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Bi-level linear programming problems involving randomness

G. C. Sasmal

Department of Mathematics, School of Applied Sciences, KIIT University, Bhubaneswar - 751 024, India.

S. K. Barik[1]

Department of Mathematics, Shree Jagannath Mahavidylaya, Naugaonhat, Jagatsinghpur, Odisha, India.

Abstract

Some of the real life decisions are made in decentralized manner under uncertainty. Stochastic bi-level linear programming problems are used to handle such decision making problems. In this paper, we have proposed stochastic bi-level linear programming problems where some of the right hand side parameters of the constraints in both first (leader) and second level (follower) as normal and log-normal random variables with known probability distributions. Rests of the model parameters are assumed to be deterministic. In order to solve the problem, deterministic models are established and solved by using fuzzy programming approach. Two numerical examples are presented to exemplify the usefulness of the proposed methodology.

AMS subject classification:
Keywords: Bi-level programming problem, Stochastic Bi-level programming problem, Fuzzy programming, Normal distribution, Log-normal distribution.

[1]Corresponding author.

1. Introduction

In a bi-level programming problem (BLPP), decisions are taken in two different levels within a optimization framework; first is upper level known as the leader and second is lower level known as follower. The leader first take the decision to optimize his objective function. The follower follows the leaders decision and take his own decision.

BLPP was first proposed by Bracken and McGill [5] to model a decentralized non-cooperative decision system with one leader and multiple followers, although Candler and Norton [7] was first used the term bi-level and multi-level programming. Incited by the game theory of Stackelberg [27], several authors have been worked on BLPPs. BLPPs can be applied to transportation (taxation, network design, trip demand estimation), management (coordination of multi-divisional firms, network facility location, credit allocation), planning (agricultural policies, electric utility), optimal design, etc.

Önal [22] presented a method for solving BLPP and obtained the global optimum by using modified simplex algorithm. A review of the features of BLPP presented by Omar Ben-Ayed[3] along with various applications and algorithms. Jan and Chern [17] developed an algorithm using parametric analysis to solve a separable integer monotone BLPP. Mathur and Puri [21] presented a non-convex BLPP with linear constraints in which the objective functions at both levels are to be minimized. An attempt has been made by Hejazi et al. [16] to develop an efficient solution procedure based on genetic algorithm. A homotopy method for solving BLPP is proposed by Zhu et al. [32] in which the developed algorithm based on constructing the homotopy equation for BLPP. Gao et al. [13] introduced a new solution algorithm to solve bi-level programming model for the discrete network design problem. Emam [10] studied a bi-level integer non-linear programming problem with linear or non-linear constraints in which the non-linear objective function at each level are to maximized. Shi et al. [26] presented a Kth-best approach for linear bi-level multi-follower programming problems with shared variables among followers.

A solution procedure of bi-level linear fractional programming problem proposed by Savita Mishra [29] and obtained a non-dominated solution set by using weighting method. Wang et al. [30] proposed a genetic algorithm to solve the linear-quadratic bilevel programming problem by using Kuhn–Tucker conditions of the lower level programming. Kuo and Huang [19] formulated a BLPP based on particle swarm optimization algorithm with swarm intelligence. Li et al. [20] proposed a biobjective BLPP and solved by using NSGA-II. Guo et al. [14] purposed a bi-level model to solve the multi-reservoir operation problem in inter-basin water transfer-supply project. Hamidi and Nehi [15] presented a method to solve a fuzzy BLPP with inexact parameters. Chen and Zhang [8] consider a variant of the knapsack problem. Payan and Keykha [23] proposed a solution procedure to solve bi-level linear fractional programming problem using k-th best method. Yue Zhenga et al. [31] suggested an interactive fuzzy decision making method for solving BLPP. Ren and Wang[24] consider a BLPP where the coefficients of both objective functions are fuzzy random variables. Suh-Wen [9] proposed a bi-level decision support system for a road network design with uncertain travel demand. Avik and Biswal [1] presented a solution procedure of both bi-level programming problem involving cost coefficients of the objective functions and some of the right hand side parameters of the constraints are multi-choice type. Gang et al. [12] presented a paper which focuses on a stone industrial park location problem with a hierarchical structure consisting of a local government and several stone enterprises under random environment.

