

Optimization of Directional and Energetic Properties of Diffraction Antenna

**Aleksandr Vitalievich Ostankov, Sergey Anatolievich Antipov
Konstantin Aleksadrovich Razinkin**

*Voronezh State Technical University,
Moskovskii prospekt, 14, Voronezh, 394026, Russia.*

Abstract

Antenna systems on the basis of open emitting transmission lines, including diffraction antennas, are very promising for SHF and EHF. Characteristic sizes of such antennas can exceed wavelength by one or two orders of magnitude, which restricts application of commercial variants of electromagnetic simulation systems and leads to necessity of development of computing algorithms for analysis and synthesis of diffraction antennas. This article develops efficient numerical model of linear diffraction antenna, its principle is based on transformation of surface wave of plane dielectric waveguide into bulk wave by comb array. Comb array is simulated by finite number of square grooves in conducting shield. Assuming excitation of dielectric waveguide by heterogeneous wave, the diffraction problem is reduced by Fourier method to system of linear equations with regard to groove waveguide modes. The model accounts for boundary effects, arbitrary location of the grooves, generally of various depth and width. Adequacy of the computing algorithm was justified by comparing of directional and energetic properties of the developed model with corresponding data obtained by other researchers as well as with results of study of antenna simulators. Calculated relationships are given which enable reasonable selection of parameters of emitting aperture with equidistant comb (thickness of dielectric waveguide, size of air gap, step and depth of grooves), which provide maximum antenna efficiency (product of radiation efficiency by aperture efficiency) upon excitation of dielectric waveguide by main E - or H -wave. Possibility and viability of the computing model in combination with genetic algorithm for optimization of characteristics of antenna radiation and parametric synthesis of its radiating aperture have been demonstrated. Results of comb profile optimization have been obtained (groove depth) in terms of criterion of antenna efficiency maximum and

minimum level of lateral lobes of angular pattern. The results confirm possibility of generation of optimum amplitude distribution on antenna aperture by means of comb profiling (at uniform gap between waveguide and comb), which can be used for implementation of diffraction antennas with improved properties.

Keywords: Diffraction Antenna, Dielectric Waveguide, Conducting Comb, Computing Algorithm, Model Adequacy, Antenna Efficiency, Angular Pattern, Level of Lateral Radiation.

INTRODUCTION

The existing approach to designing of radiating SHF and EHF systems is closely related with simulation technique. Comprehensive software packages, systems of electromagnetic simulation are being developed and improved: CST Microwave Studio, Ansoft HFSS etc. [1]. Practicability of simulation, preceding physical experiment, is undoubtful and stipulated by saving of materials and time, related with possibility to avoid the influence of errors of simulation and measurements, and so on. The question is as follows: does the use of simulators based on universal analytical methods always guarantee obtaining of reliable results as soon as possible? The answer is obviously ambiguous. What is meant here is the analysis of antenna of moderate electrical dimensions, for instance, with the sizes of $10\lambda^3$ (λ is the wavelength, radiated or received by antenna), then the answer to the question is affirmative. Another situation takes place when analyzing antennas of relatively large sizes.

Let us consider the antenna, operating principle of which is based on the use of spatial transformation of surface wave of open transmission line into bulk wave by means of diffraction grating. Let us assume a diffraction grating in the form of metallic comb: conducting shield with square grooves oriented in perpendicular to propagation of excitation wave. The transmission line of surface wave is a plane dielectric waveguide. Such antenna, due to its high radiation efficiency which can be as high as 0.95 and higher, is rather promising in millimeter range and the most efficient in resonant frequency range, where the comb step is comparable with the wavelength [2-6]. The sizes of radiating aperture of such antennas significantly depend on implemented directional properties and may be as high as $(50 \times 50 \times 2)\lambda^3$. Let us assume that antenna with radiating aperture of the mentioned sizes should be analyzed in CST Microwave Studio 3-D electromagnetic simulator. Then, reliable results can be obtained by dissection of simulated volume λ^3 into at least 50^3 elementary cubes [1], each of them (with consideration for generality of adjacent cubes) is described by 18 field components. Then, the matrix for operation of electromagnetic simulator should be comprised of 10 billion complex variables. It is obvious that the iteration series with such matrix, required for analysis of established process, cannot be executed in reasonably restricted time interval on common PC.

It should be recognized that, despite intensive development of electromagnetic simulators, analysis and especially synthesis of highly directional diffraction antennas are still troublesome. In this regard it is very important to develop mathematical models accounting for specificity of such radiating systems, reflecting main running processes and avoiding difficulties in computations, inevitable upon application of any particular electromagnetic simulators.

Such models exist. Thus, for analysis of radiating structures, containing finite periodic grating as a main element, the researchers in [7] propose a variant of finite difference solution of two-dimensional scalar initial boundary value problem, based on development of transport operators determining space–time transformation of waves in Floquet channels and segments of free space. In [8] impedance analog of electromagnetic space based on the concept of discrete grating vacuum was applied for analysis of finite comb structure with variable groove depths along aperture, excited by cylindrical wave; wide band antenna with constant phase center was developed on its basis. However, it should be recognized that the developed in [7] and [8] highly efficient algorithms require for professional software and subsequent optimization of code, that is, development of dedicated electromagnetic simulator.

The work [9] discusses the solution of problem of scattering of eigen electric waves of dielectric plane waveguide (PDW) on finite comb in ideally conducting shield by singular integral equation. Using the discrete singularity method, the obtained integral equation was reduced to the system of linear algebraic equations (SLAE) with regard to the coefficients of function introduced for description of field in groove aperture. However, upon such approach the calculation of SLAE coefficients, which are double space integrals, require for significant time expenditures, which hinders active application of optimization procedures in the frames of the developed algorithm.

A simpler and at the same time rigorous model is presented below aimed at analysis and optimization of radiating properties of linear antenna containing conducting comb covered with dielectric plate in the mode of transformation of surface wave into bulk wave. The aim of this work is to justify possibility of implementation of efficient computer aided algorithm on the basis of the obtained interrelations for analysis, and, in the case of application of genetic algorithm, synthesis of diffraction antenna.

MATHEMATICAL FORMALIZATION OF THE ANALYSIS OF DIFFRACTION ANTENNA

Let us analyze diffraction antenna in the mode of electromagnetic wave radiation on the assumption of preset surface wave of planar dielectric waveguide.

Let the radiating antenna aperture (Fig. 1), as well as the field of surface wave exciting the radiating aperture are not homogeneous along the O_y axis, then the analysis can be reduced to two-dimensional case. In addition, let us assume that the conducting surfaces of radiating aperture contacting with the surface wave field have infinite conductivity, that is, there are no thermal losses in comb at all.

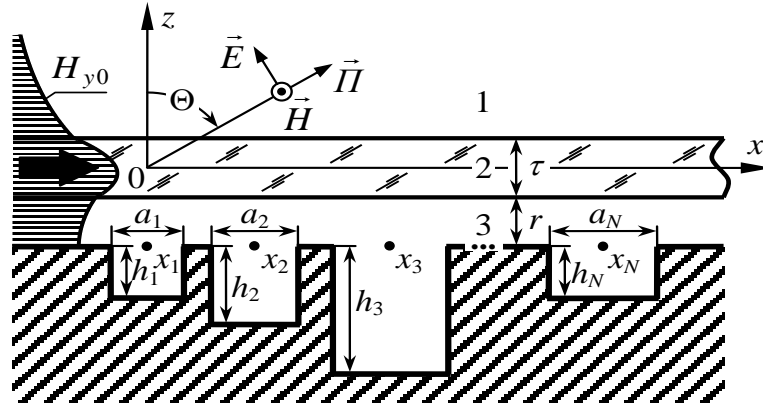


Figure 1. Simulated configuration of diffraction antenna radiating aperture in transversal cross section.

