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Mathematical Modeling of the Robot Manipulator with Four Degrees of Freedom

Tatiana Victorovna Zudilova
ITMO National Research University (ITMO University), Head of Software Development Chair, 197101 Saint Petersburg, 12 Gastello Str., office 311, Russian Federation.

Sergei Evgenievich Ivanov
ITMO National Research University (ITMO University), Department of Information Systems, 197101 Saint Petersburg, 49 Kronverksky Pr., Russian Federation.

Abstract

The nonlinear mathematical model of an industrial robot manipulator with four degrees of freedom is developed. For creation of the equations of robot manipulator is applied the matrix method and Lagrange equations of the second kind.
The robot manipulator is designed to create automated complexes for service of devices, for removing and installing the equipment, change of details and tools.
The kinematic structure of the robot manipulator and the dynamic characteristics of motion for robot manipulator are defined. The mathematical model is presented by nonlinear system of four ordinary differential equations of second order. The analytical solution of the nonlinear system is obtained by method of polynomial transformations. The power of actuators and generalized forces for move the gripping device at a given point of space for a certain period of time are defined.

Keywords: the nonlinear mathematical model, the industrial robot manipulator, the dynamic characteristics of the robot manipulator.

INTRODUCTION

The robots manipulators are widely used in many fields and spheres of human activity: assembly production, machining, heat treatment.

The industrial robot manipulator is an automatic machine, which includes the manipulator with four degrees of freedom. The robot manipulator performs functions similar to the human hand and is controlled by operator or automatically. In the industrial production are used the technological manipulators for operations of assembly and welding. The auxiliary robots manipulators are used for maintenance of the main technological equipment for lifting and transporting [1-4].

The structure of robot manipulator consists of main elements, connected with each other: the base, the stand, the mechanical arm, the gripping device. The working body of the manipulator is the gripping device. The elements connecting the base with the working body constitute a kinematic chain of the robot manipulator. The two adjacent elements constitute a kinematic pair.

The number of degrees of freedom provides the movement of the device of gripping in any point of the working area. The main function of the robot manipulator is determined by kinematic scheme.

NONLINEAR MATHEMATICAL MODEL OF THE INDUSTRIAL ROBOT MANIPULATOR

The industrial robot manipulator M20P for robotic complex is considered. The robot manipulator is shown in Figure 1 consists of a base, the stand, hands, the gripping device and actuators for moving and turns.

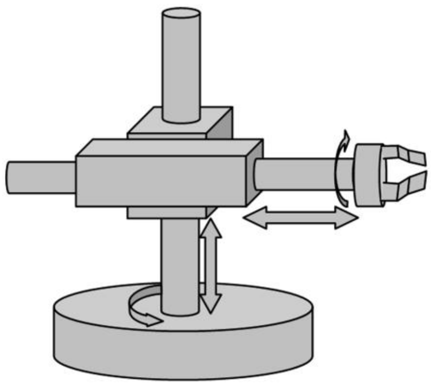

Figure 1: The industrial robot manipulator M20P

The industrial robot manipulator M20P has two translational and two rotational kinematic pairs. The elements of manipulator are numbered starting with fixed element - the base with number of zero. The motion of clamp-unclamp for the gripping device is not considered.

The kinematic scheme for industrial robot manipulator is shown in Figure 2 and consists of four moving parts.

We introduce the relative coordinate system associated with elements, with the origin at the points: O_1, O_2, O_3, O_4. The initial coordinate system O_0 we correlate with the fixed element.

For generalized coordinates we accept: the angle of rotation around the rack - q_1, lifting height - q_2, the length of arms - q_3, the angle of gripping device - q_4.

The system of dynamic equations of industrial robot manipulator is obtained.

We apply the matrix method and dynamic Lagrange equations in matrix form to produce the equations of motion [5-7].

The transition from the coordinate system O_0 to O_1 occurs by rotation around z axis at an angle q_1.

The transition from the coordinate system O_1 to O_2 occurs by rotation around z axis at an angle π, by displacement along z axis on q_2 and by rotation around x axis at an angle $\pi / 2$.

