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Cycle and Path Related Near Mean Cordial Graphs

L. Pandiselvi, S.Navaneetha Krishnan and A.Nagarajan

PG and Research Department of Mathematics, V. O. Chidambaram College, Tuticorin-628008, Tamilnadu, India.

Abstract

Let G = (V, E)be a simple graph. A **Near Mean Cordial Labeling** of G is a function in f : V(G) → {1, 2, 3, . . . , p−1, p+1} such that the induced map f* defined by

$$f^*(uv) = \begin{cases} 1 & if\big(f(u)+f(v)\big) \equiv 0 \ (mod 2) \\ 0 & else \end{cases}$$

and it satisfies the condition $|e_f(0) - e_f(1)| \leq 1$, where $e_f(0)$ and $e_f(1)$ represent the number of edges labeled with 0 and 1 respectively. A graph is called a **Near Mean Cordial Graph** if it admits a near mean cordial labeling. In this paper, It is to be proved that $P_n \times P_n$, (P_n, S_m) and C_n(When n $\equiv$ 0,1,3 $(mod\ 4)$) are **Near Mean Cordial graphs**. Also, C_n(When n $\equiv$ 2 $(mod\ 4)$) are **not Near Mean Cordial graphs.**

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Keywords and Phrases: Cordial labeling, Near Mean Cordial Labeling and Near Mean Cordial Graph.

1. INTRODUCTION

By a graph, it means a finite undirected graph without loops or multiple edges. For graph theoretic terminology, Harary[4] and G.J. Gallian[1] are referred.

A vertex labeling of a graph G is an assignment of labels to the vertices of G that induces for each edge *uv* a label depending on the vertex labels of *u* and *v*.
A graph *G* is said to be labeled if the *n* vertices are distinguished from a given set, which induces distinguish edge values satisfying certain conditions. The concept of graceful labeling was introduced by Rosa[3] in 1967 and subsequently by Golomb[2].

In this paper, It is to be proved that $P_n \times P_n$, (P_n , S_m) and C_n(When n $\equiv$ 0,1,3 $(mod\ 4)$) are **Near Mean Cordial graphs**. Also, C_n(When n $\equiv 2\ (mod\ 4)$) are **not Near Mean Cordial graphs.**

2. PRELIMINARIES

Definition 2.1: Let G = (V,E) be a simple graph. Let f:V(G)→{0,1} and for each edge uv , assign the label |f(u) − f(v)|. f is called a **cordial labeling** if the number of vertices labeled 0 and the the number of vertices labeled 1 differ by atmost 1 and also the number of edges labeled 0 and the the number of edges labeled 1 differ by atmost 1. A graph is called **Cordial** if it has a cordial labeling.

Definition 2.2: Let G = (V, E)be a simple graph. A **Near Mean Cordial Labeling** of G is a function in f : V(G) → {1, 2, 3, . . . , p−1, p+1}such that the induced map f* defined by

$$f^*(uv) = \begin{cases} 1 & if\big(f(u) + f(v)\big) \equiv 0\ (mod 2) \\ 0 & else \end{cases}$$

and it satisfies the condition $|e_f(0) - e_f(1)| \leq 1$, where $e_f(0)$ and $e_f(1)$ represent the number of edges labeled with 0 and 1 respectively. A graph is called a **Near Mean Cordial Graph** if it admits a near mean cordial labeling.

Definition 2.3 : $(P_n : S_m)$ is a graph obtained by joining the root of S_m at each vertex of P_n.

Definition 2.4 : A closed trail whose origin and internal vertices are distinct is called a cycle.

Definition 2.5 : Define the product $G_1 \times G_2$, by consider any two vertices u = (u_1, u_2) and v = (v_1, v_2) in $V_1 \times V_2$. Then u and v are adjacent in $G_1 \times G_2$.whenever ($u_1 = v_1$ and u_2 adj to v_2) or ($u_2 = v_2$ and u_1 adj to v_1).

The product $P_m \times P_n$ is called planar grids and $\mathbf{K_2 \times P_n}$ is called Ladder. The product $C_m \times P_n$ is called Grids on cylinder of order mn. In particular, $D_n = C_n \times K_2$ is called a prism and $B_m = K_{1,m} \times K_2$ is called a book.

Definition 2.6: The product $P_n \times P_n$ is called Planar Grid where P_n is a path of length $n-1$.