Let the comb is formed by finite number N of square grooves with arbitrary sizes $a_k \times h_k$, where $k = 1 \dots N$. The grooves are made in unbounded shield and located along the Ox axis, non-uniformly in general case. The coordinates of groove centers are x_k . The layer of homogeneous and non-magnetic dielectric of finite thickness τ with relative dielectric permeability ε_τ is located above the conducting comb at the distance r . The impact parameter of dielectric waveguide r is constant in any transversal cross section of the antenna. Taking this into account, the considered radiating structure is completely coordinate structure, and upon scattering of plane wave of arbitrary polarization on this structure it is sufficient to analyze two partial cases of polarization of primary wave. Let us assume subsequently that the dielectric waveguide is coordinated with excitation device, not depicted in Fig. 1.

Let us assume that heterogeneous slow E -wave of main type is propagated along the Ox axis. Let us assume that the existence of finite comb as local heterogeneity in shield does not influence on the value of longitudinal constant of propagation β_0 of primary wave. Then, it can be determined by solution of dispersion equation for the wave of E -type of plane dielectric waveguide located above solid metallic surface.

Assuming that the time dependence is preset as $\exp(-j\omega t)$, let us write expressions for magnetic field of eigen E -wave of dielectric waveguide in partial regions 1 - 3 (Fig. 1), as in [10]:

$$\begin{aligned} H_{y0}^{(1)} &= A \cdot \exp[j\gamma_0(z - \tau/2)] \cdot \exp(j\beta_0 x), \\ H_{y0}^{(2)} &= [B_1 \cdot \exp(j\eta_0 z) + B_2 \cdot \exp(-j\eta_0 z)] \cdot \exp(j\beta_0 x), \\ H_{y0}^{(3)} &= (C_1 \cdot \exp[j\gamma_0(z + \tau/2)] + C_2 \cdot \exp[-j\gamma_0(z + \tau/2)]) \cdot \exp(j\beta_0 x), \end{aligned} \quad (1)$$

where $\gamma_0 = \sqrt{k_0^2 - \beta_0^2}$ ($\text{Im } \gamma_0 > 0$, $\text{Re } \gamma_0 = 0$), $\eta_0 = \sqrt{k_0^2 \cdot \varepsilon_\tau - \beta_0^2}$ are the transversal constant of surface wave propagation in free space and in dielectric layer; $k_0 = 2\pi/\lambda$ is the constant of wave propagation in free space.

According to the Maxwell equation [10]

$$E_x^{(n)} = \frac{1}{-j\omega \cdot \epsilon_0 \cdot \epsilon^{(n)}} \cdot \frac{\partial H_y^{(n)}}{\partial z} \tag{2}$$

Let us determine tangential components of electric field. Setting the component $E_{x0}^{(3)}$ to zero in the plane $z = -\tau/2 - r$ (on ideally conducting surface), we obtain that:

$$C_1 = C_2 \cdot \exp(2j\gamma_0 r). \tag{3}$$

Let us satisfy continuity condition by the components $H_{y0}^{(n)}, E_{x0}^{(n)}$ at the region boundaries ($z = \pm\tau/2$) and exclude unknown variables A, B_1, B_2, C_1 from the obtained system of equations. The resultant equation:

$$1 - \frac{j}{2} \text{tg}(\eta_0 \tau) \cdot \left(\frac{\gamma_0 \epsilon_\tau}{\eta_0} \cdot [1 - \exp(j2\gamma_0 r)] + \frac{\eta_0}{\gamma_0 \epsilon_\tau} \cdot [1 + \exp(j2\gamma_0 r)] \right) = 0 \tag{4}$$

is a dispersion equation with regard to longitudinal propagation constant β_0 of eigen E -wave of shielded PDW. Equation (4) has the only positive root at $\tau < \lambda / (2\sqrt{\epsilon_\tau - 1})$ [10]. The mentioned correlation for the thickness τ corresponds to the condition of existence of one propagating mode of plane dielectric waveguide upon excitation by wave of E -type. Let us assume that it is satisfied. Magnetic component of primary wave field (1) in region 3 with consideration for Eq. (3) us as follows:

$$H_{y0}^{(3)} = H_0 \cdot \cos[\gamma_0(z + \tau/2 + r)] \cdot \exp(j\beta_0 x),$$

where $H_0 = 2C_2 \cdot \exp(j\gamma_0 r)$. Coefficient C_2 can be conveniently set as:

$$C_2 = e^{-j\gamma_0 r} \cdot \left(\frac{W_0 \cdot \beta_0}{k_0} \cdot \left\{ r \cdot [1 + \text{sinc}(2\gamma_0 r)] + \cos^2(\gamma_0 r) \cdot \left[\tau \cdot \frac{1 + \text{sinc}(2\eta_0 \tau)}{\epsilon_\tau} - \frac{\cos^2(\eta_0 \tau)}{j\gamma_0} \right] + \frac{\gamma_0^2 \cdot \epsilon_\tau^2}{\eta_0^2} \times \right. \right. \\ \left. \left. \times \sin^2(\gamma_0 r) \cdot \left[\tau \cdot \frac{1 - \text{sinc}(2\eta_0 \tau)}{\epsilon_\tau} - \frac{\sin^2(\eta_0 \tau)}{j\gamma_0} \right] - \tau \cdot \sin(2\gamma_0 r) \cdot [\gamma_0 \tau \cdot \text{sinc}^2(\eta_0 \tau) + j\epsilon_\tau \cdot \text{sinc}(2\eta_0 \tau)] \right\} \right)^{-\frac{1}{2}},$$

where $\text{sinc}(x) = \sin(x)/x$, $W_0 = 120\pi$; herewith, cumulative energy flow of surface wave via yOz plane

$$P_0 = \frac{1}{2} \cdot \text{Re} \left(\int_{-\tau/2-r}^{-\tau/2} E_{z0}^{(3)} \cdot H_{y0}^{(3)} dz + \int_{-\tau/2}^{\tau/2} E_{z0}^{(2)} \cdot H_{y0}^{(2)} dz + \int_{\tau/2}^{+\infty} E_{z0}^{(1)} \cdot H_{y0}^{(1)} dz \right)$$

is identically equal to unit for any geometrical parameters of radiating aperture.

Magnetic component of diffraction field above radiating structure ($z \geq \tau/2$, region 1) has continuous space spectrum and can be presented in the form of integral Fourier expansion in plane waves [11]:

$$H_y^{(1)} = \int_{-\infty}^{+\infty} A(\beta) \cdot \exp[j\gamma(\beta)(z - \tau/2)] \cdot \exp(j\beta x) d\beta,$$

where $A(\beta)$ is the spectral density of magnetic field, proportional to the amplitude of partial plane wave with the propagation constants β (along the Ox axis) and $\gamma(\beta) = \sqrt{k_0^2 - \beta^2}$ (along the Oz axis). In order to satisfy the radiation conditions it should be required that $\text{Im}\gamma(\beta) > 0$ at $\beta > k_0$ and $\text{Re}\gamma(\beta) > 0$ at $\beta < k_0$ [11]. The scattering field in dielectric layer ($|z| \leq \tau/2$, region 2) and in air gap between comb and dielectric waveguide ($-\tau/2 - r \leq z \leq -\tau/2$, region 3) should be also described by the Fourier integrals applying the Brillouin concept:

$$H_y^{(2)} = \int_{-\infty}^{+\infty} \{B_1(\beta) \cdot \exp[j\eta(\beta)z] + B_2(\beta) \cdot \exp[-j\eta(\beta)z]\} \cdot \exp(j\beta x) d\beta,$$

$$H_y^{(3)} = \int_{-\infty}^{+\infty} \{C_1(\beta) \cdot \exp[j\gamma(\beta)(z + \tau/2)] + C_2(\beta) \cdot \exp[-j\gamma(\beta)(z + \tau/2)]\} \cdot \exp(j\beta x) d\beta,$$

where $\eta(\beta) = \sqrt{k_0^2 \cdot \epsilon_\tau - \beta^2}$ is the propagation constant (along the Oz axis) of plane partial wave in dielectric layer.

