Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 12, Number 4 (2016), pp. 3641–3671

http://www.ripublication.com/gjpam.htm

# Computational study on convective flows in presence of chemical reaction and thermal radiation in porous/non-porous cavities

**Sabyasachi Mondal**[1] **and Precious Sibanda**

*School of Mathematical Sciences,*
*University of KwaZulu-Natal,*
*Private Bag X01 Scottsville 3209,*
*Pietermaritzburg, South Africa.*

**Abstract**

The present paper provide a brief review of heat and mass transfer studies in cavity. We focus particular attention on the studies which published in recent years in which thermal radiation and/or chemical reaction has been an important factor. The study is restricted mainly to cavity flows and include recent analytical, numerical and experimental studies. This paper is divided into categories related to our topic and then into sub-categories within these categories. In addition to reviewing past studies we also include new results that describe the effects of the buoyancy ratio over an unsteady double diffusive convection in an inclined rectangular enclosure.

**AMS subject classification:**
**Keywords:** Cavity flow, chemical reaction, thermal radiation, heat and mass transfer, non-uniform boundary condition, MHD, porous/non-porous media.

## 1. Introduction

Fluid dynamics deals with the science and technology of fluids in motion. Aerodynamics and hydrodynamics i.e., the study of air, gases and the liquids in motion, respectively are some disciplines of fluid dynamics. The velocity, temperature, concentration, pressure and density etc are treated as a function of time and space. The solid particles and molecules which collide with one another are the component of fluids. The Navier-Stokes equations comes from the equations of motion of the Newtonian fluids. There

[1]Corresponding Author.

are different number of ways by whom we can simplified the equations on the basis of practical physical situation. It is well known that the Navier-Stokes equations provide a complete description to a wide range of fluid flow problems and if the initial and boundary conditions are applied appropriately, then the equations can be solved accurately to estimate heat and mass transfers coefficients for different flow problems.

This paper is devoted to the review and study of cavity flows. Heat transfer within a cavity can be found in many industrial and natural applications such as in climate control, meteorological and geophysical phenomena, the cooling of electronics to name a few. The purpose of this review is to briefly present the main findings related to convection produced within cavity flows. Both experimental and numerical works are considered in porous and non-porous enclosures. New results related to the effects of the buoyancy ratio on steady/ unsteady double-diffusive convection in an inclined rectangular enclosure are also presented.

Heat transfer processes have attracted researchers, engineers and designers for its wide applications in industry (such as in gas turbines, heat exchanger, the cooling of electrical components, etc.) as well as natural aspects (such as weather broadcast, green house effects, etc). Heat transfer refers to thermal energy in transit due to temperature differences. Thus temperature difference is not only a sufficient condition but also a necessary condition for heat transfer to take place. In general, there are three different heat transfer modes. All the modes of the heat transfer need temperature gradient, but not all of them need a medium. Firstly, convection occurs when heat transfers between a surface and a moving fluid. The second mode of transmission is conduction which occurs in solids. The third mode is radiation which does not need an intervening medium with heat transmitted between two surfaces in the form of electromagnetic waves.

For these reasons a better understanding of heat transfer characteristics could lead directly to improvements in heat transfer devices and fluids. In general only a few of heat transfer problems can be solved by analytical methods. Most problems are tackled either using numerical methods or the experimental approach. When the concentrations of two or more then two components vary from one point to different points in a system is called mass transfer.

Again, studies of flow through porous medium have attracted considerable attention in the recent past few years because of the different types of important applications such flows; flow passed through packed beds, filtration of solids from liquids, the flow of liquids through ion-exchange beds and in chemical reactors for economical separation or purification of mixtures (see Fig. 1). A porous medium is a fixed solid matrix with a connected void space through which fluid can flow or consists of solid particles (which are deformable or non-deformable) so that fluid can flow through voids and passages. Let $V_v$ be the volume of voids. When a fluid flows through the interconnected voids and passages of a porous medium $V$, the walls of these voids and passages form small tunnels through which fluid can flow. The study of fluid flow at microscopic scale is complicated and unrealistic because of the complexity of the micro-geometry of porous media. A more realistic approach to study dynamics of fluid flow passed through porous media is under the assumption of continuum macroscopic phenomena. Usually, spacial averages