3. MAIN RESULTS

Theorem 3.1. *Grid* $P_n \times P_n$ is a Near Mean Cordial Graph .

Proof: Let V $(P_n \times P_n) = \{u_{i\,j} : 1 \le i \le n, 1 \le j \le n\}$.

Let E $(P_n \times P_n) = \{(u_{i\,j}u_{i\,j+1}) : 1 \le i \le n, 1 \le j \le n-1\} \cup \{(u_{i\,j}u_{i+1\,j}) : 1 \le i \le n-1, 1 \le j \le n\}$

Case(i):when n is odd:

Define $f : V(P_n \times P_n) \to \{1, 2, 3, \ldots, n^2 - 1, n^2 + 1\}$ by

when $i \equiv 1 \pmod 2$

$$f(u_{i\,2j-1}) = ni + 2j - 1, \quad 1 \le i \le n, \quad 1 \le j \le \tfrac{n+1}{2}$$

$$f(u_{i\,2j}) = n(i-1) + 2j, \quad 1 \le i \le n, \quad 1 \le j \le \tfrac{n-1}{2}$$

$$f(u_{n\,2j-1}) = n^2 - n + j, \quad 1 \le j \le \tfrac{n+1}{2}$$

$$f(u_{n\,2j}) = n^2 - j, \quad 2 \le j \le \tfrac{n-3}{2}$$

$$f(u_{n\,2}) = n^2 + 1$$

when $i \equiv 0 \pmod 2$

$$f(u_{i\,2j-1}) = ni - 2n + 2j - 1, \quad 1 \le i \le n, \quad 1 \le j \le \tfrac{n+1}{2}$$

$$f(u_{i\,2j}) = n(i-1) + 2j, \quad 1 \le i \le n, \quad 1 \le j \le \tfrac{n-1}{2}$$

Case(ii):when n is even :

Define $f : V(P_n \times P_n) \to \{1, 2, 3, \ldots, n^2 - 1, n^2 + 1\}$ by

when $i \equiv 1 \pmod 2$

$$f(u_{i\,2j-1}) = ni + 2j - 1 \quad , \quad 1 \le i \le n \quad , \quad 1 \le j \le \frac{n}{2}$$

$$f(u_{i\,2j}) = n(i-1) + 2j \quad , \quad 1 \le i \le n \quad , \quad 1 \le j \le \frac{n}{2}$$

when $i \equiv 0 \pmod 2$

$$f(u_{i\,2j-1}) = ni - 2n + 2j - 1 \quad , \quad 1 \le i \le n \quad , \quad 1 \le j \le \frac{n}{2}$$

$$f(u_{i\,2j}) = n(i-1) + 2j \quad , \quad 1 \le i \le n \quad , \quad 1 \le j \le \frac{n}{2}$$

$$f(u_{n\,2j}) = n^2 - n + 2j \quad , \quad 1 \le j \le \frac{n-2}{2}$$

$$f(u_{n\,n}) = n^2 + 1$$

In both the cases, The induced edge labelings are

$$f^*(u_{i\,j}\, u_{i\,j+1}) = \begin{cases} 1 & if \;\; f(u_{i\,j}) + f(u_{i\,j+1}) \equiv 0 \; (mod\; 2) \\ 0 & else \end{cases} \quad , \; 1 \le i \le n \, , 1 \le j \le n-1$$

$$f^*(u_{i\,j}\, u_{i+1\,j}) = \begin{cases} 1 & if \;\; f(u_{i\,j}) + f(u_{i+1\,j}) \equiv 0 \; (mod\; 2) \\ 0 & else \end{cases} \quad , \; 1 \le i \le n-1 \, , \; 1 \le j \le n$$

Here, $e_f(0) = e_f(1) = n^2 - n$

So, it satisfies the condition $|e_f(0) - e_f(1)| \le 1$.

Hence, $P_n \times P_n$ is a Near Mean Cordial Graph .

For example, the Near Mean Cordial Labeling of $P_6 \times P_6$ and $P_7 \times P_7$ are shown in Figures 3.1.1 and 3.1.2.

