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Time Truncated Group Chain Sampling Plans for Rayleigh Distribution

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Abstract

This paper introduces group chain sampling plan for Rayleigh distribution when the life test is truncated at a pre-assumed time. The design parameters such as the number of optimal groups and the operating characteristic values are calculated by satisfying the consumer's and producer's risks at a certain specified quality levels. Quality levels are defined in terms of mean with assumptions that the termination time and acceptance number are pre-fixed. An example is provided for illustrative purpose.

AMS subject classification:
Keywords: Group chain sampling plan, consumer's risk, producer's risk, Rayleigh distribution, operating characteristic values.

1. Introduction

Acceptance sampling is a quality control method used to accept or reject a lot after testing a random sample of a product. The purpose of acceptance sampling is to make a determination about lot of the product; accept the lot or reject it rather than to estimate the quality of the entire lot. Acceptance sampling is a very useful technique when a lot is so large in size or when testing is destructive. For a large lot, it is too time consuming and too costly to inspect every single of the product. Plus, checking every single product does not guarantee that the product will comply with required specification.

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Chain sampling plan was first introduced by Dodge [3] to improve the disadvantages of a single acceptance sampling plan with zero-acceptance number. Normally, a producer refuses to have the single acceptance sampling plan with zero-acceptance number due to the fact that a single defective product in the sample would lead to the rejection of the entire lot. The idea of chain sampling plan is that the current lot under inspection can be accepted if one defective product is recorded in the current sample given that no other defective product is recorded in the subsequent lots. Besides that, the chain sampling plan is really applicable in practice when we deal with extremely high quality and expensive product.

The chain sampling plan can be applied with the following four conditions [6]. The first condition is the lot should be coming from a series of continuing streams of lots. The second condition is the lot should be expected to have the same quality level. Next condition is there should be a good history of quality performance of the supplier. Final condition would be a producer should have confidence in the supplier.

Most of the chain sampling plan is assumed to have only one item to put into a tester. However in practice, there is possibility of putting more than one item into a tester. This is called group chain acceptance sampling plan. Therefore, researchers can develop the group chain sampling plan to inspect multiple products at the same time. Researchers favour the group sampling plan since it can reduce the cost and inspection time compared to the ordinary single sampling plan.

Many researchers have developed acceptance sampling plans based on truncated life tests for different lifetime distributions: Epstein [4], Goode and Kao [5], Rosaiah and Kantam [10], Tsai and Wu [11], Aslam [1], Aslam and Jun [2], Ramaswamy and Jayasri [8] and Ramaswamy and Jayasri [9]. For the group chain sampling plan, Mughal, Zain and Aziz [7] have developed the sampling plan for Pareto distribution of the 2nd kind. Their finding is consistent with Ramaswamy and Jayasri [8]. However, their proposed plan really meets economic criteria for life testing because cost, time, energy and labour of the inspection can be reduced.

There are so many lifetime distributions in the literature and none has really developed the group chain sampling plan for these distributions. Therefore, this paper will develop the group chain sampling plan for Rayleigh distribution. The objective of this research is to find the optimal number of groups and operating characteristic values at different levels of consumerâŁ™s risk. The rest of this paper is organised as follows. Section 2 lists out all notations used throughout this paper. In Section 3, we introduce cumulative distribution function (CDF) for Rayleigh distribution. Section 4 contains how group chain sampling plan is designed. Definition for operating characteristic function is provided in Section 5. Tables and example are explained in Section 6. We end this paper with a brief conclusion in Section 7.

2. Glossary of Symbols

r : Number of testers)
n : Sample size
d : Number of defective items
i : Allowable acceptance number
p : Quality level
α : Producer's risk (Probability of rejecting a good lot)
β : Consumer's risk (Probability of accepting a bad lot)
$\frac{\mu}{\mu_0}$: Mean ratio
$P_a(p)$: Probability of lot acceptance
t_0 : Test termination time

3. Rayleigh Distribution

Rayleigh distribution is normally used for fitting failure time distributions. For Rayleigh distribution, the cumulative distribution function (CDF) is given by:

$$F(t;\sigma) = 1 - \exp\left(-\frac{t^2}{2\sigma^2}\right) \tag{3.1}$$

where σ is the scale parameter. The mean of the Rayleigh distribution is:

$$\mu = \sigma\sqrt{\frac{\pi}{2}} \approx 1.2533\sigma \tag{3.2}$$

Probability of failure of a product, p during the test termination time, t_0 is given by equation (4), where t_0 is a multiple of the specified mean life, μ_0 and specified constant, a, written as:

$$t_0 = a\mu_0 \tag{3.3}$$

Therefore, p can be estimated by:

$$p = F(t;\sigma) = 1 - \exp(-\frac{1}{2}(1.2533a)^2(\mu_0/\mu)^2) \tag{3.4}$$

4. Design of the Group Chain Sampling Plan

The group chain sampling plan is applied with the following steps. The first step is for each submitted lot, we need to find the optimal number of g groups and allocate r items to each group such that the sample size is given by $n = gr$.The second step is we accept the lot when $d = 0$ and reject the lot if $d > 1$. The final step is we do accept the lot if $d = 1$ and continue the inspection if no defectives are found in the preceding i lots.

