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Determination of Parameters on Process Capability Indices Sampling Plan

D. K. Gangeshwer, T. L. Verma* and Manish Pandey*

Department of Applied Mathematics, Bhilai Institute of Technology, Durg, (C.G), India.

**Research Scholar, Department of Applied Mathematics, Bhilai Institute of Technology, Durg, (C.G), India.*

Abstract

Process capability indices (PCIs) are useful for assessing the capability of manufacturing process. The purpose of this paper is to perform a brief review of the PCIs and to determine the operating characteristic (OC) function, sample size, and acceptance constant of the PCI plan. The OC curve for testing the standard deviation using the act that $\frac{(N-1)s^2}{\sigma^2}$ has a χ^2 distribution with n-1 degrees of freedom has been considered. For practical utility of the plan the OC function, minimum sample sizes, acceptance constant are calculated, which necessary to assure quality of the product. Numerical results are given to illustrate the accuracy of the paper. The present development would be a valuable addition to the literature and a useful device for industrial engineers and quality control professionals.

Keywords: Process Capability Indices, Sample Size, Acceptance Constant, OC, Chi-square Distribution.

1. INTRODUCTION:

Wu et al [1] described that, in acceptance sampling (product inspection), small samples are extracted from large-size lots and evaluated for acceptance or rejection decisions. Moreover, to realize an effective scheme for acceptance sampling, several factors must be considered. First, the required number of samples randomly drawn from the vendor's process, which produces the products (lots), must be accurately decided. Secondly, to reflect the process capability and consolidate sample information, an adequate measure is required. Third, inherent sampling errors must be precisely evaluated without bias. Finally, the rules for accepting or rejecting the submitted lots must be fair and properly documented. The quality or reliability data most widely used in industry are attributes and variables. Gangeshwer and Shankar [5] studied the acceptance sampling plan where acceptance criteria is minimum warrantee period

Now, Mohsen and Amirhossein [15] remarked that various quality measures have been proposed to evaluate a process performance. One of the most important issues which must be considered in assessing product quality is the process capability analysis. A Process Capability Index (PCI) is a numerical summary that compares the actual process performance related to engineering specifications. So, this concept will be acceptable for both consumer and manufacturer. Jyh-Jen et al. [9] shows PCIs, as a process performance measure; have become very popular in assessing the capability of manufacturing processes in practice during the past decade. There are several capability indices, including Cp, Cpu, Cpl and Cpk, that have been widely used in manufacturing industry to provide common quantitative measures of process potential and performance. Process-capability indices are powerful means of studying the process ability for manufacturing a product that meets specifications. More and more efforts have been devoted to studies and applications of PCIs. For example, Rado [18] demonstrated how Imprecise Technology, Inc. used the PCIs for program planning and growth to enhance product development. The Cp and Cpk indices have been used in Japan and in the US automotive industry such as Ford Motor Company (see Kane, [10], [11]). For more information on PCIs, see Kotz and Johnson [12], Kotz et al. [13]. Ramakrishnan et al. [19] discussed process-capability indices and product reliability. Chen et al. [4] constructed a process-capability-monitoring chart (PCMC) for evaluating the process potential and performance for the silicon-filler product, which is designed for practical applications. Moreover, the PCIs are used to determine whether a production process is capable of producing items within a specified tolerance. See, for example Kane [10], Rado [18], Montgomery [14], Chen et al. [4]), and Wua et al. [22], Gangeshwer and Shankar [6] etc.

In many situations, capability indices can be used to relate process parameter, say μ and σ to engineering specification that may include unilateral or bilateral tolerances. The resulting indices are unit less and provide a basis for quantifying the performance of the process. Kane [10] developed two indices namely C_p and C_{p_k} as a measure of capability, which are widely used by the industry. They further discussed various applications of these indices along with their statistical sampling considerations. Subsequently, Chan et al. [2] proposed a new index, C_{pm} , to access process capability by taking into account the departures from the target value. Motivated by the inspiring papers of Kane [10] and Chan et al. [2]. This paper presents derivation of OC function and acceptance constant of the PCI plan; and illustrated numerically.

2. A BRIEF REVIEW OF PROCESS CAPABILITY INDICES:

The most popular PCIs which are defined by Jyh-Jen et al. [9] are:
C_p, C_{pk}, C_{pm} and C_{pmk}.

