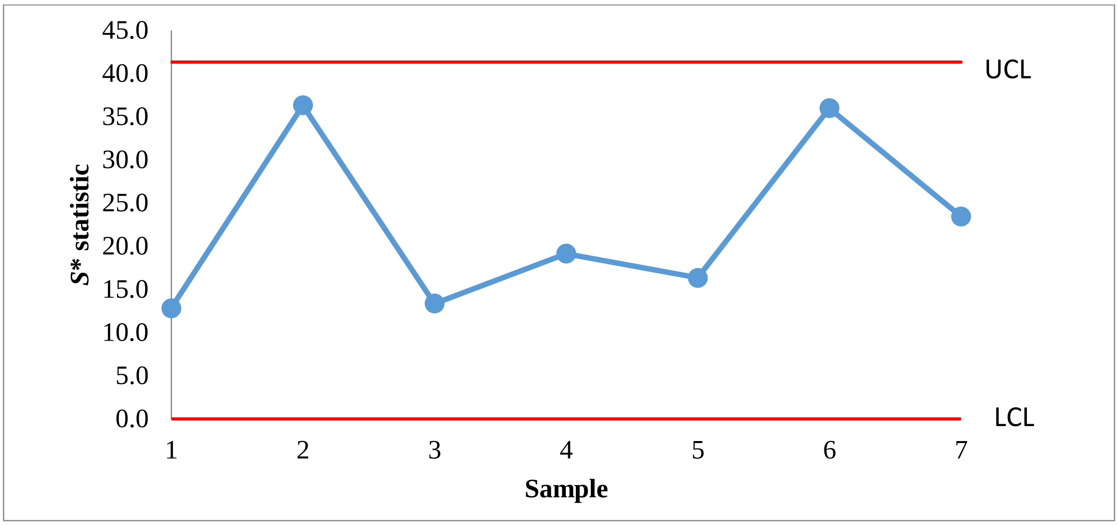

Figure. 2. Phase I S* control chart for monitoring screw quality

Phase II S control chart*

Phase II control chart is assembled similar to Phase I but using different data sets as in Appendix 2. The purpose is to test the sensitivity of the control chart when there are shifts occur in the data sets. The estimation of mean vector (7×1) and covariance matrix (7×7) based on Phase I S* control chart is used for this Phase. Table 2 and Fig. 3 present the results computed based on S* as in equation 1.

Table 2. Statistical test using S* test

Subgroup	S* statistics
1	12.813
2	36.325
3	13.359
4	19.134
5	16.328
6	43.774
7	85.707

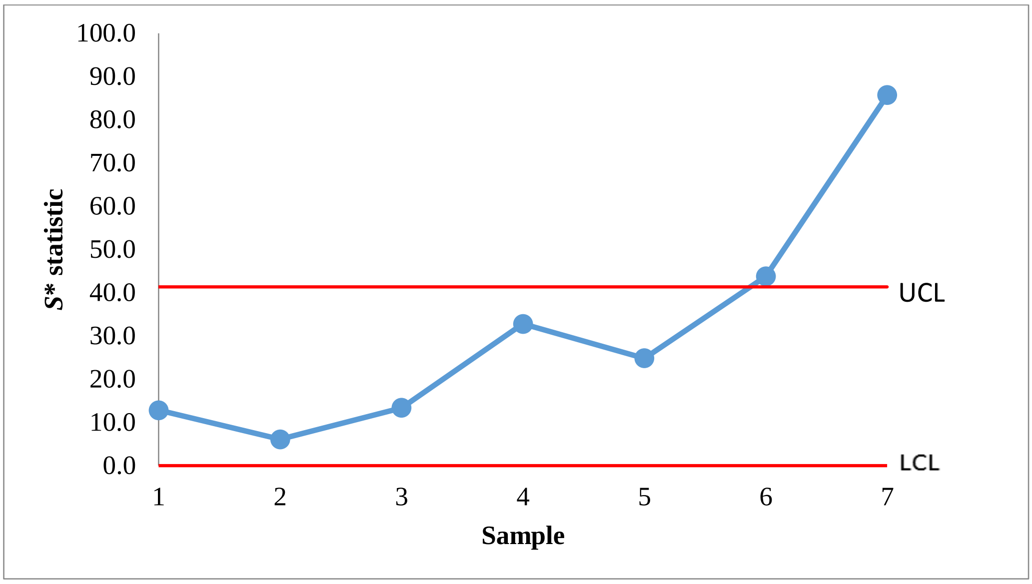

Figure 3. Phase II S* control chart for monitoring screw quality

Based on Table 2, the S* statistics values varies among sub groups and Figure 3 indicated there were two subgroups beyond the UCL. Subgroup 6 just exceeding a bit from the control limit but subgroup 7 showed the farthest.

