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A Study of Fuzzy Sets Similarity and its Application in Intelligent Transportation Systems

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Abstract

The main purpose of this paper is to develop a general analytic framework for studying of similarity of fuzzy sets which provides a new fresh look into information granulation serving as a basis for the incident awareness technique in Intelligent Transportation Systems. To this end, we propose novel concepts and two approaches to fuzzy sets similarity based on Brouwerian lattices and topology structures. Also, we go into detail about definitions and differences concerning proposed approaches including relations and operations over fuzzy sets. Then, we impose new definitions for information granulation with proximity and similarity relations. Finally, we apply these concepts in incident awareness problems to illustrate their applicability in Intelligent Transportation Systems.

AMS subject classification: 03E72.
Keywords: Fuzzy sets similarity, incident awareness, information granulation, intelligent transportation systems.

1. Introduction

The future modernization of Russian transport system requires further development of high-speed rail communications. Such strategy is not possible without modern, innovative technologies and information-controlling systems based on intelligent control approaches allowing significantly enhancing of working efficiency and productivity. The main track of intelligent controlling in *Russian Railways* is a creation of new classes of *Intelligent Transportation Systems* (*ITS*), particularly a *Multilevel Intelligent Control System* (*MICS*) with incident detection capabilities [2].

Due to the fact, that *ITS* and *MICS* accordingly are the class of safety-critical railway systems, it is necessary to register, manage and report all of the wrong events, including technical failures, human errors and other incidents [8]. To manage the incidents processing the *JSC* "*Russian Railways*" some time ago was started the system, which named the *URRAN* [5] project. In large part, the *URRAN* uses the *RAMS* (*Reliability*, *Availability*, *Maintainability* and *Safety*) methodology [16], standards *EN* 50126 and *IEC* 62278. Obviously, the methodology above is not applicable to modern trends in *ITS* development, for example to new models [20] of storages of fuzzy and semi-structured data. Moreover, now in *ITS* fuzzy methods and systems are the centrality in new classes of *ITS*: high-speed railway control systems [12]; automated railway level crossing systems [15, 4]; new railway vehicles models [17]; fault prognosis of railway track circuits [14]; improvement of automatic train operations [18]; the design of smart cars [17] to prevent intelligently against traffic accidents and many others. However, unfortunately, there is a lack of fuzzy methods in situation awareness. We consider the term situation awareness as a concept means the possibility of obtaining a complete and accurate set of required for the decision information about the situation in real time, including its nature and characteristics, the same definition as has been previously postulated in [6]. In *MICS*, the situation awareness enables minimization and the fastest way to the negative impact elimination on the transportation processes and restoration of normal functioning of railway service. One of the contemporary approaches to situation awareness providing is an information granulation presented in [13], but there is a need to revise foundations of a fuzzy sets similarity relations by modern trends and conditions of intelligent achievements.

To fill this gap, our paper proposes a general analytic framework for studying of similarity relations between fuzzy sets allowing a providing a fresh new look into information granulation serving as a basis for the incident awareness technique in *ITS* and consequently in *MICS*. The rest of our paper is organized as follows. Section 2 contains some preliminary facts needed to catch our new approach. Section 3 present the central part and a core of our study and framework definitions. Finally, we conclude the paper in Section 4 presenting some examples in the area we describe above.

2. Preliminary facts and definitions

In this section, we revise the basic concepts and foundations of the fuzzy sets theory [9, 19, 21] from the two points of view: the fundamental set-theoretic view and the algebraic structures. At first, let's remind the foundations in the suitable for us form with necessary comments and initial conditions.

Definition 1. Let U be an arbitrary Cantor set and $\mu(u)$ – be an arbitrary function, defined on U with values belonging to the [0, 1] interval. In U the set $C_\mu = \{ (u, \ \mu(u)) : u \in U \}$ is called as a fuzzy set U (in Cantor definition).

The $\mu(u)$ is a fuzzy membership function of a fuzzy set C_μ. For $\forall u \in U$ the value $\mu(u)$ is called as membership degree of u to a fuzzy set C_μ. According to Definition 1 each fuzzy set is anything other as a diagram of the membership function of this set. Further, we will actively use this identification.

Definition 2. Let U be an arbitrary Cantor set. A fuzzy set in U, the ***type 0***, we name a fuzzy set in U which membership function is continuous in U.

It is clear, that (as well as their membership functions) one can identify fuzzy sets of type 0 in any considered set with numbers from an interval [0, 1].

After Definitions 1 and 2 fuzzy sets of arbitrary natural type are entered by induction.