The transition from the coordinate system O_2 to O_3 occurs by displacement along z axis on q_3. The transition from the coordinate system O_3 to O_4 occurs by rotation around z axis at an angle q_4, by displacement along z axis on s and by rotation around x axis at an angle $-\pi / 2$.

We introduce the radius vector of points $O_i, (i = 1, 2, 3, 4)$ in the *i*-local coordinate system:

$$R_i = \begin{bmatrix} x_i & y_i & z_i & 1 \end{bmatrix}^T .$$

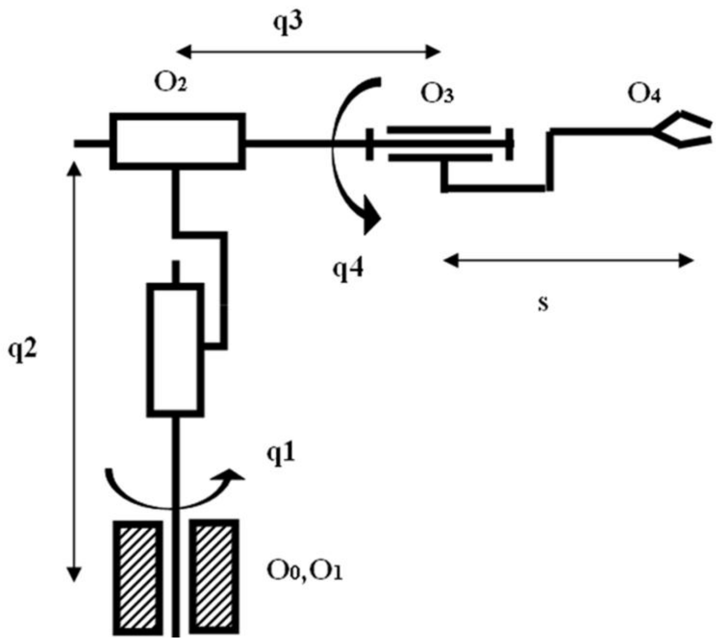

Figure 2: The kinematic scheme

The communication of radius vectors in the coordinate system *i*-1 and *i* by means of the transition matrix $A_{i-1,i}$: $R_{i-1} = A_{i-1,i} R_i$.

The transition matrix from point O_0 to point O_1 is of the form:

$$A_{01} = \begin{pmatrix} \cos(q_1) & -\sin(q_1) & 0 & 0 \\ \sin(q_1) & \cos(q_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The matrices of the transition from point O_1 to point O_2 and of the transition from point O_2 to point O_3 are of the form:

$$A_{12} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & q_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad A_{23} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The transition matrix from point O_3 to point O_4 is of the form:

$$A_{34} = \begin{pmatrix} \cos(q_4) & 0 & -\sin(q_4) & 0 \\ \sin(q_4) & 0 & \cos(q_4) & 0 \\ 0 & -1 & 0 & s \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The transition matrix from point O_0 to point O_2 is defined as the product of matrices: $A_{02} = A_{01}A_{12}$,

$$A_{02} = \begin{pmatrix} -\cos(q_1) & 0 & -\sin(q_1) & 0 \\ -\sin(q_1) & 0 & \cos(q_1) & 0 \\ 0 & 1 & 0 & q_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The transition matrix from point O_0 to point O_3 is obtained as the product of matrices: $A_{03} = A_{01}A_{12}A_{23}$,

$$A_{03} = \begin{pmatrix} -\cos(q_1) & 0 & -\sin(q_1) & -q_3\sin(q_1) \\ -\sin(q_1) & 0 & \cos(q_1) & q_3\cos(q_1) \\ 0 & 1 & 0 & q_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The transition matrix from point O_0 to point O_4 is defined as the product of matrices: $A_{04} = A_{01}A_{12}A_{23}A_{34}$,