In Appendix 2, data of subgroups 6 and 7 having two and three out of range values, respectively. Technically, if there is any change in the data, the estimation of mean vectors and covariance matrix will be shifted. The shift then can leads the results to be out-of-control signal. In this case, this analysis proof that S* chart is a sensitive chart because it can detect the out-of-control in both subgroups even though small shift in data values. Furthermore, this findings act as a warning signal in order to check the quality of all screws in subgroup 6 and 7 in identifying the reasons of defects. Therefore only good qualities of screws are produced.

5. CONCLUDING REMARKS

In this paper, we deliver the step-by-step methodology for constructing Phase I and Phase II S* control chart. It successfully shown that S* control chart can be employed in detecting defect items with high dimension data set ($n<p$) particularly in monitoring the quality of screw. One of the significant strengths of S* control chart is its ability of using small sample size in constructing the chart. The other advantage of using control chart is the easiness to interpret either in-control or out-of-control when all the statistical results can be visualized. Thus, display the crucial role of S* control chart as high dimensional quality control chart.

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Appendix 1 Data used in Phase I : S* control chart

		D	A	H	S	T	G	K
	Variable	Body Diameter X_1	Head Diameter X_2	Head Height X_3	Head Side Height X_4	Key Engagement X_5	Wall Thickness X_6	Chamfer or Radius X_7
Subgroup	Standard screw measurement	0.1870	0.308	0.188	0.171	0.090	0.065	0.005
1	1	0.188	0.303	0.186	0.17	0.1	0.066	0.0045
	2	0.187	0.304	0.187	0.169	0.09	0.065	0.004
	3	0.186	0.308	0.186	0.168	0.09	0.065	0.004
2	4	0.189	0.305	0.188	0.17	0.1	0.067	0.005
	5	0.184	0.305	0.188	0.171	0.1	0.066	0.004
	6	0.187	0.31	0.187	0.169	0.09	0.068	0.0045
3	7	0.187	0.308	0.187	0.168	0.1	0.065	0.0045
	8	0.188	0.309	0.186	0.17	0.11	0.066	0.005
	9	0.188	0.307	0.185	0.169	0.09	0.067	0.0045
4	10	0.186	0.304	0.186	0.17	0.1	0.068	0.005
	11	0.185	0.305	0.185	0.169	0.1	0.065	0.0045
	12	0.186	0.305	0.189	0.169	0.11	0.066	0.004
5	13	0.186	0.304	0.186	0.17	0.1	0.068	0.005
	14	0.185	0.305	0.185	0.169	0.1	0.065	0.0045
	15	0.186	0.305	0.189	0.17	0.11	0.066	0.004
6	16	0.186	0.306	0.185	0.169	0.1	0.066	0.004
	17	0.186	0.307	0.186	0.17	0.09	0.067	0.0045
	18	0.188	0.308	0.187	0.169	0.1	0.065	0.005
7	19	0.19	0.309	0.185	0.171	0.11	0.065	0.005
	20	0.19	0.31	0.186	0.169	0.1	0.065	0.0045
	21	0.187	0.311	0.187	0.17	0.09	0.068	0.004

Appendix 2 Data used in Phase II : S* control chart

		D	A	H	S	T	G	K
	Variable	Body Diameter X_1	Head Diameter X_2	Head Height X_3	Head Side Height X_4	Key Engagement X_5	Wall Thickness X_6	Chamfer or Radius X_7
Subgroup	Standard screw measurement	0.1870	0.308	0.188	0.171	0.090	0.065	0.005
1	1	0.188	0.303	0.186	0.17	0.1	0.066	0.0045
	2	0.187	0.304	0.187	0.169	0.09	0.065	0.004
	3	0.186	0.308	0.186	0.168	0.09	0.065	0.004
2	4	0.186	0.303	0.186	0.17	0.1	0.066	0.0045
	5	0.187	0.304	0.187	0.169	0.09	0.065	0.004
	6	0.186	0.305	0.186	0.168	0.09	0.065	0.004
3	7	0.187	0.308	0.187	0.168	0.1	0.065	0.0045
	8	0.188	0.309	0.186	0.17	0.11	0.066	0.005
	9	0.188	0.307	0.185	0.169	0.09	0.067	0.0045
4	10	0.185	0.304	0.186	0.17	0.1	0.068	0.005
	11	0.185	0.305	0.185	0.169	0.09	0.068	0.0045
	12	0.186	0.305	0.186	0.17	0.09	0.067	0.0055
5	22	0.19	0.309	0.185	0.171	0.11	0.065	0.005
	23	0.19	0.309	0.186	0.17	0.1	0.065	0.0045
	24	0.187	0.311	0.187	0.17	0.09	0.068	0.004
6	25	0.185	0.304	0.186	0.17	0.1	0.068	0.005
	26	0.185	0.305	0.185	0.169	0.09	0.065	0.0045
	27	0.186	0.305	0.189	0.17	0.11	0.067	0.0055
7	25	0.189	0.304	0.186	0.17	0.1	0.068	0.005
	26	0.185	0.305	0.185	0.169	0.09	0.065	0.0045
	27	0.186	0.305	0.189	0.17	0.11	0.067	0.0055