Biswas and De [2] presented fuzzy goal programming approach to solve fuzzy linear bilevel integer programming problems with fuzzy probabilistic constraints following Pareto distribution and Frechet distribution. Ji et al. [18] developed a class of multiobjective bilevel programs with the weights of objectives being uncertain and assumed to belong to convex and compact set.

2. Mathematical Programming Problems

BLPP consists of two decision makers in two different levels. First level decision maker, otherwise known as upper-level decision maker (leader) and second level decision maker, otherwise known as lower level decision maker (follower). But the decision vector $X \in \mathbb{R}^n$ is divided into two decision vectors as $X_1 \in \mathbb{R}^{n_1}$ and $X_2 \in \mathbb{R}^{n_2}$ such that $n_1 + n_2 = n$. The decision vectors X_1 and X_2 are handled by the first level and second level decision maker, respectively. Thus, a general model of BLPP can be represented mathematically as:

Find $X = (x_1, x_2, \ldots, x_n)^T$ so as to

$$\max_{X_1} : Z_1 = \sum_{j=1}^{n} c_j x_j \tag{2.1}$$

$$\max_{X_2} : Z_2 = \sum_{j=1}^{n} d_j x_j \tag{2.2}$$

subject to

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i, i = 1, 2, 3, \ldots, m \tag{2.3}$$

$$x_j \geq 0 \tag{2.4}$$

where $X_1 \cup X_2 = X$; $X_1 = (x_1, x_2, \ldots, x_{n_1})^T$; $X_2 = (x_{n_1+1}, x_{n_1+2}, \ldots, x_n)^T$.

Further, when some or all the parameters of a BLPP are random, then the formulated mathematical programming is know as stochastic BLPP. General model of a stochastic BLPP can be written as:

Find $X = \left(x_1, x_2, \ldots, x_n\right)^T$ so as to

$$\max_{X_1} : Z_1 = \sum_{j=1}^{n} c_j x_j \tag{2.5}$$

$$\max_{X_2} : Z_2 = \sum_{j=1}^{n} d_j x_j \tag{2.6}$$

subject to

$$\Pr\Big(\sum_{j=1}^{n} a_{ij} x_j \leq b_i\Big) \geq 1 - \gamma_i, i = 1, 2, 3, \ldots, m \tag{2.7}$$

$$x_j \geq 0 \tag{2.8}$$

where $X_1 \cup X_2 = X$; $X_1 = (x_1, x_2, \ldots, x_{n_1})^T$; $X_2 = (x_{n_1+1}, x_{n_1+2}, \ldots, x_n)^T$. $0 < \gamma_i < 1, i = 1, 2, 3, \ldots, m$ are given probabilities. Only the right hand side parameters of the problem i.e., $b_i, i = 1, 2, 3, \cdots, m$ are random variables with known probability distributions. Rest of the parameters such as x_j, c_j, d_j, and a_{ij} are assumed to be deterministic in the problem.

In this paper, we proposed a stochastic BLPP by considering only the right hand side parameters are normal and log-normal random variables with known probability distributions. Deterministic models are developed and solved by using fuzzy programming technique.

3. Methodology and Deterministic Models

In this section, we have derived the methodology to handle the randomness present in the constrained parameters of the stochastic BLPP and also established the deterministic models. The deterministic models of the stochastic BLPP where the right hand side parameters $b_i, i = 1, 2, 3, \ldots, m$ follows normal and log-normal distributions can be formulated as follows:

3.1. When $b_i, i = 1, 2, 3, \ldots, m$ follows normal distribution

It is assumed that b_i, $i = 1, 2, \ldots, m$ are independent Normal random variables with mean and variance as given by

$$E(b_i) = \mu_i, i = 1, 2, 3, \ldots, m \tag{3.1}$$

$$Var(b_i) = \sigma_i^2, i = 1, 2, 3, \ldots, m \tag{3.2}$$

where μ_i and σ_i, $i = 1, 2, \ldots, m$ are the mean and standard deviation of the b_i, $i = 1, 2, 3, \ldots, m$.

