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Mathematical Simulation in the Scope of Prefeasibility Study of Fuselages of Long Distance Aircraft

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Abstract

This article proposes mathematical model of fuselage for long для long distance aircraft at the stage of feasibility study. The structure of mathematical model has been developed which includes performance function, constraints and variables, considers for the influence of design and geometry, weight, flight conditions, strength and ergonomic properties of aircraft. In order to estimate the aircraft efficiency integral weight coefficient has been developed which accounts for the values of input characteristics. Mathematical model makes it possible to solve specific problem of feasibility study of fuselage for long distance aircraft. The dependences on the basis of the mathematical model have been used for CAD feasibility study of fuselage of long distance aircraft.

Keywords: fuselage; mathematical simulation; parametric synthesis; long distance aircraft; parametric optimization; feasibility study; computer aided design.

INTRODUCTION

Mathematical simulation is a peculiar component of engineering analysis and design of aircraft (AC). Let us consider that the mathematical model of AC fuselage at the stage of feasibility study is the formally described system which, reflecting or reproducing the analyzed object (fuselage design), can substitute it so that its study provides all required information about this object [1].

After thorough formulation of the feasibility study target it is necessary to compose the most complete list of solutions. Generated constraints on flight characteristics should be taken into consideration upon decision making. Application of analytical methods makes it possible to perform parametric study of influence of variation of design parameters and constraints on technical and economical performances of AC, and this is used upon searching for rational parameters of AC, satisfying selected estimation criterion of efficiency of design solutions. The main application field of analytical methods is not only general AC design: determination of main parameters, aerodynamic layout, configuration, center of gravity location and so on, but optimization of individual design elements of AC: searching for their best internal structure, for instance, structural layout and the like [2, 3].

Upon optimization of individual design element it is required to select criterion of its estimation. Weight is selected as such criterion. Herewith, it is assumed that parameters which determines its form and relation with other function blocks are permanent, though quite frequently it leads to erroneous results, since they are not obvious and unique. For instance, decrease in weight of AC fuselage due to improvement of its structural layout leads to violation of accepted conditions of permanence: loads on fuselage, properties of passenger (or cargo) compartment and so on are varied which were established upon general optimization of AC. It will required for variation of parameters which determine fuselage shape, and this cannot be achieved on the basis of criterion of minimum weight.

Thus, we propose to apply integral weigh coefficient as criterion for accounting of input parameters of all property groups.

EXPERIMENTAL

Upon formalization of feasibility study of fuselage for long distance AC the process is described by mathematical model which makes it possible to transfer from solution of single problems to design of unified complex system. Mathematical simulation upon feasibility study of fuselage for long distance AC is comprised of several stages:

a) apprehension of mathematical model;
b) verification of the model by experiments;
c) comparison of mathematical and theoretical studies of the model;
d) verification of the model adequacy.

The revealed fuselage properties and their parameters which influence on CAD feasibility study of fuselage form the structure of mathematical model consisting of individual sets of properties:

– design and geometry properties;
– weight properties;
– flight conditions;
– aerodynamic properties;
– strength properties;
– ergonomics properties.

Using this structure generating principle, let us arrange algorithm of feasibility study of new or modified existing fuselage of long distance AC with consideration for customer requirements (prefeasibility study of development of fuselage design concept), statistical data on this type of AC and airworthiness requirements for AC of a given category (Regulations AP-25), Figure 1.

RESULTS AND DISCUSSION

Modification of existing and development of new variants of fuselages are aimed at achievement of reasonable parameter values of property groups, whereas parametric synthesis of fuselage at preset layout is a procedure of optimization [4].

Among parameters of property groups mentioned above it is impossible to highlight a single one, which can characterize fuselage properties to the most complete extent, thus, we selected additive complex criterion which contains normalizing factor: weight coefficient for accounting of output parameters of different physical dimensionality. The additive criterion is as follows:

$$z(x) = \sum_{j=1}^{m} k_{w.i.} \cdot y_j(x), \qquad (1)$$

where $k_{w.i.}$ is the integral weight coefficient; *m* is the total number of criteria; *j* is the sequential number of partial criterion; $y_j(x)$ is the normalized value of the *j*-th partial criterion.