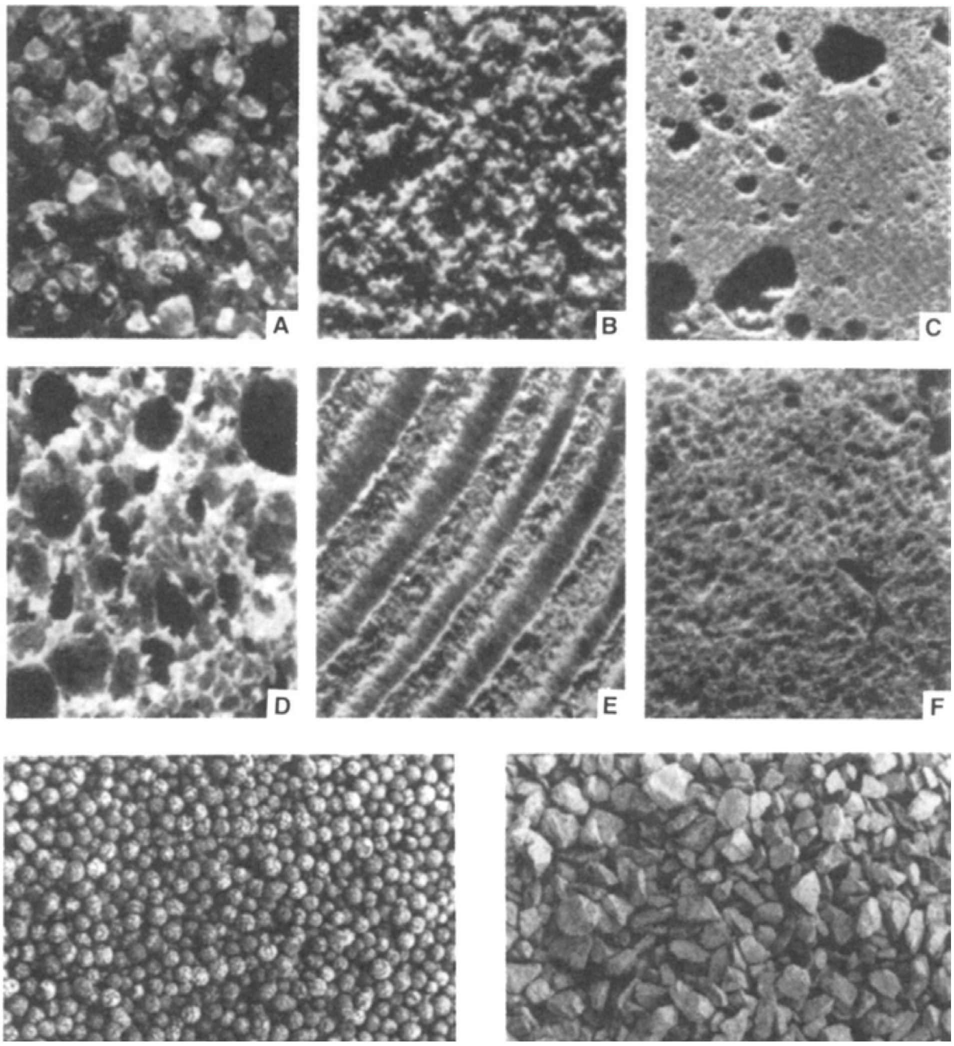


Figure 1: Examples of natural porous materials: Top row: (A) beach sand, (B) sandstone, (C) limestone; Middle row: (D) bread slice, (E) wood and (F) human lung. Bottom row: Granular porous materials used in the construction industry, 0.5-cm-diameter Liapor spheres (left), and 1-cm-size crushed limestone (right)

are used to transfer properties of porous media from microscopic to macroscopic scales.

Due to interactions, the three heat transfer mechanisms, viz., conduction, convection and radiation are inseparable. In a physical sense, the amount of heat transfer in conduction and convection depends upon the temperature difference whereas that of radiation depends upon both the temperature difference and the temperature level. Mathematically, heat transfer by conduction and convection is defined by a set of the partial differential equations, while the radiation problem is quite complicated because it requires the solution of nonlinear integro-differential equations. There exist situations in which thermal radiation is important even though the temperature may not be high. It is a fact that even under some of the most unexpected situations, the radiation heat transfer could account for a non-negligible amount of total heat transfer.