$$A_{04} = \begin{pmatrix} -\cos(q_1)\cos(q_4) & \sin(q_1) & \cos(q_1)\sin(q_4) & -q_3\sin(q_1) - s\sin(q_1) \\ -\cos(q_4)\sin(q_1) & -\cos(q_1) & \sin(q_1)\sin(q_4) & q_3\cos(q_1) + s\cos(q_1) \\ \sin(q_4) & 0 & \cos(q_4) & q_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The origin of coordinates O_4 associated with gripping device in the fixed coordinate system O_0 associated with a base of rack is defined by coordinates:

$$x_{O4} = -q_3\sin(q_1) - s\sin(q_1),\, y_{O4} = q_3\cos(q_1) + s\cos(q_1),\, z_{O4} = q_2 .$$

We denote the coordinates of the gripper fingers in a coordinate system O_4 as (x_4, y_4, z_4), then in a fixed coordinate system O_0 the coordinates of the gripper fingers can be represented in the form of:

$$x_{04} = -q_3 \sin(q_1) - x_4 \cos(q_1)\cos(q_4) + z_4 \cos(q_1)\sin(q_4) - s\sin(q_1) + y_4 \sin(q_1)$$
$$y_{04} = q_3 \cos(q_1) - x_4 \sin(q_1)\cos(q_4) + z_4 \sin(q_1)\sin(q_4) + s\cos(q_1) - y_4 \cos(q_1)$$
$$z_{04} = q_2 + x_4 \sin(q_4) + z_4 \cos(q_4)$$

The coordinates of the point x_0, y_0, z_0, given in a fixed coordinate system O_0 in the coordinate system O_4 associated with a rack, determined in the form of:

$$x_4 = -\frac{q_2 \sin^2(q_1)\sin(q_4) + q_2 \cos^2(q_1)\sin(q_4) + x\cos(q_1)\cos(q_4) +}{\left(\sin^2(q_1) + \cos^2(q_1)\right)\left(\sin^2(q_4) + \cos^2(q_4)\right)}$$
$$-\frac{y\sin(q_1)\cos(q_4) - z\sin^2(q_1)\sin(q_4) - z\cos^2(q_1)\sin(q_4)}{\left(\sin^2(q_1) + \cos^2(q_1)\right)\left(\sin^2(q_4) + \cos^2(q_4)\right)},$$

$$y_4 = -\frac{-q_3 \tan^2(q_1)\sin^2(q_4) - q_3 \tan^2(q_1)\cos^2(q_4) - s\tan^2(q_1)\sin^2(q_4)}{\left(\tan^2(q_1) + 1\right)\left(\sin^2(q_4) + \cos^2(q_4)\right)} -$$
$$\frac{-s\tan^2(q_1)\cos^2(q_4) - x\tan(q_1)\sec(q_1)\cos^2(q_4) - x\tan(q_1)\sec(q_1)\sin^2(q_4)}{\left(\tan^2(q_1) + 1\right)\left(\sin^2(q_4) + \cos^2(q_4)\right)} +$$
$$\frac{y\sec(q_1)\cos^2(q_4) + y\sec(q_1)\sin^2(q_4) - q_3\sin^2(q_4) - q_3\cos^2(q_4) - s\sin^2(q_4) - s\cos^2(q_4)}{\left(\tan^2(q_1) + 1\right)\left(\sin^2(q_4) + \cos^2(q_4)\right)},$$

$$z_4 = -\frac{q_2 \cos^2(q_1)\sec(q_4) + q_2 \sin^2(q_1)\sec(q_4) - x\cos(q_1)\tan(q_4)\sec(q_4)}{\left(\tan^2(q_4) + 1\right)\left(\sin^2(q_1) + \cos^2(q_1)\right)} +$$
$$\frac{-y\sin(q_1)\tan(q_4)\sec(q_4) - z\cos^2(q1)\sec(q4) - z\sin^2(q1)\sec(q4)}{\left(\tan^2(q4) + 1\right)\left(\sin^2(q1) + \cos^2(q1)\right)}.$$