Definition 3. Let U be an arbitrary Cantor set. A fuzzy set $C_{\mu}^{(n)} = \{ (u, \ \mu(u)) : \ u \in U \}$ in U, the ***type n***, $(n \in N)$ we name a fuzzy set U, $n \in N$, where $\mu(u)$ is any mapping of a set U to all fuzzy sets in U with the type $n-1$.

We also call a mapping $\mu(u)$ as a membership mapping of a fuzzy set $C_{\mu}^{(n)}$. As well as in a case $n = 1$ the fuzzy set in U of the ***type n***, $(n \in N)$ is anything other as the diagram of membership mapping $\mu : \ U \to C_{\tilde{\mu}}^{(n-1)}$. Follows from the Definitions 1-3 given above that the set of all fuzzy sets in U in the sense of Definition 1 coincides with a set of all fuzzy sets in U of **type 1**.

Further, according to Definition 3 fuzzy set in U of **type 2** is the set $C_{\mu}^{(2)} = \{ (u, \ \mu(u)) : \ u \in U \}$, where $\mu(u)$ - any mapping of a set U in a great number of all fuzzy sets in U of **type 1** (that is fuzzy sets in U in the sense of Definition 1). Every fuzzy set $C_{\mu}^{(2)}$ in U of the **type *2*** we uniquely assign a fuzzy set $C_{\tilde{\mu}}^{(1)}$ in U of **type 1** with membership function $\tilde{\mu}(u)$, which we define as follows. Obviously, the value of $\tilde{\mu}(u)$ belongs to interval $[0, \ 1]$ is a number, which is put in compliance to an element u by the membership function of fuzzy set in U of **type 1**, corresponding to the element through mapping $\mu(u)$. Such fuzzy set $C_{\tilde{\mu}}^{(1)}$ we name as a **cut of a fuzzy set** $C_{\mu}^{(2)}$.

The cut of the fuzzy set in U of **type 2** indirectly and numerically assigns the membership degree of every element $u \in U$ to fuzzy set $C_{\mu}^{(2)}$. The assigned degree is "managed" by the membership function $\tilde{\mu}(u)$ of appropriate fuzzy set $C_{\tilde{\mu}}^{(1)}$. By the analogy, we propose a concept of the cut of a fuzzy set $C_{\mu}^{(n)}$ for $\forall n \in N$. The set $C_{\mu}^{(1)}$ is identical to its cut. For any fixed $n \in N$ for all fuzzy sets in U of the **type *n***, one can examine the equivalence relation, meaning matching of their cuts.

The construction executed above explains the feasibility of introduction of fuzzy sets in U of the type n $(n > 1)$ as one of the forms of the process of further "fuzzification" of fuzzy sets. Because of increasing of the type n, namely, moving along "hierarchy" of fuzzy sets. The membership degree of the element u to the fuzzy sets in some sense "blurring and fuzzifying" as a result of the successive layers on top of each other intermediate maps $\tilde{\mu}$.

Now, let's turn to the considering of relations between fuzzy sets and operations over them.

For an arbitrary fixed universal set U and number $n \in N \bigcup \{0\}$ the set of all fuzzy sets (in Cantor understanding) in U of the type n we denote as a symbol $[0, 1]_n^U$. The reason for this is the famous denotation Y^X [18, 19, 20], denoting the set of all mappings from set X to set Y and also, the meaning that every fuzzy set is anything else, but some mapping of the set U to [0, 1] interval. If the $n = 1$, then we will write simply $[0, 1]^U$.

The inclusion relation between fuzzy sets is defined as it follows from [21, 9, 19].

Definition 4. Let $C_{\mu_1},\ C_{\mu_2} \in [0, 1]^U$. As it widely adopted, and the set C_{μ_1} is included into C_{μ_2} it is denoted: $C_{\mu_1} \subseteq C_{\mu_2}$, and if $\forall u \in U$: $\mu_1(u) \le \mu_2(u)$.

Following the models of Cantor's theory the following constructions on the set $[0, 1]^U$ consist of creating fuzzy analogs of intersection and union set operations, namely in the defining on $[0, 1]^U$ two binary operations, using the $\vee$ and $\wedge$ symbols. At the most general set-theoretic and algebraic approach being considered questions necessarily leads to the concept of structure [1, 3], which is a partially ordered set, each two-element subset which has exact upper and lower bounds. We note, regarding membership function of the cut of fuzzy set the partial ordering relationship is valid for sets $[0, 1]_n^U$ with $n > 1$.

Now, let's turn to consideration and the analysis of concrete definitions.