Let the probability density function of the i-th random variable b_i be given by

$$f(b_i) = \frac{1}{\sigma_i\sqrt{2\pi}} \exp^{\frac{-1}{2}\left(\frac{b_i-\mu_i}{\sigma_i}\right)^2}, -\infty < b_i < \infty, -\infty < \mu_i < \infty, \sigma_i > 0 \tag{3.3}$$

But standardized distribution function

$$f(b_i) = \phi\big(\frac{b_i - \mu}{\sigma}\big) \tag{3.4}$$

$$\Pr\left(b_i \geq \sum_{j=1}^{n} a_{ij}x_j\right) \geq (1-\gamma_i), i = 1, 2, 3, \ldots, m \tag{3.5}$$

Let $y_i = \sum_{j=1}^{n} a_{ij}x_j$, then

$$\Pr\left(b_i \geq y_i\right) \geq \left(1-\gamma_i\right) \tag{3.6}$$

$$\Rightarrow \Pr\left(b_i \geq y_i\right) = 1 - \Pr\left(y_i \leq b_i\right) \geq 1-\gamma_i \tag{3.7}$$

$$\Rightarrow 1-\phi\left(\frac{y_i-\mu_i}{\sigma_i}\right) \geq 1-\gamma_i \tag{3.8}$$

$$\Rightarrow \phi\left(\frac{y_i-\mu_i}{\sigma_i}\right) \leq \gamma_i \tag{3.9}$$

$$\Rightarrow \left(\frac{y_i-\mu_i}{\sigma_i}\right) \leq \phi^{-1}(\gamma_i) \tag{3.10}$$

$$\Rightarrow y_i \leq \sigma_i\phi^{-1}(\gamma_i) + \mu_i \tag{3.11}$$

$$\Rightarrow \sum_{j=1}^{n} a_{ij}x_j \leq \sigma_i\phi^{-1}(\gamma_i) + \mu_i \tag{3.12}$$

By using equation (3.12), the deterministic model of the stochastic BLPP (2.5)-(2.8) can be formulated as:

$$\max : z_1 = \sum_{j=1}^{n} c_jx_j \tag{3.13}$$

$$\max : z_2 = \sum_{j=1}^{n} d_jx_j \tag{3.14}$$

subject to

$$\sum_{j=1}^{n} a_{ij}x_j \leq \sigma_i\phi^{-1}(\gamma_i) + \mu_i, \quad i = 1, 2, 3, \ldots, m, \; j = 1, 2, 3, \ldots, n \tag{3.15}$$

$$x_j \geq 0 \tag{3.16}$$

3.2. When $b_i, i = 1, 2, 3, \ldots, m$ follows log-normal distribution

It is assumed that $b_i, i = 1, 2, \ldots, m$ are independent log-normal random variables with mean and variance as given by

$$M_i = E(b_i) = e^{\mu_i + \frac{\sigma_i^2}{2}}, i = 1, 2, 3, \ldots, m \tag{3.17}$$

$$S_i^2 = Var(b_i) = (e^{\sigma_i^2} - 1)e^{2\mu_i + \sigma_i^2}, i = 1, 2, 3, \ldots, m \tag{3.18}$$

where μ_i and σ_i, $i = 1, 2, \ldots, m$ are the mean and standard deviation of the $\ln(b_i)$, $i = 1, 2, 3, \ldots, m$.