The Cp index is defined as $C_p = \dfrac{USL - LSL}{6\sigma}$,

Where USL= Upper Specification Limit, LSL = Lower Specification Limit

σ = process standard deviation

The C_{pk} is then introduced to reflect the impact of μ on the process capability indices. The C_{pk} is defined as

$$C_{pk} = \min\left\{\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right\}$$

The C_{pm} Index introduced by Chan et al. [2]. This index takes into account the influence of the departure of the process mean μ from the process target T. The C_{pm} is defined as

$$C_{pm} = \min\left\{\frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}}\right\}$$

Combining the three indices C_p, C_{pk}, C_{pm} Pearn et al. [17] proposed the C_{pmk} index.

This index is defined as

$$C_{pmk} = \min\left\{\frac{USL - \mu}{6\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}}\right\}$$

3. SAMPLING PLAN AND DETERMINATION OF PARAMETERS:

The process capacity ratio [Montgomery [14]] or process capability index C_p (Kane [10]) defined as

$$C_p = \frac{\text{allowable process spread}}{\text{actual process spread}} = \frac{USL - LSL}{6\sigma} \tag{3.1}$$

A C_p of 1.0 indicates that a process is judged to be capable". A process is usually evaluated using estimated capability index as

$$\hat{C}_p = \frac{USL - LSL}{6s} \tag{3.2}$$

where, s is sample standard deviation computed from a sample of size N.

Montgomery [14] remarked that a process capability study usually measures functional parameters on the product, not the process itself. Kane [11] have shown that a test for variability is equivalent to a test for process capability index C_p . A single sampling plan for variability has been extensively studied by Burr [1]. According to Gangeshwar and Shankar [7] let a sample of n observation $x_1, x_2, ..., x_n$ be drawn from the lot. Now, acceptance criterion for capability as follows

$$\hat{C}_p > c, \text{accept the lot.}$$

$$\hat{C}_p \le c, \text{reject the lot}$$

Where, c is the acceptance constant and $\hat{C}_p$ is given in (3.2).

Further, Kane (1986) developed that the test $(\sigma_0 \succ 0)$

$$H_0 \ : (\sigma \ge \sigma_0)$$

$$H_1 \ : (\sigma \le \sigma_0)$$

Equivalent to test for process capability $(c_0 \succ 0)$

$$H_0 \ : (C_p \le c_0) \text{ (Process is not capable)}$$

$$H_1 \ : (C_p \succ c_0) \text{ (Process is capable)}$$

Burr [1] derives that OC curve for testing the standard deviation using the act that $\frac{(N-1)s^2}{\sigma^2}$ has a χ^2 distribution with n-1 degrees of freedom. Thus, the proposed plans are characterized by two parameter N and c . The OC function of the plan is given as:

$$\pi(C_p) = P(\hat{C}_p > c) = P[\chi^2 < (n-1)C_p^2 c^{-2} / C_p] \quad \ldots (3.3)$$

Using (3.3) the OC curve OC (Cp)=1- $\pi(C_p)$ can be calculated. In order to find plan parameter of single sampling plan N and c, we assume two points $(C_{p0}, 1-\alpha)$ and (C_{p1}, β) on the OC curve, where $C_{p0} = [C_p(\text{high})]$ and $C_{p1} = [C_p(\text{low})]$ are acceptable quality levels and reject able quality level respectively, and α and β are the producer's risk and consumer's risk. Now, to fulfill the above requirements on the risk, we should have the following conditions met:

$$\pi(C_p) = P[\chi^2 < (N-1)C_{p0}^2 c^{-2} / C_p = C_{p0}] = \alpha \quad (3.4)$$

$$\text{and, } \pi(C_p) = P[\chi^2 < (N-1)C_{p1}^2 c^{-2} / C_p = C_{p1}] = 1-\beta \quad (3.5)$$

where χ^2 has $\nu =$ N-1 d. f. in the both the cases. In general, the solution for N and c are obtained by solving the following pair of equations.

$$(N-1)C_{p0}^2 c^{-2} = \chi^2_{\alpha,\nu} \quad (3.6)$$

$$(N-1)C_{p1}^2 c^{-2} = \chi^2_{1-\beta,\nu} \quad (3.7)$$

Recently, Gangeshwar and Shankar [7] developed solution to determine N using the Goldberg and Levine [8] approximate relationship formula. The desired value of ν is found by taking

$$\nu = < (\sqrt{\nu})^2 > \quad \text{where v= N-1} \quad (3.8)$$

and from equation (3.7)

$$c = C_{p1}(\text{low}) \sqrt{\frac{n-1}{\chi^2_{1-\beta,\nu}}} \quad (3.9)$$

The notation < v > stands for the largest integer contained in v. Here, it may be noted that ν will be real, if and only if, the quantity under the radical is greater than or equal to zero.