We calculate the kinetic energy of each unit and the total energy.
For determine the kinetic energy of *i*-unit we use the matrix formula with transition matrices:

$$T_i = \frac{1}{2} tr(\dot{A}_{0i} H_i \dot{A}_{0i}^T), \qquad (1)$$

where H_i - the inertia matrix of *i* - unit,

$$H_i = \begin{bmatrix} J_{ixx} & J_{ixy} & J_{ixz} & m_i x_i \\ J_{iyx} & J_{iyy} & J_{iyz} & m_i y_i \\ J_{izx} & J_{izy} & J_{izz} & m_i z_i \\ m_i x_i & m_i y_i & m_i z_i & m_i \end{bmatrix},$$

where m_1, m_2, m_3 - mass of units,

x_i, y_i, z_i - coordinates of the centers of gravity of unit in the local coordinate system,

$J_{ixx}, J_{iyy}, J_{izz}$ - elements of the inertia tensor of unit, a relatively their own axes,

J_{xi}, J_{yi}, J_{zi} - moments of inertia of unit relative to the axes.

The kinetic energy for units is determined by a matrix formula (1):

$T_1 = \dot{q}_1^2(0.5J_{1xx} + 0.5J_{1yy}), T_2 = \dot{q}_1^2\left(0.5J_{2xx} + 0.5J_{2zz}\right) + 0.5\dot{q}_2^2 m_2,$

$T_3 = 0.5\dot{q}_2^2 m_3 + 0.5\dot{q}_3^2 m_3 + \dot{q}_1^2\left(0.5J_{3xx} + 0.5J_{3zz} + m_3 q_3\left(0.5q_3 + z_3\right)\right),$

$T_4 = \dot{q}_1^2\left(0.25J_{4xx} + 0.5J_{4yy} + 0.25J_{4zz} + 0.5m_4 q_3^2 + m_4 q_3 s + 0.5m_4 s^2\right) +$

$\dot{q}_1^2(0.25J4xx - 0.25J4zz)\cos(2q_4) + 0.5dq2^2 m4 + 0.5dq3^2 m4 + \dot{q}_4^2(0.5J_{4xx} + 0.5J_{4zz}).$

The total kinetic energy is equal to the sum of kinetic energies for units, taking into account the equalities:

$J_{iyy} + J_{izz} = J_{xi}, J_{ixx} + J_{izz} = J_{yi}, J_{ixx} + J_{iyy} = J_{zi}$

$T = T_1 + T_2 + T_3 + T_4 =$

$\dot{q}_1^2\left(m_4\left(sq_3 + 0.5q_3^2 + 0.5s^2 + 0.0025\right) + m_3\left(0.5q_3^2 + 0.0025\right) + 0.0025m_1 + 0.0025m_2\right) +$

$\left(0.5m_2 + 0.5m_3 + 0.5m_4\right)\dot{q}_2^2 + 0.5m_3\dot{q}_3^2 + 0.5m_4\dot{q}_3^2 + 0.0025m_4\dot{q}_4^2.$

We calculated the potential energy for each unit and the total energy.
In matrix form, the potential energy of unit is defined in the form of:

$$P_i = -m_i G^T A_i R_i,$$

where $R_i = \begin{bmatrix} x_i & y_i & z_i & 1 \end{bmatrix}^T$ - column matrix of the coordinates of the center of gravity unit,

$G_i^T = \begin{bmatrix} 0 & 0 & -g & 0 \end{bmatrix}$ - matrix row for the acceleration of free fall.

The total potential energy is defined by the form: $P = gq_2\left(m_2 + m_3 + m_4\right)$.

We apply Lagrange equations in matrix form for equations of motion:

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i}-\frac{\partial T}{\partial q_i}+\frac{\partial P}{\partial q_i}=Q_i, \qquad (2)$$

where Q_1, Q_2, Q_3, Q_4 - generalized forces created for unit.