Using (3.17) and (3.18), the parameter μ_i and σ_i can be calculated as:

$$\mu_i = \ln(M_i) - 0.5\ln(1 + \frac{S_i^2}{M_i^2}), i = 1, 2, 3, \ldots, m \tag{3.19}$$

$$\sigma_i^2 = \ln(1 + \frac{S_i^2}{M_i^2}), i = 1, 2, 3, \ldots, m \tag{3.20}$$

Let the probability density function(pdf) of the i-th log-normal random variable b_i is given by

$$f(b_i) = \frac{1}{\sqrt{2\pi} b_i \sigma_i} e^{-\frac{1}{2}\left(\frac{\ln b_i - \mu_i}{\sigma_i}\right)^2}, \quad 0 < b_i < \infty, \sigma_i > 0 \tag{3.21}$$

By the standardized distribution function, we get

$$F(b_i) = \Phi\Big(\frac{b_i - \mu}{\sigma}\Big), i = 1, 2, 3, \ldots, m$$

Using equation (2.7), we have

$$\Pr\big(b_i \geq \sum_{j=1}^{m} a_{ij}x_j\big) \geq 1 - \gamma_i, i = 1, 2, 3, \ldots, m$$

Let$y_i = \sum_{j=1}^{m} a_{ij}x_j$, then

$$\Pr(b_i \geq y_i) \geq 1 - \gamma_i, i = 1, 2, 3, \ldots, m$$

or

$$\Pr(\ln b_i \geq \ln y_i) \geq 1 - \gamma_i, i = 1, 2, 3, \ldots, m$$

By standardizing,

$$\Pr\left(\frac{\ln b_i - \mu_i}{\sigma_i} \geq \frac{\ln y_i - \mu_i}{\sigma_i}\right) \geq 1 - \gamma_i, i = 1, 2, 3, \ldots, m$$

or

$$\Pr\left(Z \geq \frac{\ln y_i - \mu_i}{\sigma_i}\right) \geq 1 - \gamma_i, i = 1, 2, 3, \ldots, m$$

or

$$1 - \Pr\left(Z \leq \frac{\ln y_i - \mu_i}{\sigma_i}\right) \geq 1 - \gamma_i, i = 1, 2, 3, \ldots, m$$

By the definition of distribution function,

$$\Phi\left(\frac{\ln y_i - \mu_i}{\sigma_i}\right) \leq \gamma_i, i = 1, 2, 3, \ldots, m$$

or

$$\frac{\ln y_i - \mu_i}{\sigma_i} \leq \phi^{-1}(\gamma_i), i = 1, 2, 3, \ldots, m$$

By simplification,

$$\sum_{j=1}^{n} a_{ij} x_j \leq e^{\mu_i + \sigma_i \Phi^{-1}(\gamma_i)}, i = 1, 2, 3, \ldots, m \tag{3.22}$$

By using equation (3.22), the deterministic model of the stochastic BLPP (2.5)-(2.8) can be formulated as:

$$\max_{X_1} : Z_1 = \sum_{j=1}^{n} c_j x_j \tag{3.23}$$

$$\max_{X_2} : Z_2 = \sum_{j=1}^{n} d_j x_j \tag{3.24}$$

subject to

$$\sum_{j=1}^{n} a_{ij} x_j \leq e^{\mu_i + \sigma_i \Phi^{-1}(\gamma_i)}, i = 1, 2, 3, \ldots, m \tag{3.25}$$

$$x_j \geq 0 \tag{3.26}$$

4. Solution Procedure

In this section, we have derived the solution procedure of the deterministic BLPP. Since BLPP have two different decision maker at two level. So, the objective function for both the decision maker are different and conflicting to each other. Therefore, we determine a compromise solution which will gratify both the decision maker. To obtain a compromise solution, we use fuzzy programming method presented by Shih et al. [25]. Steps of the fuzzy programming method is presented below:

Step 1: Find the individual optimal solution for both the leader and the follower problem as $(Z_1^U; X_1^U, X_2^U)$ and $(Z_2^L; X_1^L, X_2^L)$ respectively, over the given feasible region.

Step 2: Form the pay-off matrix of the leader and follower objective functions as:

Step 3: Set the minimum and maximum tolerance limit for the objective function from the pay-off matrix, and revise the lower tolerance limit for leader objective.