In additive criteria the performance function is generated by addition of output parameters, converted to dimensionless additives. This is carried out by introduction of normalizing factors: weight coefficients. Normalizing is required for combination of several output parameters, in general case of different physical dimensionality.

The structure of mathematical model of AC fuselage at the stage of prefeasibility study is comprised if the performance function, constraints, and variables.

The mathematical model is based on performance function, which expresses quantitatively the fuselage quality, thus, it is also known as quality function or optimality criterion. Generation of performance function was made with accounting for various output parameters of fuselage design.

Design of new AC types or modification of existing ones is an iterative procedure. Difficulties of calculation of AC weight at design stage includes contradiction: AC gross weight cannot be determined without determination of weight of all components, and weight of each component cannot be determined without known AC gross weight. Decrease in gross weight is of tactical significance for AC, since it leads also to decrease in specific consumption of fuel, materials, energy and design labor consumption of AC [5].

The task of prefeasibility study of fuselage for AC rational type with accounting for minimization of AC gross weight at the same (or increased) number of passengers is presented by the performance function:

$$f = m_0 \cdot k_{w.i.} \to \min \qquad (2)$$

where m_0 is the AC gross weight, kg, equation; $k_{w.i.}$ is the integral weight coefficient.

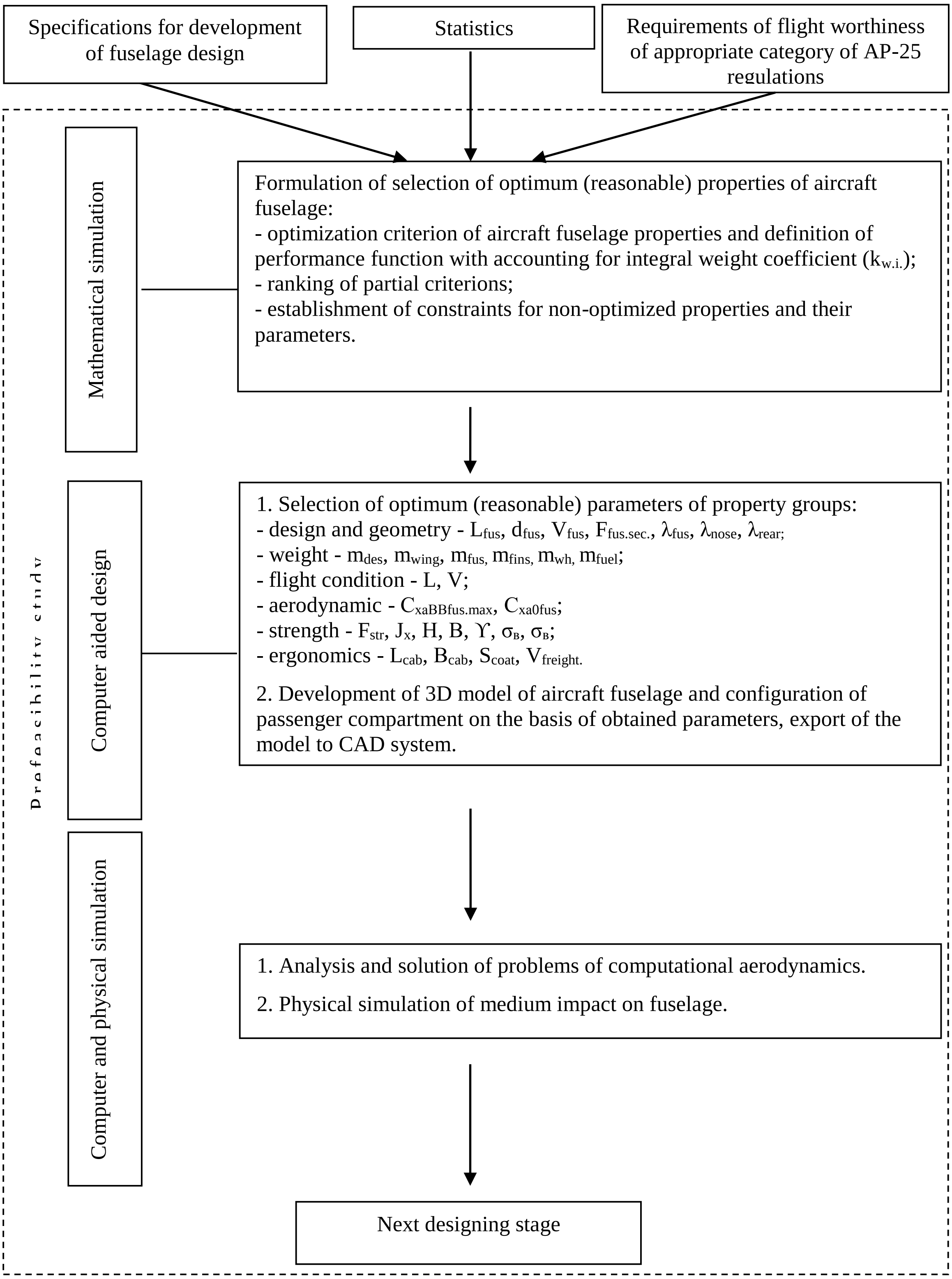

Figure 1. Algorithm of prefeasibility study of fuselage of long distance aircraft.

Efficiency factor of AC is the integral weight coefficient. The integral weight coefficient is required for accounting of non-optimized properties and their parameters at the stage of prefeasibility design of fuselage of long distance AC. The AC integral weight coefficient is a function of several properties of weight efficiency:

$$k_{w.i.} = \overline{(m_{comm.load} / m_0)} \cdot \kappa_{weight} + \overline{(m_{emp.oper.} / n_{pass})} \cdot \kappa_{weight} + \overline{\mathrm{S}_{\mathrm{wett.}}} \cdot \kappa_{weight} + \overline{T_{side}} \cdot \kappa_{weight} + \overline{T_{shell}} \cdot \kappa_{weight} \quad (3)$$