By substituting the kinetic energy, the potential energy and the generalized forces in the Lagrange equations (2), we obtain equations for robot manipulator with four degrees of freedom:

$$\ddot{q}_1(t)\left(m_4\left(-2sq_3(t)-q_3(t)^2-s^2-0.005\right)+m_3\left(-q_3(t)^2-0.005\right)-0.005m_1-0.005m_2\right)+$$
$$\dot{q}_1(t)\dot{q}_3(t)\left(\left(-2m_3-2m_4\right)q_3(t)-2m_4s\right)+Q_1=0,$$
$$\left(m_2+m_3+m_4\right)\left(\ddot{q}_2(t)+g\right)=Q_2,$$
$$m_4\left(\ddot{q}_3(t)-s\dot{q}_1(t)^2\right)+\left(-m_3-m_4\right)q_3(t)\dot{q}_1(t)^2+m_3\ddot{q}_3(t)=Q_3,$$
$$0.005m_4\ddot{q}_4(t)=Q_4.$$

For the analysis of nonlinear mathematical models apply different analytical methods [8-11]: averaging method, small parameter method, harmonic balance method, Van der Pol method, Krylov-Bogolyubov method, the Poincare perturbation method, the polynomial transformation method.

We obtained the analytical solution of equations by polynomial transformations method:

$$q_1(t)=\left(\frac{Q_1}{2\left(m_1+m_2+m_3+m_4\right)}+\frac{m_4 s}{36\left(m_3+m_4\right)}\right)t^2,$$
$$q_2(t)=\frac{Q_2-g(m_2+m_3+m_4)}{2(m_2+m_3+m_4)}t^2,$$
$$q_3(t)=\left(\frac{Q_3}{2\left(m_3+m_4\right)}+\frac{m_4^3 s^3}{18\left(m_3+m_4\right)^3}\right)t^2, \qquad (3)$$
$$q_4(t)=\frac{100Q_4}{m_4}t^2.$$

Figure 3 shows graphs of the generalized coordinates (q_1-thick line, q_2-dashed line, q_3 -dot-dashed line, q_4 -dotted line) with the following parameters for robot manipulator:

$$m_1=100, m_2=20, m_3=10, m_4=40, s=0.5, Q_1=8, Q_2=700, Q_3=5, Q_4=0.02$$

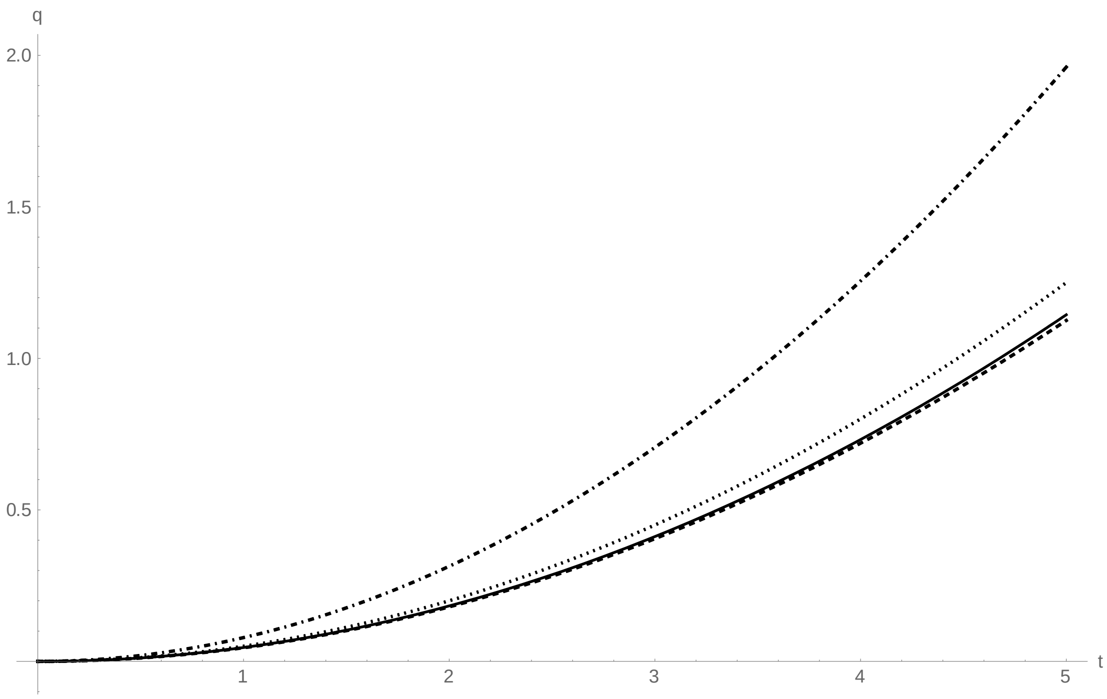

Figure 3: The dynamics of the generalized coordinates: q_1, q_2, q_3, q_4

We denote the coordinates of the object for gripping in a fixed coordinate system O_0 for (x, y, z). For bring the object to the point (x, y, z) necessary that the generalized coordinates are:

$$q_1 = 2\tan^{-1}\left(\frac{\sqrt{x^2+y^2}+y}{x}\right), q_2 = z,$$

$$q_3 = s + \sqrt{x^2+y^2}, q_4 = \pi. \quad (4)$$

We equate the solution (3) and the expression for the generalized coordinates (4), we obtain the system for determining the generalized forces: Q_1, Q_2, Q_3, Q_4.

We identified the necessary generalized forces Q_1, Q_2, Q_3, Q_4 for moving the gripper to the specified point (x, y, z) in the time interval t_k :

$$Q_1 = (m_1+m_2+m_3+m_4)\left(\frac{m_4 s}{18(m_3+m_4)} + \frac{4}{t_k^{\ 2}}\tan^{-1}\left(\frac{\sqrt{x^2+y^2}+y}{x}\right)\right),$$

$$Q_2 = 2(m_2+m_3+m_4)\frac{z}{t_k^{\ 2}} + g(m_2+m_3+m_4),$$

$$Q_3 = 2\left(m_3 + m_4\right)\frac{s+\sqrt{x^2+y^2}}{t_k^{\,2}} + \frac{m_4^3 s^3}{9\left(m_3+m_4\right)^2}, Q_4 = \frac{\pi m_4}{100 t_k^{\,2}}.$$

The powers of actuators F_1, F_2, F_3, F_4 for move the gripping device at a given point (x, y, z) of space for a certain period of time t_k are defined.

$$F_1 = \left(m_1+m_2+m_3+m_4\right)\left(\frac{m_4 s t_k}{9\left(m_3+m_4\right)} + \frac{4}{t_k^{\,2}}\tan^{-1}\left(\frac{\sqrt{x^2+y^2}+y}{x}\right)\right)\times$$

$$\left(\frac{m_4 s}{18\left(m_3+m_4\right)} + \frac{4}{t_k^{\,2}}\tan^{-1}\left(\frac{\sqrt{x^2+y^2}+y}{x}\right)\right),$$

$$F_2 = \left(m_2+m_3+m_4\right)\left(\frac{2gz}{t_k} + \frac{4z^2}{t_k^{\,3}}\right),$$

$$F_3 = \left(m_3+m_4\right)\left(s+\sqrt{x^2+y^2}\right)\left(\frac{2m_4^3 s^3}{9t_k\left(m_3+m_4\right)^3} + \frac{4\left(s+\sqrt{x^2+y^2}\right)}{t_k^{\,3}}\right),$$

$$F_4 = \frac{\pi^2 m_4}{50 t_k^{\,3}}$$

CONCLUSION

The mathematical model of industrial robot manipulator with four degrees of freedom is developed. The robot manipulator is designed to create automated systems for service devices, for removing and installing the equipment, change details and tools. For creation of the equations of robot manipulator we used the method of matrices and matrix Lagrange equations of the second kind. The mathematical model is presented nonlinear system of four ordinary differential equations of second order. The analytical solution of the nonlinear system is obtained by polynomial transformations method. The kinematic structure and dynamic characteristics of robot manipulator are defined. The power of actuators and generalized forces generated by actuators for moving the gripper at a given point in space are found.

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