Table 1: Pay-off Matrix

	$Z_1^U(X)$	$Z_2^L(X)$
$X^{(1)}$	Z_{11}^U	Z_{12}^L
$X^{(2)}$	Z_{21}^U	Z_{22}^L

Step 4: Set the positive and negative deviation for the leader's control variables be P and N respectively, where $P, N \in R$, and they need not be same.

Step 5: Build the membership function for both the objective function and leader's control variable i.e. Z_1, Z_2 and X_1 as:

$$\mu_{Z_1}(X) = \begin{cases} 1, & Z_1 \geq Z_1^U. \\ \dfrac{Z_1 - Z_1^L}{Z_1^U - Z_1^L}, & Z_1^L < Z_1 < Z_1^U; \\ 0, & Z_1 \leq Z_1^U. \end{cases} \tag{4.1}$$

$$\mu_{Z_2}(X) = \begin{cases} 1, & Z_2 \geq Z_2^L. \\ \dfrac{Z_2 - Z_2^U}{Z_2^L - Z_2^U} & Z_2^U < Z_2 < Z_2^L; \\ 0, & Z_2 \leq Z_2^U. \end{cases} \tag{4.2}$$

$$\mu_{X_1}(X_1) = \begin{cases} \dfrac{(X_1^U + P) - X_1}{P}, & X_1^U \leq X_1 \leq X_1^U + P \\ \dfrac{X_1 - (X_1^U - N)}{N}, & X_1^U - N \leq X_1 \leq X_1^U \end{cases} \tag{4.3}$$

Step 6: Using max-min operator by Bellman and Zadeh [4], we formulate the fuzzy programming models of the deterministic BLPP problems (3.13)-(3.16) and (3.23)-(3.26) as:
Model 1:

$$\max : \lambda \tag{4.4}$$

$$\sum_{j=1}^{n} c_j x_j - (Z_1^U - Z_1^L)\lambda \geq Z_1^L \tag{4.5}$$

$$\sum_{j=1}^{n} d_j x_j - (Z_2^L - Z_2^U)\lambda \geq Z_2^U \tag{4.6}$$

$$\mu_{X_1} \geq \lambda I \tag{4.7}$$

$$\sum_{j=1}^{n} a_{ij}x_j \leq \sigma_i\phi^{-1}(\gamma_i) + \mu_i, \;\; i = 1, 2, 3, \ldots, m, j = 1, 2, 3, \ldots, n \tag{4.8}$$

$$\lambda \in [0, 1] \tag{4.9}$$

$$x_j \geq 0, j = 1, 2, 3, \ldots, m \tag{4.10}$$

where I is a column vector with the same dimension as X_l and all of its entries are equal to 1. $\lambda = min\{\alpha, \beta, \overline{\gamma}\}$, α, β, and $\overline{\gamma}$ be the minimum acceptable degree of satisfaction for the objective Z_1, Z_2 and X_1, respectively. Hence, $\mu_{Z_1} \geq \alpha$ and $\mu_{Z_2} \geq \beta$, and $\mu_{X_1} \geq \overline{\gamma}$.

Model 2:

$$\max : \lambda \tag{4.11}$$

$$\sum_{j=1}^{n} c_j x_j - (Z_1^U - Z_1^L)\lambda \geq Z_1^L \tag{4.12}$$

$$\sum_{j=1}^{n} d_j x_j - (Z_2^L - Z_2^U)\lambda \geq Z_2^U \tag{4.13}$$

$$\mu_{X_1} \geq \lambda I \tag{4.14}$$

$$\sum_{j=1}^{n} a_{ij}x_j \leq e^{\mu_i + \sigma_i \Phi^{-1}(\gamma_i)}, i = 1, 2, 3, \cdots, m \tag{4.15}$$

$$\lambda \in [0, 1] \tag{4.16}$$

$$x_j \geq 0, j = 1, 2, 3, \cdots, m \tag{4.17}$$

where the symbols and notations are same as above model.