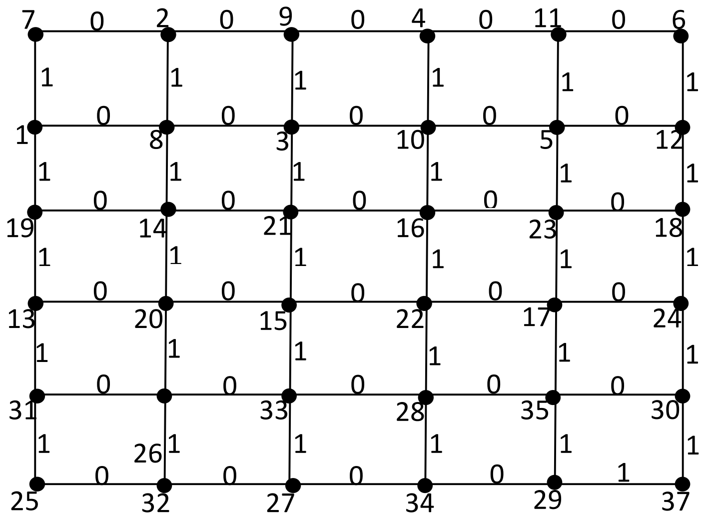

Figure 3.1.1

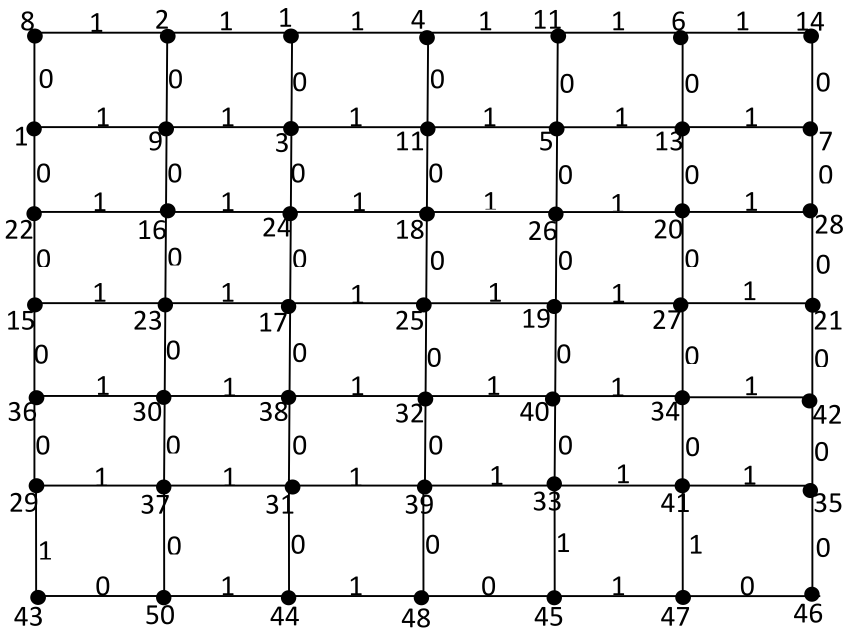

Figure 3.1.2

Theorem 3.2. (P_n, S_m) is a Near Mean Cordial Graph.

Proof: Let $V(P_n, S_m) = \{u_i : 1 \le i \le n,\ u_i^j : 1 \le i \le n,\ 1 \le j \le m\}$.

Let $E(P_n, S_m) = \{(u_i u_{i+1}) : 1 \le i \le n-1\} \cup \{(u_i u_i^j) : 1 \le i \le n, 1 \le j \le m\}$.

When n is even and m is even :

Define f : V(P_n ,S_m) → {1, 2, 3, . . . , mn+$n-1$, mn+n+1} by

f(u_{2i-1}) = $mn+i$, $1 \leq i \leq \frac{n}{2}$

f(u_{2i}) = $mn+\frac{n}{2}+i$, $1 \leq i \leq \frac{n-2}{2}$

f(u_n) = n(m+1)+1

f(u_i^j) = $m(i-1)+j$, $1 \leq i \leq n$, $1 \leq j \leq m$

When n is odd and m is even :

Define f : V(P_n ,S_m) → {1, 2, 3, . . . , mn+$n-1$, mn+n+1} by

f(u_{2i-1}) = $mn+i$, $1 \leq i \leq \frac{n+1}{2}$

f(u_{2i}) = $mn+\frac{n+1}{2}+i$, $1 \leq i \leq \frac{n-3}{2}$

f(u_{n-1}) = n(m+1)+1

f(u_i^j) = $m(i-1)+j$, $1 \leq i \leq n$, $1 \leq j \leq m$

When n is odd and m is odd :