Earlier works on heat transfer in Newtonian fluids considered convection and conduction and overlooked the effect of thermal radiation (Siegel and Howell [1]; Howell and Menguc [2]). The available literature barely delineates the part played by convection in a fluid combined with radiation. The formulation of heat transfer by conduction and convection leads to differential equations while that by radiation leads to integral equations. Thus, the complexity involved in the solution of the integro-differential equations resulting from the combined convection and radiation problem warrants the use of several simplifying assumptions. The convection in presence of radiation parameter are used in many environmental and scientific processes. Astrophysical flows, cooling and heating of chambers, water evaporation from open reservoirs and solar power technology are some example. Radiative effects on heat transfer in nonporous/ porous medium can be well estimated by using the radiative flux model or Rosseland model (Raptis [3]).

## 2. Basic equations of maximum problems in this paper

The basic equation for the problems investigated in the paper are mainly continuity, momentum, energy and concentration equations.

### 2.1. The equations of flow of an incompressible viscous fluid

Equation of continuity:

$$\frac{\partial u_j}{\partial x_j} = 0. \tag{1}$$

Momentum equation

(i) for flow through porous medium:

$$\rho\left(\frac{\partial u_i}{\partial t} + \frac{u_j}{\epsilon}\frac{\partial u_i}{\partial x_j}\right) = -\frac{\partial p}{\partial x_i} + \mu\frac{\partial^2 u_i}{\partial x_j \partial x_j} + \rho X_i - \frac{\mu\epsilon}{K}u_i - \frac{1.75\sqrt{\sum u_i^2}}{\sqrt{150K\epsilon}}u_i.$$

(ii) for flow through non-porous medium:

$$\rho\left(\frac{\partial u_i}{\partial t} + u_j\frac{\partial u_i}{\partial x_j}\right) = -\frac{\partial p}{\partial x_i} + \mu\frac{\partial^2 u_i}{\partial x_j \partial x_j} + \rho X_i.$$

Energy equation:

$$\rho C_p\left(\frac{\partial T}{\partial t} + u_j\frac{\partial T}{\partial x_j}\right) = k\frac{\partial^2 T}{\partial x_j \partial x_j} + 2\mu e_{ij}e_{ij} + Q(T - T_c) - \frac{\partial q_r}{\partial x_2}, \tag{2}$$

where

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right).$$

The species concentration is governed by the equation

$$\frac{\partial C}{\partial t} + u_j \frac{\partial C}{\partial x_j} = D \frac{\partial^2 C}{\partial x_j \partial x_j}. \tag{3}$$

The equation of state is

$$\rho = \rho_0 \left[1 - \alpha_1 (T - T_0) + \alpha_2 (C - C_0)\right], \tag{4}$$

where $\rho_0$, $T_0$ and $C_0$ denote density, temperature and concentration at a reference state with $\alpha_1 > 0$ and $\alpha_2 > 0$. Here $u_i$, $\rho$, $\mu$, $p$, $X_i$, $T$, $C_p$, $k$, $C$, $D$ are velocity components, fluid density, viscosity, pressure, ith component of external force per unit mass, specific heat at constant pressure, temperature, thermal conductivity, species concentration and the coefficient of solutal diffusivity respectively.

### 2.2. The equations of flow of an electrically conducting fluid

With the displacement currents ignored, Maxwell's equations are

$$\nabla.\mathbf{H} = 0, \tag{5}$$

$$\nabla \times \mathbf{H} = 4\pi \mathbf{J}, \tag{6}$$

$$\nabla \times \mathbf{E} = -\mu_e \frac{\partial \mathbf{H}}{\partial T}, \tag{7}$$

where $\mathbf{H}$, $\mathbf{E}$, $\mathbf{J}$ and $\mu_e$ are magnetic field, electric field, electric current density and magnetic permeability respectively.