Definition 5. The union of the fuzzy set C_{μ_1} with fuzzy set C_{μ_2} is the fuzzy set, denoted as $C_{\mu_1} \bigcup C_{\mu_2}$, and its membership function is denoted as $(\mu_1 \vee \mu_2)(u)$ and $\forall u \in U$ defined as

$$(\mu_1 \vee \mu_2)(u) = \max\{\mu_1(u), \mu_2(u)\}. \tag{2.1}$$

Definition 6. The intersection of the fuzzy set C_{μ_1} with fuzzy set C_{μ_2} is the fuzzy set, denoted as $C_{\mu_1} \bigcap C_{\mu_2}$, and its membership function is denoted $(\mu_1 \wedge \mu_2)(u)$ and $\forall u \in U$ defined as

$$(\mu_1 \wedge \mu_2)(u) = \min\{\mu_1(u), \mu_2(u)\}. \tag{2.2}$$

The resulting definitions 5 and 6, the structure is a distributive lattice [10, 1, 3] with zero (the smallest element of this structure), which is a empty set (with membership function equal to zero at all points of U), and unity (the greatest element), which is a U (with membership function equal to 1 respectively).

In fact, the considered structure "remains non-adequate" to the Boolean algebra. However, it is a Brouwerian lattice [10]. Indeed, for each pair of fuzzy sets C_{μ_1} and C_{μ_2} exists a pseudo-difference [10], which is a fuzzy set $C_{\mu_1 \underline{*} \mu_2}$ with membership function $\left(\mu_1 \underline{*} \mu_2\right)(u)$, defined as following:

$$\left(\mu_1 \underline{*} \mu_2\right)(u) = \begin{cases} 0, & if\ \ \mu_2(u) \ge \mu_1(u), \\ \mu_1(u), & if\ \ \mu_2(u) < \mu_1(u). \end{cases} \tag{2.3}$$

Consequently, pseudo-complement [18] of arbitrary fuzzy set $C_\mu \in [0, 1]^U$ is a fuzzy set C_{μ^*}, which membership function is defined as following:

$$\mu^*(u) = \begin{cases} 1, & if\ \ \mu(u) < 1, \\ 0, & if\ \ \mu(u) = 1. \end{cases} \tag{2.4}$$

For each fuzzy set $C_\mu \in [0, 1]^U$ the following equality is valid $\left(\mu \vee \mu^*\right)(u) = 1$ $(\forall u \in U)$, which means $C_\mu \bigcup C_{\mu^*} = U$. Pseudo-complement of set U is a $\emptyset$. The set U is a pseudo-complement of any fuzzy set C_μ, so that $\mu(u) < 1 \; \forall u \in U$. Thus, the intersection of a fuzzy set with its pseudo-complement may be non-empty and moreover be matching with it. In this case from the fact that the intersection of fuzzy sets with their pseudo-complement empty, it does not follow that at least, one of these sets is empty. If we pass from Brouwerian lattices to a dual theory of pseudo-Boolean algebra [1], the logical interpretation of the relevant situation is expressed in non-compliance with the law of the excluded middle and implementation of the law of contradiction (which takes place in intuitionistic logic Brouwer) [10, 11]. Returning to the fuzzy sets, we note that the non-empty intersection of these sets with their pseudo-complements just represents a resource for all sorts of constructions "fuzzy" theory.

Further, from (4) it is follows

$$\mu^{**}(u) = \begin{cases} 1, & if \;\; \mu(u) = 1, \\ 0, & if \;\; \mu(u) < 1. \end{cases}$$

Hence, $\mu^{**}(u) \leq \mu(u) \; \forall u \in U$. In this way, $C_{\mu^{**}} \subseteq C_\mu$ for any fuzzy set C_μ. It is also $\mu^{**}(u) = \mu(u)$ only for those $u \in U$, for which $\mu(u) = 0$ or $\mu(u) = 1$.

For pseudo-complement operation are true the following equations $(\forall u \in U)$:

$$(\mu_1 \wedge \mu_2)^*(u) = \mu_1^*(u) \vee \mu_2^*(u)$$

and

$$(\mu_1 \vee \mu_2)^*(u) = \mu_1^*(u) \wedge \mu_2^*(u),$$

meaning implementation of both de Morgan's laws.

Now, let's pay attention to the operation used almost everywhere with fuzzy sets and positioned as a set complement operation [9, 19, 21]. As a complement of a fuzzy set with membership function $\mu(u)$ they considered a fuzzy set with following membership function

$$\bar{\mu}(u) = 1 - \mu(u). \tag{2.5}$$

While such definition $\forall u \in U$ and for $0 < \mu(u) < 1$, we've got the equality:

$$(\mu \vee \bar{\mu})(u) = \max\{\mu(u), 1 - \mu(u)\} \neq 1.$$

Thus, the main purpose of complement operation is not executed, while the equality $\bar{\bar{\mu}}(u) = \mu(u)$ is valid and both de Morgan's laws are valid too.