Define f : V(P_n ,S_m) → {1, 2, 3, . . . , mn+$n-1$, mn+n+1} by

f(u_1) = $mn+n+1$

f(u_{2i-1}) = $(n-1)(m+1)-(m+1)(i-1)$, $1 \leq i \leq \frac{n-1}{2}$

f(u_{2i}) = $(m+1)+(m+1)(i-1)$, $1 \leq i \leq \frac{n-1}{2}$

f(u_{2i-1}^j) = $(m+1)(i-1)+j$, $1 \leq i \leq \frac{n+1}{2}$, $1 \leq j \leq m$

f(u_{2i}^j) = $mn+n-1+(m+1)(i-1)-(j-1)$, $1 \leq i \leq \frac{n-1}{2}$, $1 \leq j \leq m$

When n is even and m is odd :

Define f : V(P_n ,S_m) → {1, 2, 3, . . . , mn+$n-1$, mn+n+1} by

f(u_1) = $mn+n+1$

f(u_{2i-1}) = $(n-1)(m+1)-(m+1)(i-1)$, $1 \leq i \leq \frac{n-2}{2}$

f(u_{2i}) = $(m+1)+(m+1)(i-1)$, $1 \leq i \leq \frac{n}{2}$

$f(u_{2i-1}^{j}) = (m+1)(i-1)+j$, $1 \le i \le \frac{n}{2}$, $1 \le j \le m$

$f(u_{2i}^{j}) = mn+n-1+(m+1)(i-1)-(j-1)$, $1 \le i \le \frac{n}{2}$, $1 \le j \le m$

From all the cases, The induced edge labelings are

$$f^*(u_i\,u_{i+1}) = \begin{cases} 1 & \text{if } f(u_i) + f(u_{i+1}) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases} , \ 1 \le i \le n-1$$

$$f^*(u_i\,u_i^{j}) = \begin{cases} 1 & \text{if } f(u_i) + f(u_i^{j}) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases} , \ 1 \le i \le n , \ 1 \le j \le m$$

(i)Let $n = 2k$, $(k \equiv 1 \ (mod\ 2))$ and m is even

Here, $e_f(0) = \frac{mn}{2} + k - 1$ and $e_f(1) = \frac{mn}{2} + k$.

(ii)Let $n = 2k$, $(k \equiv 1 \ (mod\ 2))$ and m is odd

Here, $e_f(0) = \frac{mn}{2} + k$ and $e_f(1) = \frac{mn}{2} + k - 1$.

(iii)Let $n = 2k, (k \equiv 0 \ (mod\ 2))$

Here, $e_f(0) = \frac{mn}{2} + k$ and $e_f(1) = \frac{mn}{2} + k - 1$.

(iv)Let $n = 2k+1, (k\epsilon N)$ $and\ m\ is\ even$

Here, $e_f(0) = \frac{mn}{2} + k = e_f(1)$.

(v)Let $n = 2k+1, (k\epsilon N)$ $and\ m\ is\ odd$

Here, $e_f(0) = \frac{mn+1}{2} + k$ and $e_f(1) = \frac{mn-1}{2} + k$.

So, it satisfies the condition $|e_f(0) - e_f(1)| \le 1$.

Hence, (P_n , S_m) is a Near Mean Cordial Graph .

For example, the Near Mean Cordial Labeling of $(P_4 , S_6), (P_3 , S_8), (P_6 , S_5)$ and (P_5 , S_7) are shown in Figures 3.2.1 and 3.2.4.