The Ohm's law for the flow of electric current, neglecting Hall effect, is given by

$$\mathbf{J} = \sigma(\mathbf{E} + \mu_e \mathbf{u} \times \mathbf{H}), \tag{8}$$

where velocity vector is $\mathbf{u}$ and $\sigma$ is electrical conductivity of the fluid. After using Maxwell's equations, we have the following equations governing the flow of an incompressible fluid which is electrically conducted as

Conservation of mass:

$$\frac{\partial u_j}{\partial x_j} = 0. \tag{9}$$

Momentum equation:

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} - \frac{\mu_e}{4\pi\rho} H_j \frac{\partial H_i}{\partial x_j} = -\frac{\partial}{\partial x_i}\left(\frac{p}{\rho} + \mu_e \frac{|\mathbf{H}|^2}{8\pi\rho}\right) + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j}. \tag{10}$$

The energy equation:

$$\rho C_p \left(\frac{\partial T}{\partial t} + u_j \frac{\partial T}{\partial x_j}\right) = k \frac{\partial^2 T}{\partial x_j \partial x_j} + 2\mu e_{ij} e_{ij} + \frac{|\mathbf{J}|^2}{\sigma}. \tag{11}$$

Magnetic induction equation:

$$\frac{\partial H_i}{\partial t} + u_j \frac{\partial H_i}{\partial x_j} = H_j \frac{\partial u_i}{\partial x_j} + \eta \frac{\partial^2 H_i}{\partial x_j \partial x_j}, \tag{12}$$

where $\eta$ is the magnetic diffusivity given by $1/4\pi\mu_e\sigma$.

# 3. Boundary Conditions

## 3.1. Boundary conditions for the flow of incompressible viscous fluid

At a rigid surface the relative velocity of fluid with respect to the surface is zero, i.e., $u_i = 0$. The boundary conditions at an interface between two fluids(in which the upper fluid occupies the region 1 and the lower fluid occupies the region 2) are

a) The tangential stress is continuous at the interface,

b) The normal stress difference is

$$(\tau_{ij}n_j)_1 - (\tau_{ij}n_j)_2 = -\gamma\left(\frac{1}{R_1} + \frac{1}{R_2}\right)n_i, \tag{13}$$

where $\gamma$ is the surface tension (assumed constant) and $\mathbf{n}$ (with components $n_i$) is the unit vector in the direction of normal. When the surface tension is nonuniform, the normal stress difference is

$$\left(\tau_{ij}n_j\right)_1 - \left(\tau_{ij}n_j\right)_2 = -\gamma\left(\frac{1}{R_1} + \frac{1}{R_2}\right)n_i + (\nabla\gamma)_i\,, \tag{14}$$

where $\nabla\gamma$ is the vector gradient of $\gamma$ in the interfacial surface. In (13) and (14), $R_1$ and $R_2$ are the radii of curvature of the interface in any of teh two orthogonal planes containing $\mathbf{n}$, being considered positive when the corresponding centre of curvature lies on the side of the interface to which $\mathbf{n}$ points.

c) Temperature is continuous at the interface.

d) Heat flux is also continuous at the interface.

e) At a rigid boundary temperature may be kept constant/nonuniform or if the boundary is thermally insulated the heat flux will vanish at the boundary.

f) Concentration is continuous at the interface.

g) At a rigid boundary concentration may be kept constant/nonuniform or concentration gradient may vanish at the boundary.

## 3.2. Boundary conditions for the flow of an electrically conducting fluid in presence of a magnetic field

In addition to the boundary conditions on velocity and temperature given above the following boundary conditions on the magnetic field and the electric field must hold.

a) At an interface between two fluids the tangential component of the magnetic field must be continuous.

b) The normal component of the magnetic induction must be continuous.

c) The tangential component of the electric field must be continuous at the interface.

## 4. Literature Review

The present paper is intended to present a survey of recent studies on heat and mass transfer in cavities in the presence of thermal radiation. In this survey, although extensive cannot include every published article, hence some selection is necessary. Many articles are reviewed here which are relate to the science of heat and mass transfer, including analytical, numerical and experimental studies. Related applications where heat and mass transfer play an important role in natural systems are discussed here. New results related to the influence of buoyancy ratio in cavity flows are given.

Heat transfer is important to many environmental and industrial processes as well as to conversion and energy production. Natural convection in cavities has gained importance in many electronic applications. Natural convection cooling is desirable because it doesn't require energy source for cooling and hence more reliable. Air is taken as the cooling medium for cooling electronic components due to its simplicity and low cost. There are numerous studies in the literature regarding natural convection heat transfer in cavities.