where $\overline{(m_{comm.load} / m_0)}$ is the load ratio in relative units, one of performances of AC efficiency, the ratio of useful load to calculated gross weight, that is, to that at which the AC strength satisfies the requirements of Regulations of flight worthiness (AP-25); $\overline{(m_{emp.oper.} / n_{pass})}$ is the ratio of AC operational empty weight to the number of passenger seats, weight flow rate per one passenger seat in relative units; $\overline{\mathrm{S}_{\mathrm{wett.}}}$ is the wetted surface area of airframe, in relative units, accounts for interrelation between geometrical properties and AC gross weight; $\overline{T_{side}}$ is the line tangential force in fuselage side plates, relative units; $\overline{T_{shell}}$ – is the line tangential force in fuselage shell, relative units; k_{weight} is the weight coefficient (rank) of each individual parameter.

The weight coefficients are determined by the rule of tank normalization [5]

$$\sum_{i=1}^{n} \kappa_{weight} = 1 \quad (4)$$

where *n* is the total number of ranks; *i* is the rank consecutive number.

Therefore, the importance of each component of integral weight coefficient is determined with regard to minimization of gross weight. In order to determine weight coefficients we will use the rule of ranking, applied in the cases when it is not required to perform accurate measurements for estimation of objects. The weight coefficients (ranks) are determined according to the following stages, Table 1 [5]:

- arrangement of components of integral weight coefficient in descending order, its numbers define only the sequence order of components from the most important to the least important one;
- establishment of degree of preference of one component in comparison with another (ranked series of criteria) — the bottom line of Table 1 and establishment of their position k_i in the ranked series (column 2, Table 1);
- determination of significance coefficients r_i of each component. For ranked series of components in descending order with at least one equivalence sign $\approx$ the values of significance coefficients r_i for equivalent components are calculated as arithmetic average:

$$r_4 = r_5 = \frac{4+5}{2} = 4.5 \quad (5)$$

For other components the significance coefficients equal to the value of their position in ranked series:

$$r_i = k_i \qquad (6)$$

the results are written into column 3, Table 1;
– for each significance coefficient r_i the coefficient t_i (I = 5), column 4, Table 1, is calculated as follows:

$$t_i = \exp(\frac{-r_i}{I}) \qquad (7)$$

– the sum T is calculated as follows using data in Table 1:

$$T = \sum_{i=1}^{i=I} t_i \qquad (8)$$

where *i* is the component of integral weight coefficient, *I* is the consecutive number of component of integral weight coefficient;
– the ranks (weight coefficients) k_{weight} are calculated as follows

$$k_{weight} = t_i / T \qquad (9)$$

Let us determine parametric, discretizing, and functional constraints applied to fuselage parameters in order to provide preset functions [6, 7].