With introducing of union and intersection operation for fuzzy sets is useful to mean the following. It is previously known [3] that the set with two binary operations $a \vee b$ and $a \wedge b$ if and only if be the structure, if the operation satisfies the relations:

$$a \wedge a = a, a \vee a = a;$$

$$a \wedge b = b \wedge a, a \vee b = b \vee a;$$

$$(a \wedge b) \wedge c = a \wedge (b \wedge c)\,,$$
$$(a \vee b) \vee c = a \vee (b \vee c)\,;$$
$$a \wedge (a \vee b) = a;$$
$$a \vee (a \wedge b) = a.$$

The criterion above allows us to identify from candidates those operations, which are not capable of creating the structure on the set $[0, 1]^U$. Next, we give two such determination [9, 19, 21].

Definition 7. The union of the fuzzy set C_{μ_1} with fuzzy set C_{μ_2} is a fuzzy set, while $\forall u \in U$ and having a membership function

$$(\mu_1 \vee \mu_2)(u) = \min\{1,\ \mu_1(u) + \mu_2(u)\}\,. \tag{2.6}$$

Definition 8. The intersection of the fuzzy set C_{μ_1} with fuzzy set C_{μ_2} is a fuzzy set, while $\forall u \in U$ and having a membership function

$$(\mu_1 \wedge \mu_2)(u) = \max\{0,\ \mu_1(u) + \mu_2(u) - 1\}\,. \tag{2.7}$$

For binary operations $\vee$, $\wedge$, defined through (2.6) and (2.7), the idempotence laws, for example, are not implemented. Such obstacle does not preclude, however, examined the feasibility of using the definitions in the theory and applications of fuzzy sets.

In switching to "fuzzy" side, it is reasonable to consider in sets $[0, 1]_n^U$ the quasi-ordering relation [1].

Definition 9. Let's $C_{\mu_1},\ C_{\mu_2} \in [0, 1]^U$. We define, that C_{μ_1} is fuzzy-included into C_{μ_2} (denotation: $C_{\mu_1} \tilde{\subseteq} C_{\mu_2}$), only if inequality $\mu_1(u) \le \mu_2(u)$ is implemented for all $u \in U$, except finite set.

A significant exception to the generalization set of measure zero.

3. Proposed analytical framework for fuzzy sets similarity and information granulation

In this Section, we propose two novel approaches, regarding the concept of fuzzy sets similarity.

The first approach is applicable in situations, where we deal with a distinct discrete character (for example if the universal set is finite and not numerous). Let's U – is arbitrary universal set and $k,\ n \in N$.

Proposition 1. Fuzzy set $C_{\mu_1}^{(n)}$ is *k-similar* to a fuzzy set $C_{\mu_2}^{(n)}$, if their fuzzy cuts have, at least, k similar elements.

The relation of *k-similarity* is a **tolerance** relation in $[0, 1]_n^U$. It is not difficult to make sure, that for *k-similarity* relation is also an equivalence relation too in $[0, 1]_n^U$, there is a necessary and sufficient, that set U be finite, and the number of its elements is equaled to k. We note that situations, where tolerance is also being an equivalence should not be excluded from considering, however formally we have the formally a "fuzzy sets weakening".

Proposition 2. Let's define that fuzzy set $C_{\mu_1}^{(n)}$ has *k-proximity* with the fuzzy set $C_{\mu_2}^{(n)}$, if cuts of the sets are differing no more, than $k - 1$ elements.

The *k-proximity* relation also is a tolerance relation in $[0, 1]_n^U$. If $k = 1$ the fuzzy sets *proximity* means the matching of their cuts. For finite and universal sets, the concepts of *similarity* and *proximity* are equivalent. Really, if N – the number of all elements of U, then to sets $C_{\mu_1}^{(n)}$ and $C_{\mu_2}^{(n)}$ (for an arbitrary and fixed n) were *k-similar*, there is a necessary and sufficient then they have an $(N - k)$*-proximity*. If set U is infinite, then two sets $C_{\mu_1}^{(n)}$ and $C_{\mu_2}^{(n)}$ have k_1*-proximity* while some k_1 and may not be k_2*-similar* for any k_2. At the same time two k_2*-similar* while some k_2 sets have a k_1*-proximity* for any k_1. Thus, the *similarity* relation is a narrowing, generally speaking, the *proximity* relation. It is appropriate also to refer to the notion of Hamming distance. Exactly, *k-similarity* of $C_{\mu_1}^{(n)}$ and $C_{\mu_2}^{(n)}$ means, that Hamming distance between those sets is not exceeded $k - 1$.