Solar collectors, building insulation, double-glazed windows, cooling and heating devices for electronic instruments and gas-filled cavities around nuclear reactor cores etc. are some important examples of natural convection flow in different type of enclosures (Kelkar and Patankar [4]). Isotherms and streamlines are first caiculated and examined by Poots [5] by hand.

A typical model of convective driven by lateral thermal gradient consists of a two dimensional rectangular cavity with the two vertical end walls held at different constant temperatures. In order to determine the flow structure and heat transfer across cavities with different physical properties, numerous analytical, experimental and computational techniques have been used (Batchelor [6]; Bejan and Tien [7]). Natural convection in a square cavity analyzed numerically using a control volume approach by Barakos et al. [8]. Rohsenow et al. [9] wrote a book containing the new findings on radiation, convection, conduction, multi-phase and microscale heat transfer, in porous media in natural processing. Recently, Gagawane et al. [11] studied the effect of Rayleigh number on natural convection by scalar thermal lattice Boltzmann method with two-dimensional and nine-velocity link.

Since the early work of Darcy in the nineteenth century, extensive investigations have been conducted on flow and heat transfer through porous media, covering a broad range of different fields and applications such as ground-water hydrology, petroleum reservoir and geothermal operations, packed-bed chemical reactors, transpiration cooling and building thermal insulation. Early work on porous media used the Darcy's law that neglected important effects such as boundary and inertia effects. Cheng [10] provided a comprehensive review of the literature on free convection in fluid-saturated porous medium focusing on geothermal system. Vafai and Tien [12] reported a pioneering work on the boundary and inertia effects of porous media on convective flow and heat transfer situations. Darcy-Brinkman-Forchheimer model describes the effects of inertia and viscous forces in the porous media as used by Poulikakos and Bejan [13], Lauriat and Prasad [14].

Nithiarasu et al. [15] investigated natural convection heat transfer in a fluid saturated variable porosity medium. Medeiros et al. [16] studied numerically heat transfer by natural convection in a porous cavity under a non-Darcian approach for uniform porosity using Darcy-Brinkman-Forchheimer model. They compared the numerical results with the works that considered uniform porosity and found to have excellent accuracy. Again, Nithiarasu et al. [17] considered Darcy-Brinkman-Forchheimer model in their study. Marcondes et al. [18] studied numerical analysis of natural convection in cavities with variable porosity. The fundamentals of heat transfer in porous media also are presented in Bejan [19] and Bejan[20]. Nield [21] proposed that the Brinkman term be treated in the same way as the Darcy and Forchheimer terms, so that the total viscous dissipation remains equal to the power of the total drag force.

Nield [22] suggested that the Brinkman equation may break down in this limit. The convective heat transfer mechanism is studied by Nield and Bejan [23]. Buoyancy-driven convection in a cavity with differentially heated isothermal walls is a prototype of many industrial application such as energy efficient design of buildings and rooms, operation and safety of nuclear reactor and convective heat transfer associated with boilers. The sinusoidal wall temperature variation produces uniform melting of materials such as glass. Roy and Basak [24] solved the nonlinear coupled partial differential equations for flow and temperature fields with both uniform and non-uniform temperature distributions prescribed at the bottom wall and at one vertical wall. Basak et al. [25] have investigated natural convection flow in a square cavity filled with a porous medium for both uniform and non-uniform heating from below by using Darcy-Forchheimer model.

Flows and heat transfer in a lid-driven cavity with buoyancy or without buoyancy effect, for steady or unsteady cases, have been an important topic because of its wide application area in engineering and science. Some of these applications include oil extraction, cooling of electronic devices and heat transfer improvement in heat exchanger devices. Shankar and Deshpande [26] reviewed and showed its applications from a scientific and engineering point of view. Lid-driven cavities with filled saturated porous medium are another important application because of its wide applications in engineering such as heat exchangers, solar power collectors, packed-bed catalytic reactors, nuclear energy systems and so on. The involvement of both natural and forced convection, referred as mixed convection, in porous media has been an important topic because of its wide range of application in engineering and science. The lid-driven cavity problems have also been used as a benchmark configuration for the evaluation of numerical solution of Navier Stokes equations (Schreiber and Keller [27]; Thompson and Ferziger [28]). A number of experimental and numerical studies have been conducted to investigate the flow field and heat transfer characteristics of lid-driven cavity flow in the past several decades.