Let us describe the diffraction field inside each k -th comb groove by discrete combination of waveguide modes with amplitudes $D_m^{(k)}$ and propagation constants $\zeta_m^{(k)} = \sqrt{k_0^2 - (m\pi/a_k)^2}$ (along the Oz axis):

$$H_y^{(4,k)} = \sum_{m=0}^{+\infty} D_m^{(k)} \cdot \cos[\zeta_m^{(k)}(z + \tau/2 + r + h_k)] \cdot f_m^{(k)}(x),$$

where $f_m^{(k)}(x)$ is the modal function providing automatic execution of boundary conditions on metal walls of groove:

$$f_m^{(k)}(x) = \begin{cases} \cos[(m\pi/a_k) \cdot (x - x_k + a_k/2)], & |x - x_k| \leq a_k/2, \\ 0, & |x - x_k| > a_k/2. \end{cases} \quad (5)$$

Tangential components of electric field above the comb $E_x^{(n)}$ and its grooves $E_x^{(4,k)}$ are determined by Eq. (2).

Tangential components of total magnetic and electric field (with consideration for primary wave) should satisfy the continuity conditions at the boundaries of partial regions:

$$\text{at } z = \tau/2 \quad (-\infty < x < \infty) \quad (6)$$

$$A(\beta) = B_1(\beta) \cdot \exp[j\eta(\beta)\tau/2] + B_2(\beta) \cdot \exp[-j\eta(\beta)\tau/2],$$

$$A(\beta) \cdot \gamma(\beta) = \{B_1(\beta) \cdot \exp[j\eta(\beta)\tau/2] - B_2(\beta) \cdot \exp[-j\eta(\beta)\tau/2]\} \cdot \eta(\beta) / \varepsilon_\tau,$$

at $z = -\tau/2$ ($-\infty < x < \infty$)

$$B_1(\beta) \cdot \exp[-j\eta(\beta)\tau/2] + B_2(\beta) \cdot \exp[j\eta(\beta)\tau/2] = C_1(\beta) + C_2(\beta), \tag{7}$$

$$\{B_1(\beta) \cdot \exp[-j\eta(\beta)\tau/2] - B_2(\beta) \cdot \exp[j\eta(\beta)\tau/2]\} \cdot \eta(\beta) / \varepsilon_\tau = [C_1(\beta) - C_2(\beta)] \cdot \gamma(\beta),$$

at $z = -\tau/2 - r$

$$\int_{-\infty}^{+\infty} \{C_1(\beta) \cdot \exp[-j\gamma(\beta)r] + C_2(\beta) \cdot \exp[j\gamma(\beta)r]\} \cdot \exp(j\beta x) d\beta + H_0 \cdot \exp(j\beta_0 x) = \tag{8}$$

$$= \sum_{m=0}^{+\infty} D_m^{(k)} \cdot \cos(\zeta_m^{(k)} h_k) \cdot f_m^{(k)}(x), \quad x \in [x_k - a_k/2, x_k + a_k/2], \quad k = 1 \dots N,$$

$$\int_{-\infty}^{+\infty} \{C_1(\beta) \cdot \exp[-j\gamma(\beta)r] - C_2(\beta) \cdot \exp[j\gamma(\beta)r]\} \cdot \gamma(\beta) \cdot \exp(j\beta x) d\beta = \tag{9}$$

$$= j \sum_{k=1}^N \sum_{m=0}^{+\infty} D_m^{(k)} \cdot \zeta_m^{(k)} \cdot \sin(\zeta_m^{(k)} h_k) \cdot f_m^{(k)}(x), \quad -\infty < x < \infty.$$

Components of the field of primary surface wave in Eqs. (6), (7) ($z = \pm\tau/2$) are mutually compensated, the constituent $E_{x0}^{(3)}$ in Eq. (9) ($z = -\tau/2 - r$) is identically equal to zero.

Eliminating of coefficients $B_{1,2}(\beta), C_{1,2}(\beta)$ makes it possible to reduce Eqs. (6) - (9) to pair system of functional equations of the following type:

$$\left\{ \int_{-\infty}^{+\infty} A(\beta) \cdot \nu(\beta) \cdot \exp(j\beta x) d\beta + H_0 \cdot \exp(j\beta_0 x) = \sum_{m=0}^{+\infty} D_m^{(k)} \cdot \cos(\zeta_m^{(k)} h_k) \cdot f_m^{(k)}(x), \quad k = 1 \dots N, \tag{10} \right.$$

$$\left. \int_{-\infty}^{+\infty} A(\beta) \cdot \mu(\beta) \cdot \gamma(\beta) \cdot \exp(j\beta x) d\beta = j \sum_{k=1}^N \sum_{m=0}^{+\infty} D_m^{(k)} \cdot \zeta_m^{(k)} \cdot \sin(\zeta_m^{(k)} h_k) \cdot f_m^{(k)}(x), \tag{11} \right.$$

where

$$\begin{aligned} \left\{ \begin{array}{l} \nu(\beta) \\ \mu(\beta) \end{array} \right\} &= \cos[\eta(\beta)\tau] \cdot \exp[-j\gamma(\beta)r] \times \\ &\times \left[1 - j \frac{1}{2} \operatorname{tg}[\eta(\beta)\tau] \cdot \left(\frac{\gamma(\beta)\varepsilon_\tau}{\eta(\beta)} \cdot [1 \pm \exp(j2\gamma(\beta)r)] + \frac{\eta(\beta)}{\gamma(\beta)\varepsilon_\tau} \cdot [1 \mp \exp(j2\gamma(\beta)r)] \right) \right]. \end{aligned}$$

Using orthogonality of eigen functions from Eqs. (10) - (11), it is possible to obtain the system of equations which contain only coefficients $D_m^{(k)}$ [12]. With this aim let us multiply both members of Eq. (11) by the function $\exp(-j\beta'x)$ and integrate over x in infinite limits. Accounting for subsequent substitution of β' by β qw obtain the equation for spectral density $A(\beta)$ of field above the structure:

$$A(\beta) = \frac{j}{2\pi \cdot \mu(\beta) \cdot \gamma(\beta)} \cdot \sum_{k=1}^N \sum_{m=0}^{+\infty} D_m^{(k)} \cdot \zeta_m^{(k)} \cdot \sin(\zeta_m^{(k)} h_k) \cdot I_m^{(k)}(\beta), \quad (12)$$

where

$$\begin{aligned} I_m^{(k)}(\beta) &= \int_{-\infty}^{+\infty} f_m^{(k)}(x) \cdot \exp(-j\beta x) dx = \\ &= 0.5a_k \cdot \exp[-j(\beta x_k + m\pi/2)] \cdot (\text{sinc}[(\beta a_k + m\pi)/2] + (-1)^m \cdot \text{sinc}[(\beta a_k - m\pi)/2]). \end{aligned}$$

Let us multiply Eq. (10) by the function $f_s^{(q)}(x)$ determined by Eq. (5) and integrate over x in the limits from $x_q - a_q/2$ to $x_q + a_q/2$. We obtain as follows:

$$\int_{-\infty}^{+\infty} A(\beta) \cdot \nu(\beta) \cdot I_s^{(q)}(\beta) d\beta + H_0 \cdot I_s^{(q)}(\beta_0) = 0.5a_q (1 + \Delta_s^0) \cdot D_s^{(q)} \cdot \cos(\zeta_s^{(q)} h_q), \quad (13)$$

where $q=1\dots N$, $s=0\dots\infty$, Δ_s^i is the Kronecker symbol;

$\bar{I}_s^{(q)}(\beta)$ is the function, complexly conjugated with $I_m^{(k)}(\beta)$, where the second index m corresponds to s , and k to index q .