Table 3.1: Value of OC function for various c

CP	**c =1.33**		**c= 1.46**	
	N=30	**N=50**	**N=50**	**N=70**
0.5	0.96	0.93	0.94	0.92
1.0	0.84	0.72	0.77	0.68
1.1	0.80	0.66	0.72	0.61
1.2	0.76	0.60	0.67	0.53
1.3	0.72	0.53	0.61	0.45
1.4	0.68	0.46	0.55	0.37
1.5	0.63	0.38	0.48	0.27
1.6	0.58	0.29	0.41	0.17
1.7	0.53	0.20	0.34	0.06
1.8	0.47	0.10	0.26	0.00
1.9	0.41	0.00	0.17	0.00
2.0	0.34	0.00	0.08	0.00

Table 3.2 Acceptance constant determination for testing Cp

Sample Size	$\alpha = 0.10, \beta = 0.10$		$\alpha = 0.05, \beta = 0.05$	
	Cp (High)=1.66, Cp(Low)=1.33		Cp (High)=1.6, Cp(Low)=1.2	
	Cp (High)/Cp(Low)	c	Cp (High)/Cp(Low)	c
30	1.25	1.58	1.33	1.54
50	1.25	1.51	1.33	1.43
70	1.25	1.49	1.33	1.38
100	1.25	1.45	1.33	1.35

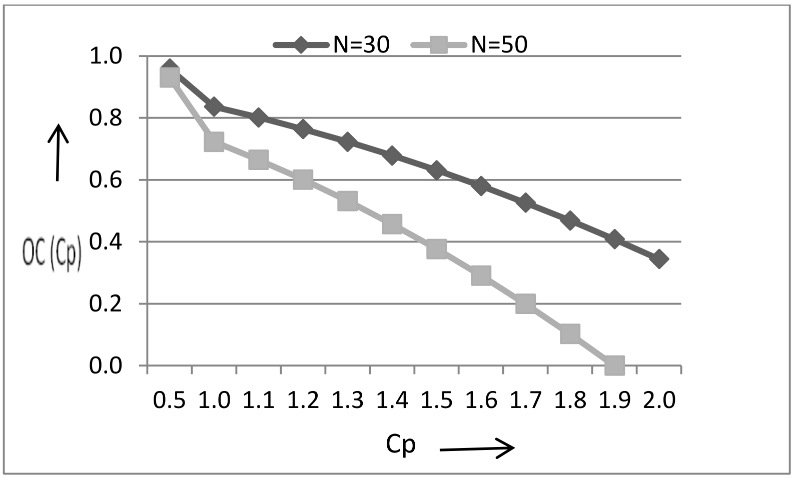

Figure 3.1: OC curve for sampling plan that rejects process capability if Cp<c where c=1.33