Step 7: Solve the above fuzzy programming models and obtained the satisfactory optimal solution.

5. Numerical Examples

In this Section, we have consider two different numerical examples to verify the above developed methodologies to solve a stochastic BLPP. For this, we have presented the following stochastic BLPP with right hand side parameters follows independent normal and log normal random variables as below:

5.1. Example 1:

Let us consider the stochastic BLPP with right hand side parameters follows independent normal random variables:

$$\max_{x_1} Z_1 = 15x_1 + 7x_2 \tag{5.1}$$

$$\max_{x_2} Z_2 = 7x_1 + 10x_2 \tag{5.2}$$

subject to

$$\Pr(6x_1 + 10x_2 \leq b_1) \geq 0.95 \tag{5.3}$$

$$\Pr(12x_1 + 8x_2 \leq b_2) \geq 0.92 \tag{5.4}$$

$$x_1, x_2 \geq 0 \tag{5.5}$$

Where b_1 and b_2 are independent normal random variables with known means and variences. Further, $\gamma_1 = 0.05$ and $\gamma_2 = 0.08$ are specified probability levels.

Assume that the mean and variance values of b_1 and b_2 are given by $\mu_1 = E(b_1) = 20$, $\mu_2 = E(b_2) = 25$, $\sigma_1^2 = 4$ and $\sigma_2^2 = 9$. Now using the mean and variance values of b_1 and b_2, we establish the deterministic BLPP as:

$$\max_{x_1} Z_1 = 15x_1 + 7x_2 \tag{5.6}$$

$$\max_{x_2} Z_2 = 7x_1 + 10x_2 \tag{5.7}$$

subject to

$$6x_1 + 10x_2 \leq 16.71 \tag{5.8}$$

$$12x_1 + 8x_2 \leq 20.785 \tag{5.9}$$

$$x_1, x_2 \geq 0 \tag{5.10}$$

The above problem is solved separately and obtain the solution for both the level of decision maker. The solution for leader is given by $Z_1^U = 25.98$ at $x = (1.732; 0)$. The follower's solution is given by $Z_2^L = 17.74$ at x = (1.03; 1.05). Using the pay-off matrix as constructed in Table 1, we get $Z_1^L = 22.8$ and $Z_2^U = 12.124$. The decision variable x_1 is controlled by the leader, so the leader set the positive and negative deviation for the variable x_1 as $P = 1$ and $N = 3$ respectively. By using (4.1), (4.2) and (4.3), we construct the membership functions for the problem. Then the crisp equivalent model for the problem can be stated as:

$$\max : \lambda \tag{5.11}$$

subject to

$$15x_1 + 7x_2 - 3.18\lambda \geq 22.8 \tag{5.12}$$

$$7x_1 + 10x_2 - 5.616\lambda \geq 12.124 \tag{5.13}$$

$$6x_1 + 10x_2 \leq 16.71 \tag{5.14}$$

$$12x_1 + 8x_2 \leq 20.785 \tag{5.15}$$

$$x_1 + \lambda \leq 2.732 \tag{5.16}$$

$$x_1 - 3\lambda \geq -1.268 \tag{5.17}$$

$$\lambda \in [0, 1] \tag{5.18}$$

$$x_1, x_2 \geq 0 \tag{5.19}$$

By solving the above model using LINGO 11.0 [28], we obtain the compromise optimal solution as $\lambda = 0.50$, $(Z_1, Z_2) = (24.3951, 14.9416)$ and $(x_1, x_2) = (1.3798, 0.5283)$.

5.2. Example 2:

$$\max_{x_1} Z_1 = 10x_1 + 12x_2 \tag{5.20}$$

$$\max_{x_2} Z_2 = 7x_1 + 20x_2 \tag{5.21}$$

subject to

$$\Pr(6x_1 + 10x_2 \leq b_1) \geq 0.95 \tag{5.22}$$

$$\Pr(12x_1 + 8x_2 \leq b_2) \geq 0.92 \tag{5.23}$$

$$x_1, x_2 \geq 0 \tag{5.24}$$

Where b_1 and b_2 are independent log-normal random variables with known means. Further, $\gamma_1 = 0.05$ and $\gamma_2 = 0.08$ are specified probability levels.