The second approach is based on topology structures, that particularly more general and have more naturally expressed of the proximity concept.

Proposition 3. Let's define that fuzzy sets $C_{\mu_1}^{(n)}$ and $C_{\mu_2}^{(n)}$ have ε*-likeness* ($\varepsilon > 0$), if the distance between the cuts of these sets in a considered metric function space does not exceed ε.

The relation of ε*-likeness* also a tolerance relation in $[0, 1]_n^U$. For example, referring to one of the standard metrics, we can say that the fuzzy sets $C_{\mu_1}^{(n)}$ and $C_{\mu_2}^{(n)}$ have ε*-likeness*, if in the cuts of these sets the absolute value of the difference of membership degree of any element $u \in U$ does not exceed ε. This form of proximity refers to uniform convergence topology.

The relations proposed above, namely *k-proximity*, *k-similarity* and ε*-likeness* we join to the one concept – *resemblance*.

Based on *resemblance*, below we propose the main result, influencing to information granulation and situation awareness accordingly.

Proposition 4. *Information granule* in $[0, 1]_n^U$ is its subset Gr, such as any element belonging to it have a *resemblance*.

Consequence 1. Any subset of information granule, containing, at least, two elements constitutes an information granule too.

Proposition 5. Information granule in $[0, 1]_n^U$ defined as *closed* if it is not contained in any other information granule.

Obviously, those closed information granules are maximum elements in partial ordered set (relatively to set-theoretic inclusion) of all granules in $[0, 1]_n^U$, that is, the subject and only those subsets of $[0, 1]_n^U$, which coincide with the intersection of classes of tolerance of all elements belonging to them.

The set of all fuzzy sets in any three-element set is geometrically identified with a single cube in R^3. For the *n – similarity* relation the class of tolerance of any fuzzy set representing some point of a cube is an association of three pieces which are lying in this cube, passing through this point and such that one of them is parallel to abscissa axis, another – ordinate axes, the third – axes of *z*-coordinates. The closed information granules are various intervals of the detailed view.

4. Some applications to incident awareness in *ITS*

As a linguistic variable ℵ [21], associated with some incident in *ITS* we will consider, for example, the risk of failure from one of the existing local area network (*LAN*), where the attacker impacts through a set of malware, in an antagonistic strategic zero-sum game. Universal set U here can be composed of N divisions (elements of) the scale of the danger of the incident. Therm-set T (ℵ) is a collection of X linguistic variable values ℵ, each of them is associated with its meaning ! (X), which constitutes a fuzzy set. The membership function $\mu\ (u)$ of fuzzy set ! $(X)$for each division u levels of incident danger U assigns a value of the compatibility of this item (its membership degree of fuzzy sets). For example, if $N = 10$ the scale interval $u = 5$ compatible with the value of the linguistic variable can "***significantly dangerous"*** to have an expert assessment in the form of 0.8, and compatible with the value of the division $u = 8$ is "***extremely dangerous***" - the number of 0.9.

It is natural to consider as a linguistic variable representing the accuracy of the presence of unauthorized violations into the targeted *ITS LAN* network. Here, alone and in the first place should be held gradation against "evil" intent, and so on. The validity of incident and its awareness we define as the linguistic scale: "***not worthy***", "***weak alert***", "***border alert***", "***weak danger***". "***danger***", and so on. We can consider these questions separately only for attempts to enter the *LAN* or in the context held penetrations.

As the second example, we mention the analysis of hardware failures in *MICS* referring to 1st ("***critical***") and 2nd ("***dangerous***") categories, which took place on the railway transport. As a linguistic variable, may appear a quality of hardware operation. The values of the linguistic variable can be: "***unacceptably low level***", "***low level of acceptable quality***", and so forth.

5. Conclusion

We conclude our paper with several findings. At first we analyze various scientific papers in fuzzy sets area and found that despite a large amount of research this area has a lack of fuzzy methods in situation awareness. Also, we have revealed a need for the new fuzzy sets foundations suited to applications to new classes of *ITS* developed *JSC* "*Russian Railways*". At the second, in the paper, we revise the foundation concepts of fuzzy sets similarity. Moreover, finally, we propose a novel analytical framework for formal definitions of fuzzy sets similarity and as the main result, we formulated a resemblance relation between fuzzy sets in a fresh formal way and based on algebraic Brouwerian lattices and topology structures.

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