After elimination of spectral density $A(\beta)$ described by Eq. (12) from Eq. (13) we obtain the system of linear algebraic equations with regard to amplitudes of waveguide modes $D_m^{(k)}$, excited in comb grooves:

$$\sum_{k=1}^N \sum_{m=0}^{\infty} D_m^{(k)} \cdot [\zeta_m^{(k)} \cdot \sin(\zeta_m^{(k)} h_k) \cdot \sigma_{m,s}^{(k,q)} + j0.5a_k \Delta_k^q \Delta_m^s (1 + \Delta_s^0) \cdot \cos(\zeta_m^{(k)} h_k)] = jH_0 \cdot \bar{I}_s^{(q)}(\beta_0), \quad (14)$$

where $\sigma_{m,s}^{(k,q)}$ are the coefficients determining interrelation between the grooves [12]:

$$\sigma_{m,s}^{(k,q)} = \frac{1}{j2\pi} \cdot \int_{-\infty}^{+\infty} \Gamma_{m,s}^{(k,q)}(\beta) d\beta, \quad (15)$$

$$\text{where } \Gamma_{m,s}^{(k,q)}(\beta) = \frac{\Xi_{m,s}^{(k,q)}(\beta)}{\mu(\beta)} = \frac{j\nu(\beta)}{\mu(\beta) \cdot \gamma(\beta)} \cdot I_m^{(k)}(\beta) \cdot I_s^{(q)}(\beta).$$

The system of equations (14) is reduced by restriction of number of groove waveguide modes ($m, s=0\dots M$). Amplitudes of waveguide modes $D_m^{(k)}$ determined by Eq. (14) are applied for calculation of spectral density $A(\beta)$ of scattered field and radiation field on its basis.

Let us determine relations suitable for calculation of angular pattern in far zone and power radiated by antenna. The latter can be determined as energy flow of scattered field via the xOy plane:

$$P_{\Sigma} = \frac{1}{2} \cdot \text{Re} \int_{-\infty}^{+\infty} E_x^{(1)} \cdot H_y^{(1)} dx,$$

this integral is the Fourier transformation (spectrum) of product $\mu(x)$ by $v(x)$:

$$P_{\Sigma} = \frac{W_0\pi}{k_0} \cdot \text{Re} \int_{-\infty}^{+\infty} \mu(x) \cdot v(x) \cdot \exp(-j\xi \cdot x) \Big|_{\xi=0} dx,$$

where

$$\begin{aligned} \mu(x) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} A(\beta) \cdot \gamma(\beta) \cdot \exp[j\gamma(\beta)(z - \tau/2)] \cdot \exp(j\beta x) d\beta = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} G_{\mu}(\beta) \cdot \exp(j\beta x) d\beta; \\ v(x) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \bar{A}(-\beta) \cdot \exp[-j\gamma(\beta)(z - \tau/2)] \cdot \exp(j\beta x) d\beta = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} G_{\nu}(\beta) \cdot \exp(j\beta x) d\beta. \end{aligned}$$

The spectrum of product $\mu(x)$ by $v(x)$ can be substituted by integral convolution of spectra $G_{\mu}(\beta)$, $G_{\nu}(\beta)$ of these functions [13]:

$$P_{\Sigma} = \frac{W_0\pi}{k_0} \cdot \text{Re} \int_{-\infty}^{+\infty} G_{\mu}(\beta) \cdot G_{\nu}(\xi - \beta) \Big|_{\xi=0} d\beta = \frac{W_0\pi}{k_0} \cdot \text{Re} \int_{-\infty}^{+\infty} |A(\beta)|^2 \cdot \gamma(\beta) d\beta.$$

Since $\gamma(\beta)$ has real part only at $|\beta| \leq k_0$ ($\beta = k_0 \sin \Theta$, Θ is the observation angle counted clockwise from normal to comb in cylindrical coordinates $\rho = \sqrt{x^2 + z^2}$, $\Theta = \text{arctg}(x/z)$, y), then

$$P_{\Sigma} = \pi k_0 W_0 \cdot \int_{-\pi/2}^{\pi/2} |A(k_0 \sin \Theta)|^2 \cdot \cos^2 \Theta d\Theta, \tag{16}$$

hence, non-normalized angular pattern of antenna in terms of power is

$$F^2(\Theta) = \pi k_0 W_0 \cdot |A(k_0 \sin \Theta)|^2 \cdot \cos^2 \Theta. \tag{17}$$

For the case of excitation of radiating aperture of dielectric waveguide by H -wave all required interrelations were obtained by the same approach.

IMPLEMENTATION OF MAIN COMPUTATIONAL PROCEDURES

After setting of initial data the dispersion Eq. (4) is solved numerically and propagation constant β_0 of surface wave is determined. Infinite number of equations (and unknown variables, respectively) in Eq. (14) requires for its correct reduction. In order to provide the solution accuracy with the error of power balance less than 0.1 % it is sufficient to consider for all waveguide modes, propagating in the grooves, and three–four attenuating modes. Thus obtained dimensionality of SLAE is not high, in resonant frequency range ($\lambda \approx x_{k+1} - x_k$) it is not higher than $5N \times 5N$. The generated system of equations is solved by conventional approach.

The main time expenditures upon generating of Eq. (14) fall into calculation of the array of coefficients $\sigma_{m,s}^{(k,q)}$ determining electrodynamic interrelation of the grooves. It follows from Eq. (15) that $\sigma_{m,s}^{(k,q)} = \sigma_{s,m}^{(q,k)}$; in addition, for equidistant comb $\sigma_{m,s}^{(k,q)} = \sigma_{m,s}^{(k+i,q+i)}$ is valid (i is an integer number), which agrees with the reciprocity principle for antenna arrays. These circumstances make it possible to reduce the time

of calculation of the array $\sigma_{m,s}^{(k,q)}$ by $2 \dots N$ times. The required coefficients $\sigma_{m,s}^{(k,q)}$ are calculated by direct numerical integration of complex function $\Gamma_{m,s}^{(k,q)}(\beta)$ over real axis of complex variable β . Infinite integration path is reduced according to asymptotics $|\Gamma_{m,s}^{(k,q)}(\beta)|$ (recommended interval $|\beta| \leq 10k_0$), dissected into intervals stipulated by specificity $\Gamma_{m,s}^{(k,q)}(\beta)$, the fact is taken into account that at $|\beta| > k_0\sqrt{\varepsilon_\tau}$ $\Gamma_{m,s}^{(k,q)}(-\beta) = \Gamma_{m,s}^{(k,q)}(\beta)$, then integration with automatic step selection and open ends is applied. While integrating in the intervals $(-k_0\sqrt{\varepsilon_\tau}, -k_0)$, $(k_0, k_0\sqrt{\varepsilon_\tau})$, the singularities of the function $\Gamma_{m,s}^{(k,q)}(\beta)$ should be preliminary eliminated in poles $|\beta| = \beta_0$, corresponding to the roots of Eq. (4). In order to satisfy radiation conditions the pole $\beta = -\beta_0$ should be bypassed by semi circumference from above, and $\beta = \beta_0$ — from below [14]. In particular, the integral along interval $\beta_0 \mp \Delta\beta$, containing the pole β_0 , can be calculated as:

$$\int_{\beta_0 - \Delta\beta}^{\beta_0 + \Delta\beta} \Gamma_{m,s}^{(k,q)}(\beta) d\beta = \int_{\beta_0 - \Delta\beta}^{\beta_0 + \Delta\beta} \left[\Gamma_{m,s}^{(k,q)}(\beta) - \frac{\Phi_0}{\beta - \beta_0} \right] d\beta + j\pi \cdot \Phi_0, \quad (18)$$

where $\Phi_0 = \Xi_{m,s}^{(k,q)}(\beta_0) / [d\mu(\beta_0) / d\beta]$ is the remainder of function $\Gamma_{m,s}^{(k,q)}(\beta)$ in pole β_0 . At low $\Delta\beta$ ($\Delta\beta \rightarrow 0$) the first integral in Eq. (18) can be considered as zero. A peculiar feature of the coefficients $\sigma_{m,s}^{(k,q)}$ is their independence on the groove depth, which facilitates optimization of groove profile without recalculation of array $\sigma_{m,s}^{(k,q)}$, that is, as soon as possible.