Assume that the mean and variance values of b_1 and b_2 are given by $M_1 = E(b_1) = 40$, $M_2 = E(b_2) = 50$, $S_1 = 4$ and $S_2 = 5$. Now, the parameter values of b_1 and b_2 are calculated by using the equation (3.17)and (3.18) as: $\mu_1 = 3.68390$, $\mu_2 = 3.907048$,$\sigma_1^2 = 0.00995$ and $\sigma_2^2 = 0.00995$.

Now using the mean and varience values and the values of b_1 and b_2, we establish the deterministic BLPP as:

$$\max_{x_1} Z_1 = 10x_1 + 12x_2 \tag{5.25}$$

$$\max_{x_2} Z_2 = 7x_1 + 20x_2 \tag{5.26}$$

subject to

$$6x_1 + 10x_2 \leq 33.77818 \tag{5.27}$$

$$12x_1 + 8x_2 \leq 43.24576 \tag{5.28}$$

$$x_1, x_2 \geq 0 \tag{5.29}$$

The above problem is solved separately and obtain the solution for both the level of decision maker. The solution for leader is given by $Z_1^U = 46.84285$ at x = (2.253; 2.025). The follower's solution is given by $Z_2^L = 67.55637$ at x = (0.0; 3.37781). Using the pay-off matrix as constructed in Table 1, we get $Z_1^L = 40.53372$ and $Z_2^U = 56.29014$. The decision variable x_1 is controlled by the leader, so the leader set the positive and negative deviation for the variable x_1 as $P = 1$ and $N = 3$ respectively. By using (4.1), (4.2) and (4.3), we construct the membership functions for the problem. Then the crisp equivalent model for the problem can be stated as:

$$\max : \lambda \tag{5.30}$$

subject to

$$10x_1 + 12x_2 - 6.30913\lambda \geq 40.53372 \tag{5.31}$$

$$7x_1 + 20x_2 - 11.26623\lambda \geq 56.29014 \tag{5.32}$$

$$6x_1 + 10x_2 \leq 33.77818 \tag{5.33}$$

$$12x_1 + 8x_2 \leq 43.24576 \tag{5.34}$$

$$x_1 + \lambda \leq 3.25322 \tag{5.35}$$

$$x_1 - 3\lambda \geq -0.74678 \tag{5.36}$$

$$\lambda \in [0, 1] \tag{5.37}$$

$$x_1, x_2 \geq 0 \tag{5.38}$$

By solving the above model using LINGO 11.0 [28], we obtain the compromise optimal solution as $\lambda = 0.50$, $(Z_1, Z_2) = (43.68832, 61.92332)$ and $(x_1, x_2) = (1.126609, 2.701853)$.

Table 2: Optimal Solution of Example 1 and Example 2

Examples	Decision Variables	Value of the Objective Functions	Satisfaction Level (λ)
Example 1	$x_1 = 1.3798, x_2 = 0.5283$	$(Z_1, Z_2) = (24.3951, 14.9416)$	$\lambda = 0.50$
Example 2	$x_1 = 1.126609, x_2 = 2.701853$	$(Z_1, Z_2) = (43.68832, 61.92332)$	$\lambda = 0.50$

6. Conclusion

In this paper, a stochastic BLPP has been developed involving the right hand side parameters in both the leader and follower constraints as normal and log-normal random variables. Deterministic models are established and solved by using an efficient and powerful compromise technique known as fuzzy programming method. Optimal compromise solutions are obtained with level of satisfaction 0.50 each in both the normal and log-normal cases as shown in Table 2. This model can be applied in various decision making problems due its robustness. Also, this model can be solved directly by using genetic algorithms without transforming into deterministic.

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