Functional constraints leads to further decrease in the allowable design subspace and make it shape more complicated. These conditions provide required values of these or those properties and economic performances. Functional constraints in this case are the strength conditions applied to thickness of covering of side plates and shells of fuselage I calculated cross sections and cross section of stringers.

Table 1: Weight coefficients (ranks)

Components of integral weight coefficient		position k_i	r_i	t_i	Rank k_{weight}
1		2	3	4	5
i = 1	$\overline{(m_{comm.load} / m_0)}$ – load ratio	1	1	0.81	0.28
i = 2	$\overline{(m_{emp.oper.} / n_{pass})}$ – weight flow per one passenger seat	2	2	0.67	0.23
i = 3	$\overline{S_{wett.}}$ – wetted surface area of airframe	3	3	0.54	0.19
i = 4	$\overline{T_{side}}$ – line tangential force in fuselage side plates	4	4.5	0.40	0.14
i = 5	$\overline{T_{shell}}$ – line tangential force in fuselage shell	5	4.5	0.40	0.14
			Total	2.81	1.0

$$\{ (\overline{m_{comm.load} / m_0}) \succ (\overline{m_{emp.oper.} / n_{pass}}) \succ \overline{S_{wett.}} \succ (\overline{T_{side}} \approx \overline{T_{shell}}) \}$$

The covering thickness of side plates and shell of fuselage in calculated cross section is determined as follows:

$$\delta \geq \frac{T}{\tau^{cov.}_{destr.}} \quad (10)$$

where $\tau^{cov.}_{destr.}$ – is the destructing tangential stress of covering, $\tau^{cov.}_{destr.} = 0.3 \cdot \sigma^{cov.}_{destr.}$; T is the line tangential force in side plates or shells of.

Stringer cross section is determined as follows:

$$m\left(F_{str.} + \phi_{cov.} \cdot b_{str.} \cdot \delta\right) \geq \frac{M^p_z}{\sigma^{str.}_{destr.} \cdot H} \quad (11)$$

where *m* is the number of shell stringers, assumed to be $m = 19$, $\varphi_{cov.}$ is the reducing coefficient of covering, $\varphi_{cov.} = 0.7$, $b_{str.}$ is the stringer step, $b_{str.} = 0.25\, m$

Let us introduce parametric constraints for coordinates of freights placed in fuselage:

$$0 \leq x_i \leq l_{\Phi} \quad (12)$$

where x_i is the coordinate of the *i*-th freight along the fuselage length onto horizontal projection of distance ($L_{takeoff}$, km), covered by AC in the time $t_{takeoff.}$

$$500 \leq L_{takeoff.} \leq 1000 \quad (13)$$

Seat pitch (L_{seat}, m) is selected from the following range:

$$0{,}5 \leq l_{seat} \leq 1 \quad (14)$$

Coordinates of freight position in fuselage are selected from the following range :

$$0 \leq x_i \leq l_{fus.} \quad (15)$$

where x_i is the coordinate of freight position in fuselage.

Discretizing constraints are as follows:

$$x_j = \{x_{j1}, x_{j2}, ..., x_{jm}\}, \quad (16)$$

where x_j is the *j*-th parameter of technical object; x_{j_k} are the allowable values of the *j*-th parameter (*k=1,2,..m*).

Such constraints are applied to the parameters either in relation with their physical essence or according to the requirements of valid state standards. In this work the discretizing constraints are applied to the following parameters [8]:

1) Coefficients used in calculations and stipulated by Federal flight regulations (AP-25: Airworthiness of aircrafts, strength standards, requirements to flight properties, steadiness and controllability, requirements to aircraft designs), for instance:

- k_{beam}*={1,0, 1,1}*, where k_{beam} is the coefficient accounting for the number of beams; *1,0; 1,1* are the allowable values of the coefficient;

- k_4*={0,0; 0,003}*, where k_4 is the coefficient accounting for luggage transportation type; *0,0; 0,003* are the allowable values of the coefficient;

2) Those stipulated by stringer geometrical properties:

- surface area of cross section (F_{0str}, cm^2), stringer inertia moment (Jx, cm^4), height (H, mm) and width (B, mm) of its flanges, since they depend on the selected profile, and parameters of covering materials (elasticity modulus (E), material ultimate strength (σ_u) and so on).