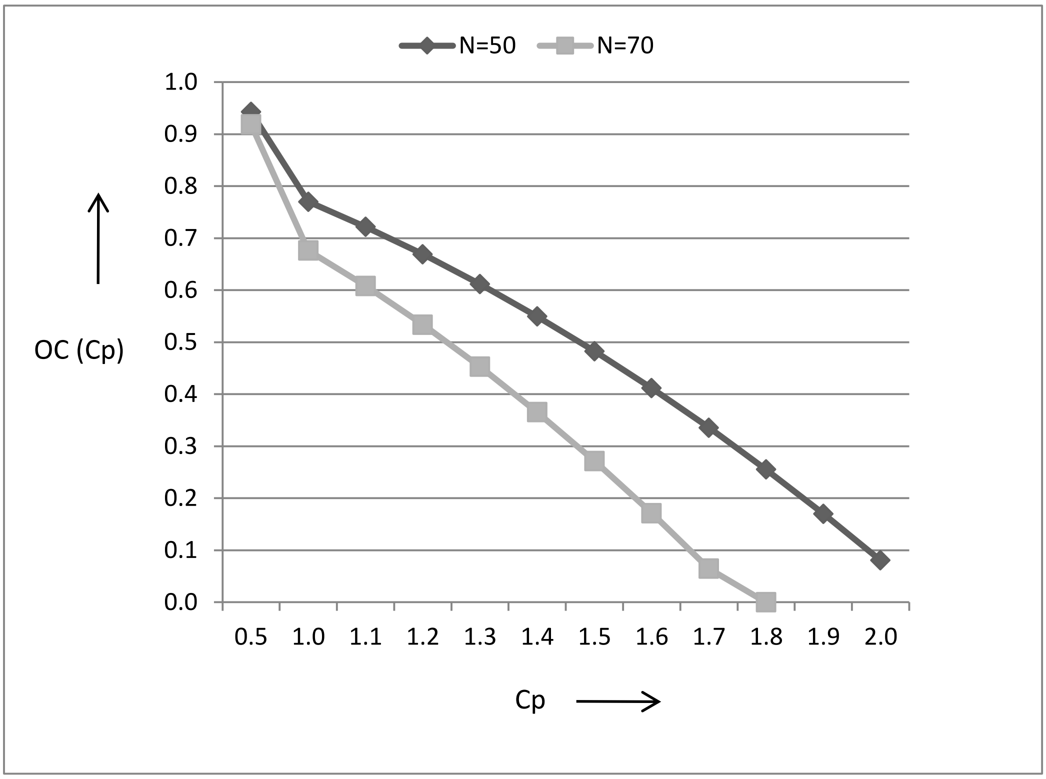

Figure 3.2: OC curve for sampling plan that rejects process capability if Cp<c where c=1.46

4. NUMERICAL ILLUSTRATION AND DISCUSSION OF RESULTS:

Wu et al. [22] concluded that process capability index has been the most popular one used in the manufacturing industry dealing with problems of measuring reproduction capability of processes to enhance product development with very low fraction of defectives. In the manufacturing industry, lower confidence bound estimates the minimum process capability providing pivotal information for quality engineers to monitoring the process and assessing process performance for quality assurance. In recent years, process-capability analysis has become an important integrated part in the applications of statistical techniques for quality assurance. Quality assurance in mass production is achieved using statistical process-control techniques. The process-capability analysis helps to determine the ability for manufacturing between tolerance limits and engineering specifications. The capability analysis gives information about the changes and tendencies of the systems during production [Sagbas [20]].

In this paper, suppose we wish to evaluate the whether a process is capable at the c_0 =1.33 level using N=30 samples and sample rejection limit (critical value) of c=1.33, the OC curve in table 3.1 shows that OC (1.33) =0.72 which implies that there is a 72% chance of incorrectly judging the process not capable (accepting H0). Figure (3.1) and figure (3.2) shows the OC curve for sampling plan that rejects process capability if Cp<c where c=1.33 and c= 1.46 respectively. Gangeshwar and Shankar [7] shows for $\alpha = \beta = 0.05$ and $\text{Cp (High)} / \text{Cp (Low)} = 2.1238$ and calculated $\sqrt{\nu} = 6.2851$ giving $\nu = < 39.50 > = 39$. Once the degrees of freedom ν is known, the sample size N = 40 and using (3.8). The Table 3.2 Shows the values of Cp (High) / Cp (Low) and c for varying sample size and $\alpha = \beta$ equl to 0.10 and 0.05, the critical value c can be determined by for any specified Cp (High) and Cp (Low). For example, if, $\alpha = \beta = 0.05$, Cp (High) =1.6 and Cp (Low)=1.2, then Cp (High)/Cp(Low) =1.33 , and from table 3.2 , c=1.38 when N=70. Similarly, for example, if $\alpha = \beta = 0.10$, Cp (High) =1.66 and Cp (Low) =1.33, then Cp (High)/Cp (Low) =1.25, and from table 3.2, c=1.49 when N=70. These results are agreed with Kane [6]. For practical utility of the plan the OC function and acceptance constant are calculated which are necessary to assure the quality of product.

The present development would be a valuable addition to the literature and a useful device for industrial engineers and quality control professionals. An important key to higher rules and greater profits in any industry centre on the best quality of the product to customer's satisfaction at minimum cost. However, Kane [10] pointed out that, experience to date has shown that there are potential problems in using the Cp on a routine basis. These drawbacks generally stem from users having the incomplete understanding of statistical principles rather than from problems with the indices.

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