After solution of SLAE (14) the angular pattern $F^2(\Theta)$ of antenna and its main parameters are calculated according to Eq. (17): boresight Θ_0 , half-power bandwidth $\Delta\Theta_{0.5}$, maximum level of lateral lobes ξ_m (LLL). Then, using Eq. (16), we calculate radiation power P_Σ , radiation efficiency $\eta = P_\Sigma / P_0$, standing wave ratio k_{SW} (SWR), stipulated by existence of surface wave reflected from comb, "plane" directional factors (DF) and aperture efficiency ν (AE) [15], and, finally, total efficiency of antenna determined as $\Xi = \eta \cdot \nu = \lambda \cdot F^2(\Theta_0) / (L \cdot \cos \Theta_0)$, where L is the comb length.

While selecting software for implementation of computing algorithm, we, at the expense of calculation speed, preferred MathCAD, user friendly system of computer mathematics not requiring for knowledge of professional languages. Herewith, the time of analysis of 50λ antenna using 2 GHz single core PC was less than one minute. Upon implementation of the model as executable code the analysis time can be reduced by 2–3 times.

For multiextremal optimization of antenna properties and its parametric synthesis it would be reasonable to apply modified genetic algorithm proposed in [16]. Software support of the algorithm is available in Internet [17], it is a stand-alone module (function subprogram), readily integrated into user program. A peculiar feature of the modified algorithm is its capability to reach global minimum with the least number of accesses to objective function at required probability. Objective function is the function supplementing total efficiency Ξ to unit, or LLL ξ_m , either the function obtained by additive combination of requirements to Ξ and ξ_m .

JUSTIFICATION OF ADEQUACY OF THE COMPUTING ALGORITHM AND RESULTS OBTAINED ON ITS BASIS

Reliability of data on the basis of the presented model was verified by their comparison with calculations by other researchers and experiments. Comparison of the results with similar calculations in [9, 18] demonstrated their complete identity (in the ranges of graphical accuracy), disagreements with experimental data [19] were in the range of the measurement error with all assumptions and restrictions of the developed model.

In Fig. 2 solid lines illustrate calculated properties of *H*-polarized radiation of diffraction antenna obtained on the basis of method of singular integral equations [9]. Radiating aperture of the considered antenna at the wavelength of $\lambda = 4.1$ mm contained ideally conducting comb with 25 equal grooves ($N=25$) with the width of $a = 0.421\lambda$ and the depth of $h = 0.426\lambda$, located equidistantly at the step of $d = x_{k+1} - x_k = 0.891\lambda$. Plane waveguide was made of ideal dielectric material with $\varepsilon_\tau = 2.5$, its thickness was $\tau = 1$ mm. Dots in the same figure depict simulated results on the basis of the above computing algorithm. It can be seen in the figure that the calculated data obtained by both electrodynamic models coincide nearly completely. It should be mentioned that a portion of data (at $r > 0.5$ mm) corresponds to the most complicated for analysis mode of Bragg diffraction with radiation along normal to antenna aperture in the range of width of angular pattern.

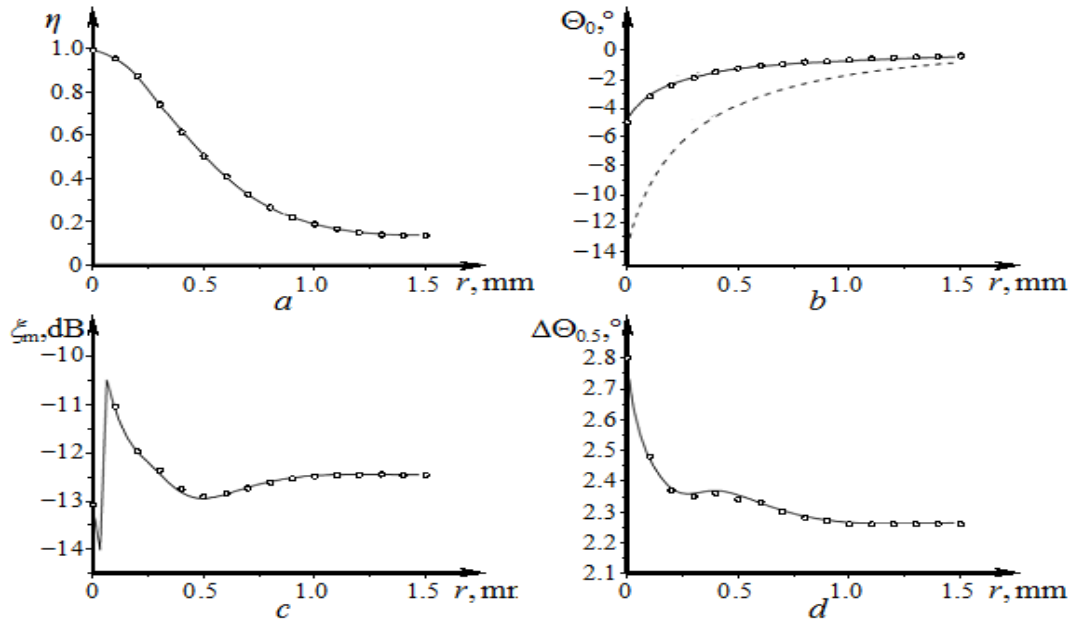


Figure 2. Test properties of diffraction antenna as a function of PDW impact parameter [9].

Figure 3 compares the calculated data of attenuation of surface wave as a function of comb groove depth [18]. Attenuation of surface wave in this case is the value equaling to $10 \cdot \lg(1/k_{\rightarrow}^2)$, where k_{\rightarrow}^2 is the relative portion of power of surface wave passing to the aperture periphery and not radiated by comb. We consider variants of equidistant comb with various number of identical grooves ($a = 0.2\lambda$, $h = (0.1 - 0.3)\lambda$) and step d selected so that the electrical distance of aperture is constant and equals to 20λ . The PDW parameters are as follows: $\epsilon_r = 3$, $\tau = 0.228\lambda$, $r = 0.5\lambda$. Dotted results, obtained on the basis of developed computing algorithm, with sufficient accuracy coincide with the data in [18].

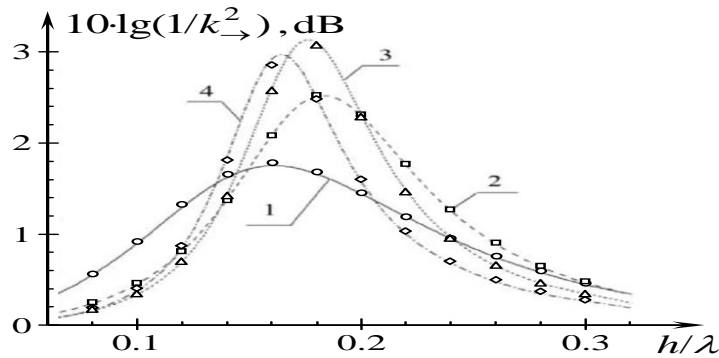


Figure 3. Test resonant curves for diffraction antenna: 1 — $d/\lambda=0.5$; 2 — 0.6; 3 — 0.7; 4 — 0.8 [18].

Dots in Figs. 4 and 5 depicts experimental data of diffraction antenna simulators obtained by regular procedure [19], and solid lines depicts simulating results obtained by the developed algorithm. Parameters of radiating aperture were as follows: equidistant comb step $d = 24$ mm, number of grooves $N = 20$, groove width $a = 8$ mm, depth $h = 4; 6.6$ and 9.2 mm, thickness of polystyrene PDW ($\epsilon_r = 2.56$) $\tau = 6$ mm, impact parameter $r = 10$ (Fig. 4) and 7 mm (Fig. 5). Experimental determination of attenuation constant α of surface wave in the considered antenna was performed in the mode of radio wave radiation on the basis of attenuation introduced into high frequency section by the considered antenna being an open transmission line. Upon simulation the attenuation constant α was calculated using the expression $\alpha = \ln(1/k_{\rightarrow}^2)/(2L)$ without accounting for reflection of surface wave at comb. Comparison of experimental data and simulating results definitely confirms reliability of the latter.