When n is even and m is even

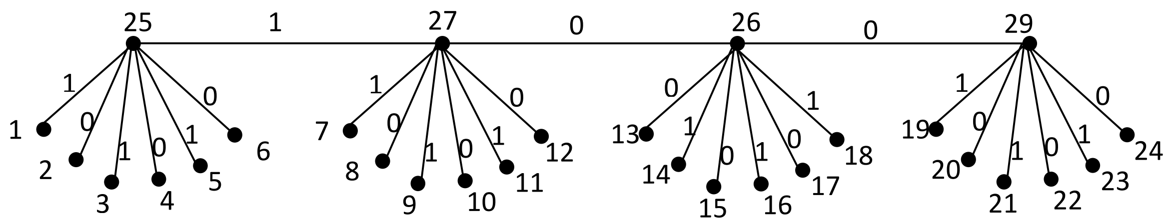

Figure 3.2.1

When n is odd and m is even

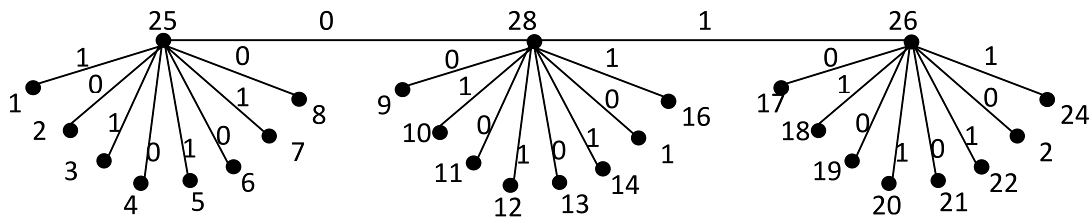

Figure 3.2.2

When n is even and m is odd

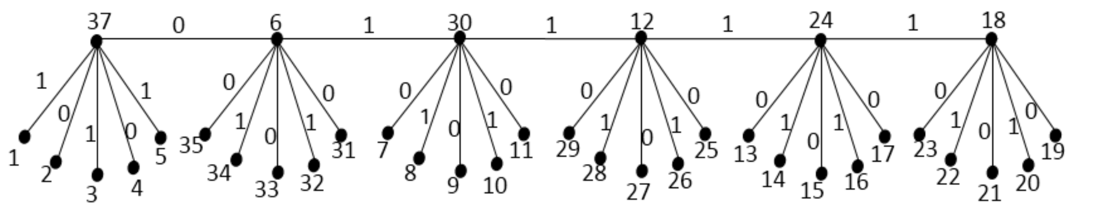

Figure 3.2.3

When n is odd and m is odd

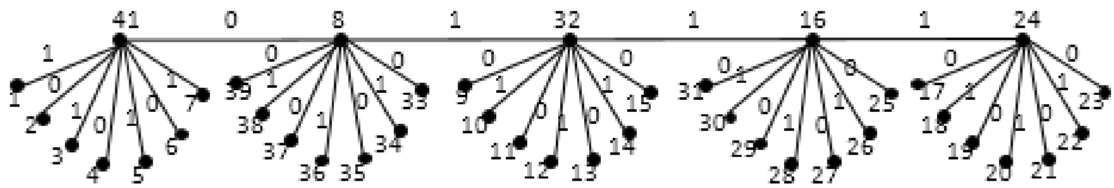

Figure 3.2.4

Theorem 3.3. C_n is a Near Mean Cordial Graph ,when n ≡ 0,1,3 (mod 4).

Proof: Let V(C_n) = $\{u_i : 1 \leq i \leq n\}$.

Let E (C_n) = $\{(u_i u_{i+1}) : 1 \leq i \leq n-1\} \cup \{u_n u_1\}$.

Case (i): when n is odd

Define f : V(C_n) → {1, 2, 3, . . . , n−1, n+1} by

$$f(u_{2i-1}) = i \quad , \quad 1 \leq i \leq \frac{n-1}{2}$$

$$f(u_{2i}) = \frac{n+3}{2} + (i-1) \quad , \quad 1 \leq i \leq \frac{n-3}{2}$$

$$f(u_{n-1}) = n+1 \quad , \quad f(u_n) = \frac{n+1}{2}$$

The induced edge labelings are

$$f^*(u_i\, u_{i+1}) = \begin{cases} 1 & \text{if } f(u_i) + f(u_{i+1}) \equiv 0 \text{ (mod 2)} \\ 0 & \text{else} \end{cases} \quad , \ 1 \leq i \leq n-1$$

$$f^*(u_n\, u_1) = \begin{cases} 1 & \text{if } f(u_n) + f(u_1) \equiv 0 \text{ (mod 2)} \\ 0 & \text{else} \end{cases}$$

Here, $e_f(0) = \frac{n-1}{2}$ and $e_f(1) = \frac{n+1}{2}$, when $n \equiv 1 (mod\ 4)$

Also $e_f(0) = \frac{n+1}{2}$ and $e_f(1) = \frac{n-1}{2}$, when $n \equiv 3\ (mod\ 4)$

Therefore, it satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.