The group chain sampling plan is characterized by the parameters g and i. The probability of rejecting a good lot is known as producer's risk, while the probability of accepting a bad lot is known as consumer's risk. When deciding the parameters of the proposed sampling plan, we make use of the consumer's risk. Normally, the consumer's risk is expressed by the consumer's confidence level. Suppose the confidence level is p^*, then the consumer's risk becomes $\beta = 1 - p^*$. We will determine the number of groups in the proposed sampling plan so that the consumer's risk does not exceed β:

$$P_a(p) \leq \beta \tag{4.5}$$

We assume that the lot size is large enough so that Binomial distribution can be used to find the probability of lot acceptance. The probability of lot acceptance in the chain sampling plan is given by:

$$P_a(p) = (1-p)^n + np(1-p)^{n-1}(1-p)^{ni} \tag{4.6}$$

The above formula is modified to suit with the group chain sampling plan: replace n with gr since $n = gr$. Therefore, the probability of lot acceptance for the group chain sampling plan is given by:

$$P_a(p) = (1-p)^{gr} + grp(1-p)^{gr-1}(1-p)^{gri} \tag{4.7}$$

In Table 1, the number of optimal groups g is presented satisfying equation for $\beta = 0.25, 0.10, 0.05, 0.01$; $a = 0.7, 0.8, 1.0, 1.2, 1.5, 2.0$; $r = 2(1)5$ and $i = 1(1)4$. These values are consistent with Mughal, Zain and Aziz [6] for developing the proposed plan using Rayleigh distribution. Once the optimal number of groups is obtained, the probability of lot acceptance can be calculated using different values of quality level. For a fixed r and i, the operating characteristics values as a function of the mean ratio, $\frac{\mu}{\mu_0}$, are calculated in Table 2.

5. Operating Characteristic Functions

The probability of lot acceptance can be defined as a function of the deviation of specified mean life from the true mean life. This function is called operating characteristic (OC) function of the sampling plan. Once the optimal number of groups is obtained, the probability of lot acceptance can be calculated at different values of quality level.

6. Illustrative Example

Table 1 shows the number of optimal groups for the proposed plan for Rayleigh distribution at different quality levels. For instance, if the consumerâŁ™s risk is set at 0.25, number of testers is 2, allowable acceptance number is 1, specified constant is 1.0, then the number of optimal groups is 2.

Table 1: Number of optimal groups for the proposed plan for Rayleigh distribution.

			a					
β	r	i	0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	1	3	2	2	1	1	1
	3	2	2	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.10	2	1	4	3	2	2	1	1
	3	2	3	2	1	1	1	1
	4	3	2	2	1	1	1	1
	5	4	2	1	1	1	1	1
0.05	2	1	5	4	3	2	1	1
	3	2	3	2	2	1	1	1
	4	3	2	2	1	1	1	1
	5	4	2	2	1	1	1	1
0.01	2	1	7	5	3	3	2	1
	3	2	4	4	2	2	1	1
	4	3	3	3	2	2	1	1
	5	4	3	2	2	1	1	1

Meanwhile, Table 2 illustrates the operating characteristics values when number of testers is 4 and the allowable acceptance number is 3. For example, the probability of lot acceptance is 0.9927 when mean ratio is 8, number of optimal groups is 1 and specified constant is 1.

Suppose that μ and μ_0 are true and specified mean life of a product respectively. That is, a lot of items is considered good if the true mean life, μ is greater than the specified mean life, μ_0. Assume that the lifetime of the item follows Rayleigh distribution. An experimenter is interested to find the true mean life of an item, that is at least 1000 hours with confidence level of 0.90. If the experimenter designs the test on the basis of $a = 0.7$, the number of testers $r = 4$ and preceding samples $i = 3$, then the number of optimal groups would be $g = 2$. Therefore, the design parameters for the group chain sampling plan are $(a, r, i, g) = (0.7, 4, 3, 2)$. This means the experimenter needs to select a random sample of size 8 from the lot and put 4 items to each of the 2 groups. The lot will be accepted if not more than one defective is recorded within 700 hours and no defective items are found in the next 3 subsequent samples. If the test has the same design parameters, the probability of lot acceptance increases from 0.5003 to 0.9152 when the mean ratio increases from 2 to 4, as shown in Table 2.

Table 2: Operating characteristics values having $r = 4$, $i = 3$ for the proposed plan for the Rayleigh distribution.