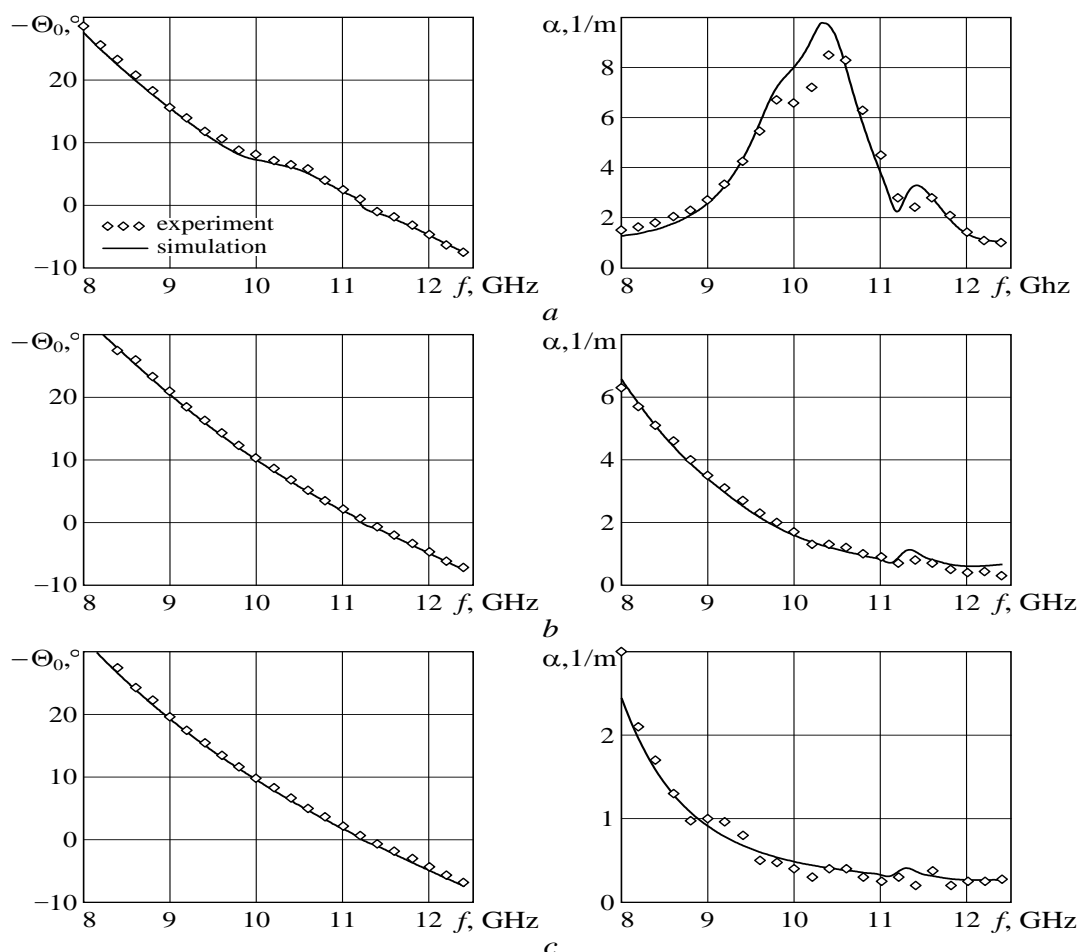


Figure 4. Experimental and calculated frequency properties of diffraction antenna at $r = 10$ mm, $h = 4$ (a), 6.6 mm (b) and 9.2 mm (c)

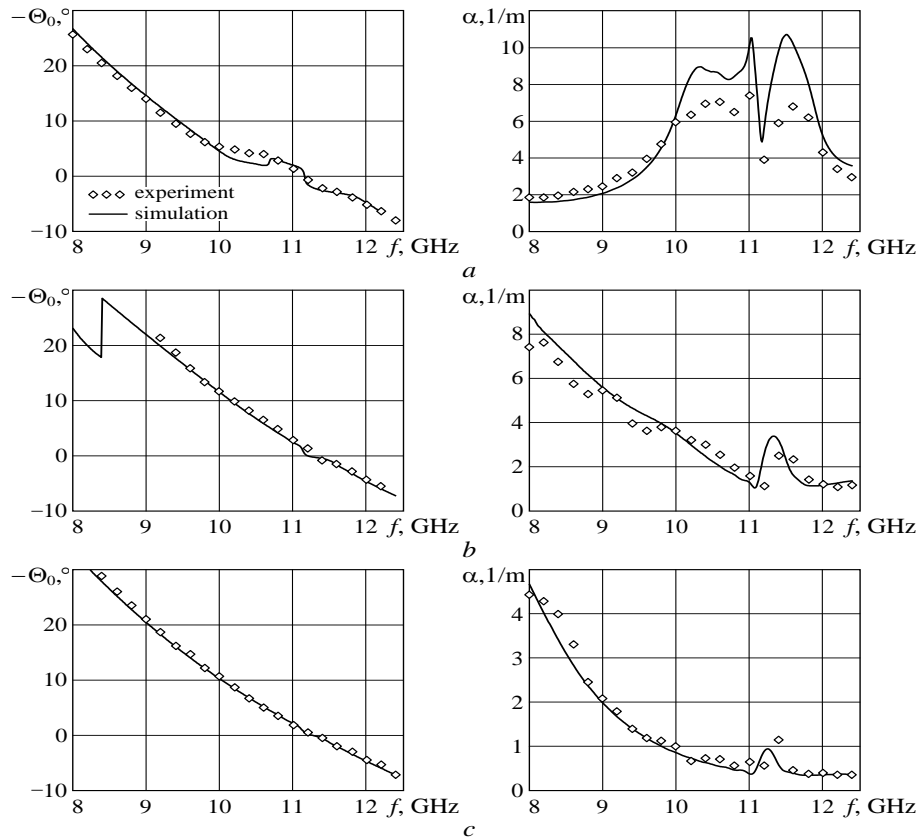


Figure 5: Experimental and calculated frequency properties of diffraction antenna at $r=7$ mm, $h=4$ (a), 6.6 mm (b) and 9.2 mm (c).

RESULTS OF NUMERICAL ANALYSIS AND PARAMETRIC SYNTHESIS OF DIFFRACTION ANTENNA

In Figs. 7 and 8 isolevel lines show efficiency distribution surfaces ($\Xi \times 100\%$) of antenna containing equidistant comb with 25 identical grooves and excited by the lowest E - and, respectively, H - wave of PDW ($\epsilon_r = 2.56$). Asterisks indicate at positions of global maximums. Horizontal axis shows groove depth h , vertical axis shows impact parameter r in the fractions of comb period ($d = x_{k+1} - x_k$), reference to wavelength is provided by dimensionless frequency parameter $\kappa = d/\lambda$. The groove width upon excitation by wave of E -type is set to $a = 0.3d$, that of H -type — to $0.8d$.

It can be seen in Figs. 7 and 8 that for each set of initial parameters τ , κ there exist optimum region of geometrical sizes h , r , in the limits of which the diffraction antenna efficiency is maximum. With increase in the frequency parameter κ and PDW thickness τ the zone of increased efficiency is displaced to the region of lower values of impact parameter r , optimum groove depth weakly depends on the thickness of dielectric τ .

Peculiar feature of the surfaces is their quasi-periodicity over groove depth h , the quasi-period is 0.5λ (E -) and $0.5\lambda/\sqrt{1-(0.5\lambda/a)^2}$ (H -wave) (in Figs. 7 and 8 the surfaces are shown in lower limits). This circumstance makes it possible to restrict search area of preferred depths upon optimization of comb profile. It is practically important to implement high antenna efficiency (0.7 and higher) with no gap between PDW and comb in the mode of wave radiation of vertical polarization (Fig. 7). The data illustrated in Figs. 7 and 8 can be used for direct implementation of radiating aperture or for further optimization of its configuration.