Hence, C_n is a Near Mean Cordial Graph .

For example, the Near Mean Cordial Labeling of C_9 and C_{11} are shown in Figures 3.3.1 and 3.3.2.

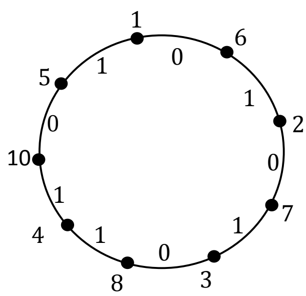

Figure 3.3.1

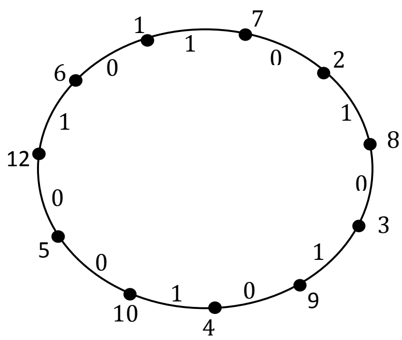

Figure 3.3.2

Case (ii): when n ≡ 0 (mod 4)

Define f : V(C_n) → {1, 2, 3, . . . , n−1, n+1} by

$f(u_{2i-1}) = i \quad , \quad 1 \le i \le \frac{n}{2}$

$f(u_{2i}) = \frac{n+2}{2} + (i-1) \quad , \quad 1 \le i \le \frac{n}{2} - 1$

$f(u_n) = n + 1$

The induced edge labelings are

$$f^*(u_i\, u_{i+1}) = \begin{cases} 1 & \text{if } f(u_i) + f(u_{i+1}) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases} \quad , \ 1 \le i \le n-1$$

$$f^*(u_n\, u_1) = \begin{cases} 1 & \text{if } f(u_n) + f(u_1) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}$$

Here, $e_f(0) = e_f(1) = \frac{n}{2}$,

So, it satisfies the condition $|e_f(0) - e_f(1)| \le 1$.

Hence, C_n is a Near Mean Cordial Graph .

For example, the Near Mean Cordial Labeling of C_8 is shown in the Figure 3.3.3.

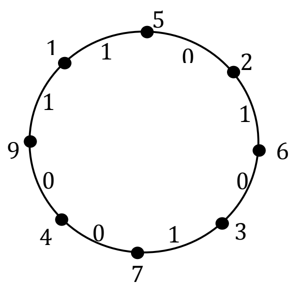

Figure 3.3.3

Theorem 3.4. $C_n(when\ n \equiv 2\ (mod\ 4))$ is not a Near Mean Cordial Graph.

Proof: Let V(C_n) = $\{u_i : 1 \le i \le n\}$.

Let E (C_n) = $\{(u_i u_{i+1}) : 1 \le i \le n-1\} \cup \{u_n u_1\}$.

Consider C_{10} . Now, the vertex labels are 1,2,3,4,5,6,7,8,9,11.

Out of which 4 are even numbers and six odd numbers.

Further 3! = 6 pairs consisting of even numbers only and 5! = 120 pairs consisting of odd numbers only. Also, there are 24 pairs consisting of one odd and one even number.

If a pair consisting of same parity, it gives edge labeling 1 otherwise the edge labeling is 0.

i. All the 5 pair consisting of the same parity, then the possibility that there are exactly 2 pairs with only even numbers and so the remaining 3 pairs consisting odd numbers only(since there are 4 even numbers in the given label). Clearly, In this case, $|e_f(0) - e_f(1)| > 1$.
ii. Further it is noted that there are exactly 4 pair consisting of both odd and even numbers. Then we should choose one pair with odd number only (since there are 4 even numbers in the given label). Clearly, In this case, $|e_f(0) - e_f(1)| > 1$.

In general, if there are odd number of pairs of labels (when n = 6, 10, 14 ,….,) as in the above discussion, we get, $|e_f(0) - e_f(1)| > 1$.

In general, From the above discussion we may conclude that it does not satisfy the edge condition of Near Mean Cordial Labeling . Hence, $C_n(when\ n \equiv 2\ (mod\ 4))$ is not a Near Mean Cordial Graph.

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