β	g	a	2	4	6	8	10	12
0.25	1	0.7	0.7672	0.9745	0.9944	0.9981	0.9992	0.9996
	1	0.8	0.6767	0.9591	0.9907	0.9969	0.9987	0.9994
	1	1.0	0.4934	0.9135	0.9787	0.9927	0.9969	0.9985
	1	1.2	0.3369	0.8483	0.9591	0.9855	0.9937	0.9969
	1	1.5	0.1727	0.7227	0.9135	0.9674	0.9855	0.9927
	1	2.0	0.0432	0.4934	0.7956	0.9135	0.9591	0.9787
0.10	2	0.7	0.5003	0.9152	0.9791	0.9928	0.9970	0.9985
	2	0.8	0.3851	0.8712	0.9663	0.9882	0.9949	0.9975
	1	1.0	0.4934	0.9135	0.9787	0.9927	0.9969	0.9985
	1	1.2	0.3369	0.8483	0.9591	0.9855	0.9937	0.9969
	1	1.5	0.1727	0.7227	0.9135	0.9674	0.9855	0.9927
	1	2.0	0.0432	0.4934	0.7956	0.9135	0.9591	0.9787
0.05	2	0.7	0.5003	0.9152	0.9791	0.9928	0.9970	0.9985
	2	0.8	0.3851	0.8712	0.9663	0.9882	0.9949	0.9975
	1	1.0	0.4934	0.9135	0.9787	0.9927	0.9969	0.9985
	1	1.2	0.3369	0.8483	0.9591	0.9855	0.9937	0.9969
	1	1.5	0.1727	0.7227	0.9135	0.9674	0.9855	0.9927
	1	2.0	0.0432	0.4934	0.7956	0.9135	0.9591	0.9787
0.01	3	0.7	0.3272	0.8414	0.9568	0.9846	0.9933	0.9967
	3	0.8	0.2252	0.7707	0.9321	0.9750	0.9890	0.9945
	2	1.0	0.2111	0.7589	0.9276	0.9732	0.9882	0.9941
	2	1.2	0.1045	0.6291	0.8712	0.9492	0.9769	0.9882
	1	1.5	0.1727	0.7227	0.9135	0.9674	0.9855	0.9927
	1	2.0	0.0432	0.4934	0.7956	0.9135	0.9591	0.9787

7. Conclusion

The proposed sampling plan is an extension from the ordinary chain sampling plan. A group chain sampling plan is developed for truncated life test in the case of Rayleigh distribution. The number of optimal groups is determined when the consumer's risk and the other plan parameters are specified. The results have shown that the optimal number of groups decreases as the specified constant increases. The operating characteristics value increases when the quality level improves. The group chain sampling plan can be used when we deal with extremely high quality and expensive product. It has shown that by applying the group chain sampling plan, the cost, time and labour charge can be reduced.

References

[1] Aslam, M. (2007). Double Acceptance Sampling based on Truncated Life Tests in Rayleigh Distribution. *European Journal of Scientific Research*, 17(4), 605–606.

[2] Aslam, M. & Jun, C. H. (2009). A Group Acceptance Sampling Plan for Truncated Life Test Having Weibull Distribution. *Journal of Applied Statistics*, 36(9), 1021–1027.

[3] Dodge, H. F. (1955). Chain Sampling Inspection Plan. *Industry Quality Control*, 11(4), 10–13.

[4] Epstein, B. (1954). Truncated Life Tests in the Exponential Case. *The Annals of Mathematical Statistics*, 25(3), 555–564.

[5] Goode, H. P. & Kao, J. H. K. (1961). Sampling Plans based on the Weibull Distribution. *Proceedings of the Seventh National Symposium on Reliability and Quality Control*, 24–40.

[6] Montgomery, D. C. (2009). *Statistical Quality Control:* A Modern Introduction (6th ed.). Arizona: Wiley.

[7] Mughal, A. R., Zain, Z. & Aziz, N. (2015). Time Truncated Group Chain Sampling Strategy for Pareto Distribution of the 2nd Kind. *Research Journal of Applied Sciences, Engineering and Technology*, 10(4), 471–474.

[8] Ramaswamy, A. R. S. & Jayasri, S. (2014). Time Truncated Chain Sampling Plans for Generalized Rayleigh Distribution. *International Refereed Journal of Engineering and Science*, 3(2), 49–53.

[9] Ramaswamy, A. R. S. & Jayasri, S. (2015). Time Truncated Chain Sampling Plan for Weibull Distribution. *International of Engineering Research and General Science*, 3(2), 59–67.

[10] Rosaiah, K. & Kantam, R. (2005). Acceptance Sampling based on the Inverse Rayleigh Distribution. *Economic Quality Control*, 20(2), 277–286.

[11] Tsai, T. R. & Wu, S. J. (2006). Acceptance Sampling based on Truncated Life Tests for Generalized Rayleigh Distribution. *Journal of Applied Statistics*, 33(6), 595–600.