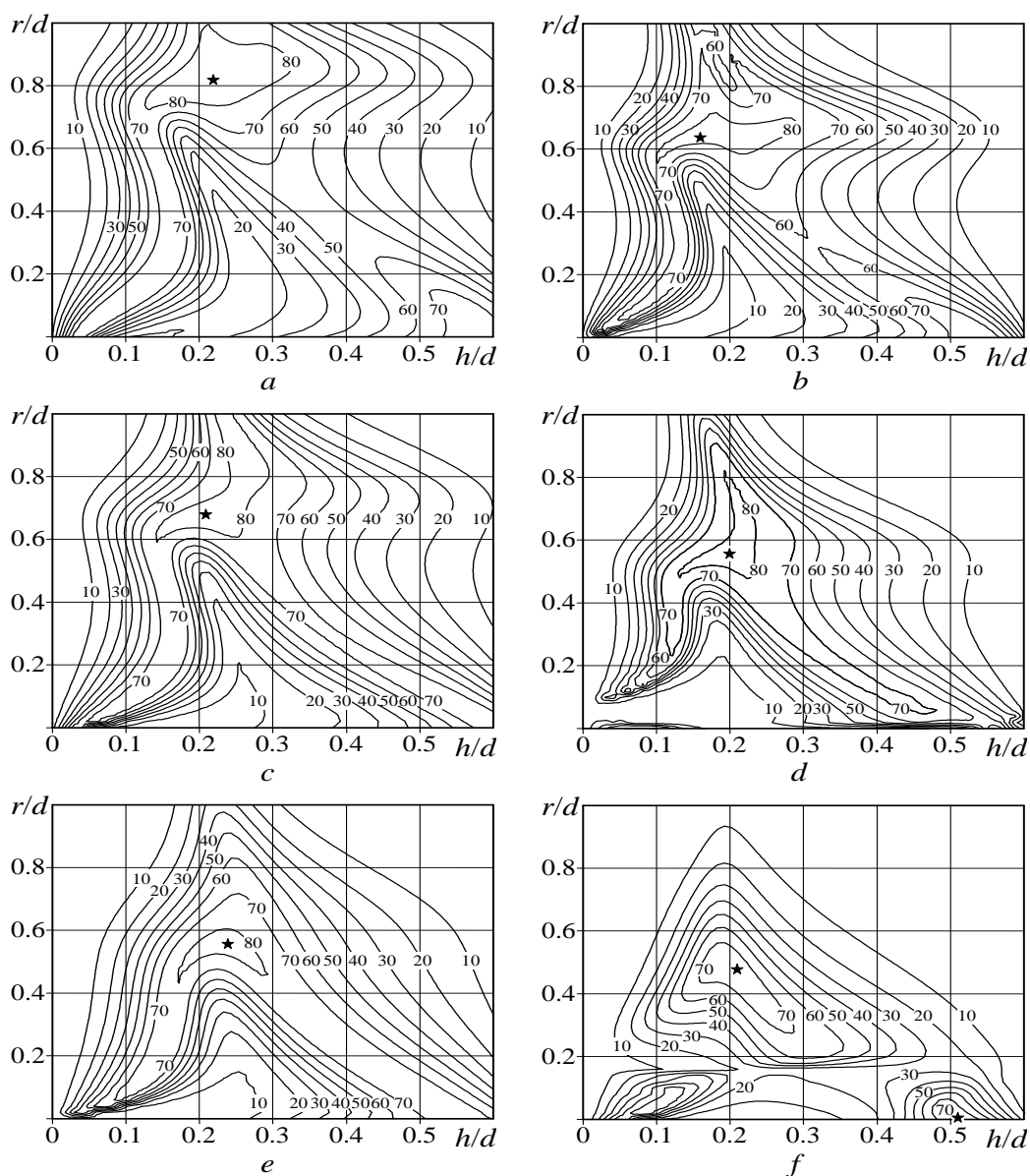


Figure 7. Calculated antenna efficiency upon excitation by wave of E -type:
 a) $\tau = 0.2d$, $\kappa = 0.7$; b) $\tau = 0.2d$, $\kappa = 0.8$; c) $\tau = 0.3d$, $\kappa = 0.7$;
 d) $\tau = 0.3d$, $\kappa = 0.8$; e) $\tau = 0.4d$, $\kappa = 0.7$; f) $\tau = 0.4d$, $\kappa = 0.8$.

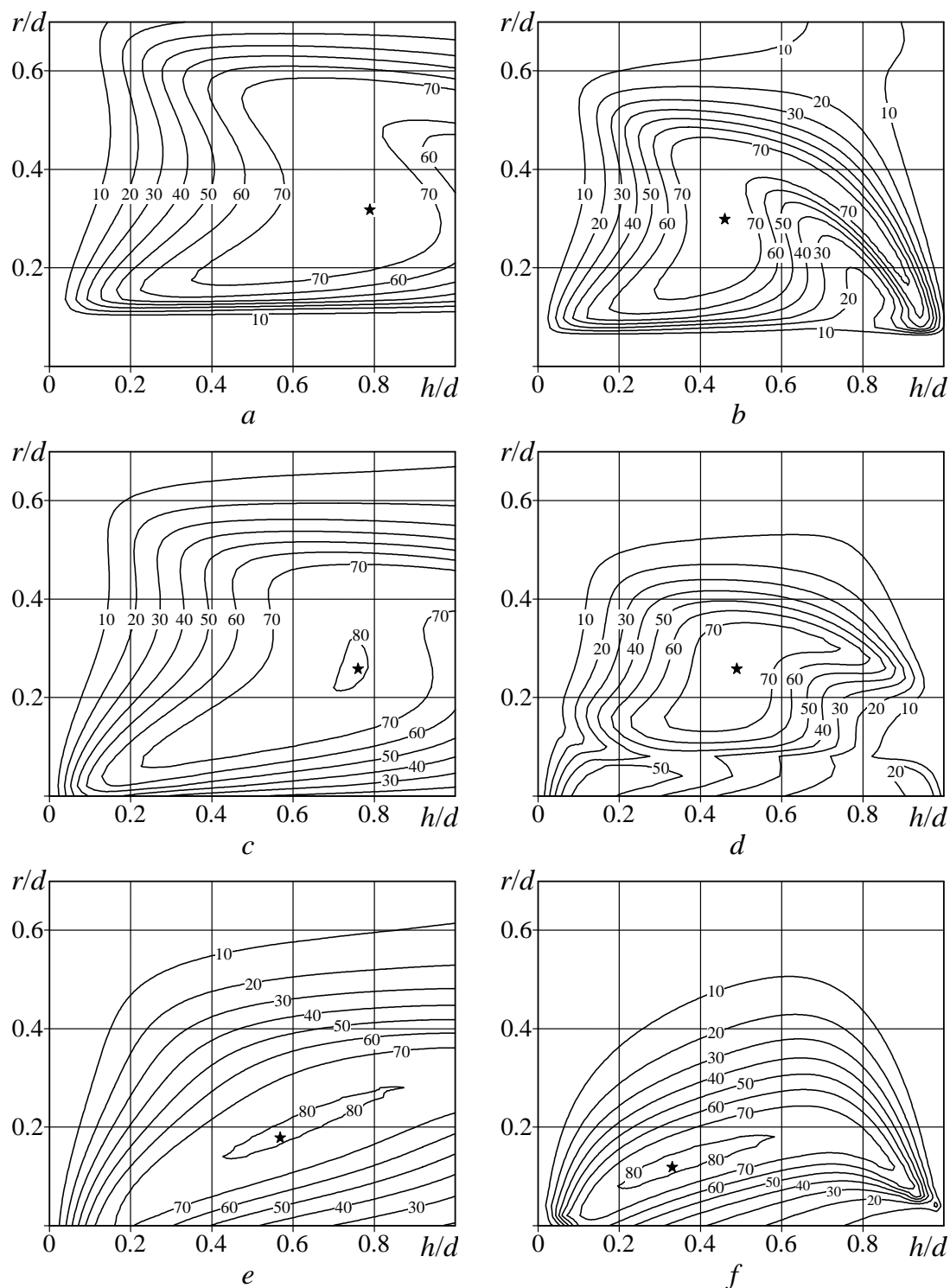


Figure 8. Calculated antenna efficiency upon excitation by wave of H -type:

a) $\tau = 0.2d$, $\kappa = 0.7$; b) $\tau = 0.2d$, $\kappa = 0.8$; c) $\tau = 0.3d$, $\kappa = 0.7$;

d) $\tau = 0.3d$, $\kappa = 0.8$; e) $\tau = 0.4d$, $\kappa = 0.7$; f) $\tau = 0.4d$, $\kappa = 0.8$.