Covering of fuselage, spar webs, stringers are designed on the basis of regular sheets of aluminum alloys (Russian Standard GOST 21631-76E) [9].

3) Those stipulated by physical constraints characterizing properties of environment (the parameters are selected according to ISA International Standard Atmosphere, Russian Standard GOST 4404-81):

– air density (ρ, kg/m^3) at cruise altitude;
– speed of sound (a, km/h) at cruise altitude;
– air kinematic viscosity ($\upsilon_н$, m^2/s) at cruise altitude;
– air temperature (T, K or t, °C) at cruise altitude.

Let us write the constraints as follows:

$$
Constraints \begin{cases}
\delta \geq \dfrac{T}{\tau_{destr.}^{cov.}}; \\
m(F_{str.} + \phi_{cov.} b_{str.} + \delta) \geq \dfrac{M_z^p}{\sigma_{destr.} H}; \\
\tau_{destr.}^{cov.} = 0.3\sigma_{destr.}^{cov.}; \\
0 \leq x_i \leq l_{fus.}; \\
500 \leq L_{takeoff} \leq 1000; \\
0,5 \leq l_{seat} \leq 1; \\
\kappa_{beam} = \{1,0;\ 1.1\}; \\
\kappa_4 = \{0,0;\ 0,003\} \\
F_{str}^0;\ J_x; H;\ .B;\ \delta; \\
\sigma_u;\ \sigma_p;\ E;\ \gamma.
\end{cases} \qquad (17)
$$

Variables are the parameters determined in input data:

- design and geometry (length — l_{fus}, m; diameter (d_{fus}, m), volume (V_{fus}, m^2) of fuselage);
- weight (weight of AC units, relative and absolute (m_{des}, m_{wing}, m_{fus}, m_{fins}, m_{wheel}, m_{fuel}, kg));

- flight conditions (flight distance (L, km) and cruise speed (V, km/h));
- aerodynamic (wave ($C_{xa\ BBfus.max}$) and drag ($C_{xa\ 0fus.}$) run);
- strength (stringer cross section properties (F_{str}^0, J_x, H, B, δ) and covering thickness (σ_B, $\sigma_{destr.}$, γ));
- ergonomic (length (L_{cab}, m) and width (B_{cab}, m) of passenger compartment, coat room surface area (S_{coat}, m^2), volume ($V_{freight}$, m^3), required for freights and so on).

Let us write the variables as follows:

$$Variables\begin{cases} d_{fus.};V_{fus.};l_{fus.}; \\ m_{des};m_{wing};m_{fus.};m_{fins};m_{fuel}; \\ L;V \\ C_{xaBBfus.max};C_{xa0fus.}; \\ F_{str.}^{0};J_{x};H;B;\delta; \\ L_{cab};B_{cab};S_{coat};V_{freight.} \end{cases} \quad (18)$$

In order to use the mathematical model in the course of prefeasibility study of fuselage of long distance AC it is required that the model describes correctly in term of quality and quantity properties of simulated object (fuselage), that is, it should be adequate to the simulated object [10, 11]. In order to verify the adequacy of the mathematical model to the actual object it is required to compare output values of this object with output values of the model. Hence, prior to solution of optimum task it is necessary to verify the adequacy of the available model.

Adequacy of the above mathematical model is verified experimentally upon study of properties of fuselage of long distance AC using computer and physical simulation.

The results of the researches are adopted for implementation in OAO Aviatekhpriemka (Moscow, Russia) and as training program of Chair of Aircrafts, Aerospace Institute, OGU.

CONCLUSIONS

1. Mathematical model is developed distinguishing from the existing ones by accounting for interrelations between AC properties revealed upon studies and exerting the highest influence of AC fuselage, including:
   - design and geometry;
   - weight;
   - aerodynamic;
   - flight conditions;
   - strength;
   - ergonomics.

2. The applied integral weight coefficient upon prefeasibility study of fuselage of long distance AC makes it possible to consider for non-optimized parameters and is a function of several properties of weight efficiency.

3. Dependences obtained on the basis of the mathematical model make it possible to develop software modules of structure optimization.

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