Let us consider an example of such optimization. Let the comb is excited by surface wave of E -type, the frequency parameter is $\kappa=0.7$, the thickness of plane dielectric waveguide is $\tau=0.2d$. According to Fig. 7a the best values of impact parameter r and groove depth h are $h=0.22d$ and $r=0.82d$, respectively. Herewith, the calculated properties of antenna radiation are as follows: $\Xi = 0.864$, $\eta = 0.939$, $k_{SW} = 1.04$, $\nu = 0.920$, $\xi_m = -12.4$ dB. If other than in Figs. 7 and 8 values of thickness of dielectric waveguide τ and frequency parameter κ are used, then it would be reasonable to apply genetic MGA algorithm (with objective function $(1-\Xi)$) for searching of r and h values. A next step may be selection of optimum values of groove width (a) and comb length (N). However, as shown by calculations, such optimization is not required for the above parameters. Let the depth of comb grooves is not the same. Smooth variation of groove depth along the region of interaction between surface wave and comb can lead to generation of amplitude distribution on antenna aperture, which is optimum with regard to AE or LLL. let us consider linear law of variation of depths $h_k = \delta \cdot k + \chi$, described by two parameters δ and χ . Optimum values of δ and χ were sought by means of genetic MGA algorithm for the above parameters of radiating aperture. While applying objective function $(1-\Xi) \rightarrow \min$ linear comb profile, close to optimum, was determined, in accordance with which the groove depth should increase from $0.15d$ to $0.25d$ along propagation of surface wave. The determined profile provides not only for increase in efficiency ($\Xi = 0.879$), but decrease in maximum level of lateral radiation as well ($\xi_m = -15.0$ dB). At that, the other antenna properties are as follows: $\eta = 0.910$, $k_{SW} = 1.03$, $\nu = 0.966$. Optimization of comb profile in terms of the criterion $\xi_m \rightarrow \min$ demonstrated possibility of lateral radiation up to -30 dB. However, the absence of radiation upon optimization of control of energy properties results in that the antenna efficiency decreases significantly. Thus, upon minimization of LLL it is reasonable to apply objective function obtained by combination (addition, for instance) of requirements to the efficiency Ξ and maximum level of lateral lobes of angular pattern ξ_m : $(1-\Xi - \mathcal{G} \cdot |\xi_m|) \rightarrow \min$, where $\mathcal{G} \leq 0.1$ is the weight factor. As a consequence of optimization by MGA algorithm the linear profile was determined ($h_1 = 0.05d$, $h_N = 0.38d$), which guarantees maximum LLL $\xi_m = -19.2$ dB at the efficiency $\Xi = 0.746$. The radiation properties of antenna with such profile are as follows: $\eta = 0.817$, $k_{sw} = 1.01$, $\nu = 0.914$. Since nearly one fifth of power supplied to antenna is not radiated, and the power reflected by comb towards excitation device is relatively low, then it would be reasonable to increase the number of grooves in order to increase power takeoff. Calculations demonstrate that for the determined comb profile the antenna efficiency is maximum at $N = 43$ and equals to $\Xi = 0.825$. Herewith, $\eta = 0.947$, $k_{sw} = 1.02$, $\nu = 0.871$, $\xi_m = -19.2$ dB, $\Theta_0 = -21.7^\circ$, $\Delta\Theta_{0.5} = 2.2^\circ$; normalized angular pattern of the antenna is illustrated in Fig. 9.

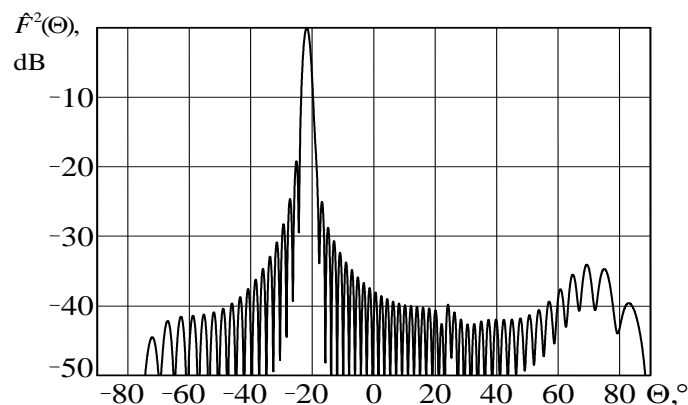


Figure 9. Angular pattern of antenna with optimized comb linear profile upon excitation by surface wave of E -type.

Analysis of the presented results evidences possibility of application of the developed computing algorithm for synthesis of optimum profile of comb aperture used in diffraction antenna. Such synthesis can be implemented on the basis of application of multiextremal optimization algorithm, genetic algorithm in particular. Efficiency of application of genetic algorithm significantly increases, for instance, in comparison with dynamic programming [20] upon the use of non-trivial approximations of profile (piecewise linear, polynomial, exponential sinusoidal and so on), described by several (from 3 and higher) sought coefficients. Approbation demonstrated that independent variant of searching for optimum depths of each individual groove in terms of preset criterion; Herewith, the number of sought coefficients (groove depths) was 25–50.

Approbation of computing algorithm also demonstrated that in addition to synthesis of comb profile there is actual possibility of genetic optimization of antenna properties due to combination of other geometrical properties of aperture (thickness and impact parameter of PDW, groove width, comb length, as well as step of groove positioning). However, if in the first case the most time consuming calculations of coefficients $\sigma_{m,s}^{(k,q)}$ is performed only once, then in other cases at each variation of sought parameter, which leads to noticeable increase in time for synthesis (up to several hours). In order to accelerate synthesis it is required to implement model and genetic algorithm in the form of executable code and its optimization.

CONCLUSIONS

Therefore, a new electrodynamic model has been developed aimed at analysis of antenna radiating properties on the basis of transformation of surface wave into bulk wave. Antenna radiating aperture in the model is simulated by two-dimensional structure covered by plane dielectric waveguide in the form of finite combination of square grooves in shield. Assuming that the structure is excited by heterogeneous wave of plane dielectric waveguide and applying presentation of scattered field in the

form of continuous Fourier spectrum the diffraction problem is reduced to SLAE of relatively low order. The generated computing algorithm makes it possible to consider for non-equidistant location of grooves, in general case of different depth and width, and it differs from known methods by relative simplicity of implementation. The approach, used upon the model generation, is further development of analysis of open radiating electrodynamic structures.

Comparing the directional and energetic properties of antennas, obtained by means of the developed model, with identical data, obtained by other researches, as well as with experimental studies of antenna simulators, we justified adequacy of the presented computing algorithm and reliability of the results on its basis.

By means of calculations new data on complete efficiency (product of aperture efficiency by radiation efficiency) of diffraction antenna have been obtained, making it possible to select reasonably parameters of aperture containing equidistant comb (PDW thickness, impact parameter, step and depth of grooves) upon excitation of dielectric waveguide by main E - and H -wave, including the case of gap absence between PDW and comb.

Possibility and viability of the computing model in combination with genetic algorithm for optimization of characteristics of antenna radiation and parametric synthesis of its comb aperture have been demonstrated. Results of comb profile optimization have been obtained (groove depth) in terms of criterion of antenna efficiency maximum and minimum level of lateral lobes of angular pattern. results confirm possibility of generation of optimum amplitude distribution with regard to aperture efficiency or level of lateral lobes by means of comb profiling (at uniform PDW impact parameter), which can be used for implementation of diffraction antennas with improved operating and engineering properties.

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