Khanafer and Chamkha [29] studied the unsteady mixed convection flow in a lid-driven encloser filled with Darcian fluid-saturated uniform porous medium with internal heat generation. Al-Amiri [30] investigated the momentum and energy transfer in a lid-driven cavity filled with a porous medium. In this study, the general formulation of the momentum equation was used by incorporating both the inertial and viscous effects whereas the forced convection was induced by sliding top constant-temperature wall. Oztop [31] investigated, numerically, combined convection heat transfer and fluid flow in a partially heated porous lid-driven cavity. Kandaswamy et al. [32] studied the mixed convection in lid-driven square cavity filled with porous medium numerically. Oztop et al. [33] performed numerical simulations of the conduction-combined forced and natural convection (mixed convection) heat transfer and fluid flow for 2-D lid-driven square enclosure divided with a finite thickness and finite conductivity. Basak et al. [34] analyzed mixed convection in a lid driven porous square cavity with linearly heated side wall(s) and Muthtamilselvan et al. [35] studied the characteristics of a lid-driven flow in a two dimensional square cavity filled with heat-generating porous medium.

Most of the studies in the existing literature addressed mixed convection in lid-driven cavities in horizontal configurations, where the sidewalls are aligned with the direction of gravity. In some applications, it may be necessary to keep the cavity inclined and there has not been any investigation concerning this configuration. Depending on the inclined orientation, the lid-driven shear may be assisting or opposing the buoyancy force. In such cases the resulting flow and thermal field as well as the heat transfer process will be very different from the horizontal configuration case. Understanding the mixed convection heat transfer process in inclined cavities is thus very important for design purposes in the event of inclined orientation. Oztop [36] investigated numerically natural convection heat transfer in a partially cooled and inclined rectangular enclosure filled with saturated porous medium. A numerical study was performed by Oztop and Varol [37] to obtain combined convection field in an inclined porous lid-driven enclosure heated from one wall with a non-uniformly heater. Mixed convection in an inclined lid-driven enclosure with a constant heat flux using differential quadrature method was examined by Ogut [38].

Study of the flow of electrically conducting fluid in the presence of magnetic field is important from a technical point of view and such types of problems have received much attention by many researchers. There has been widespread interest in the study of effect of magnetic field on natural convection in fluid saturated cylindrical porous annulus/annuli. Most of the studies found in literature on the effect of magnetic field on natural convection are mainly confined to rectangular enclosures or single cylindrical annulus in the presence and absence of porous media. Magnetohydrodynamics deals with dynamics of an electrically conducting fluid, which interacts with a magnetic field. The study of heat transfer and flow, through and across porous media, is of great theoretical interest because it has been applied to a variety of geophysical and astrophysical phenomena. Practical interest of such study includes applications in electromagnetic lubrication, boundary cooling, bio-physical systems and in many branches of engineering and science.

Oreper and Szekely [39] reported numerical simulation of natural convection in a rectangular enclosure with horizontal magnetic field. Their computational results show that the magnetic field can suppress natural convection depends on the strength of magnetic field. Kandaswamy and Kumar [40] studied the effect of a magnetic field on the buoyancy-driven flow of water inside a square cavity with differentially heated side walls numerically. Sellers and Walker [41] studied the steady liquid-metal flow through a rectangular duct with electrically insulated walls in the presence of variable magnetic field. Sankar et al. [42] investigated the effect of direction of magnetic field in a vertical cylindrical annulus. They showed that the radial magnetic field is more effective in suppressing the convection in tall cavities, while the axial magnetic field is effective in shallow cavities. However, some comments on the MHD convection in a porous medium have been done very recently by Nield [22]. Barletta et al. [43] have studied the mixed convection with heated effect in a vertical porous annulus with the radially varying magnetic field. Shivashankar et al. [44] studied the effect of magnetic field on natural convection in a vertical double cylindrical annular cavity filled with a fluid saturated porous media. The main objective of the present numerical study is to comprehend the effect of magnetic field on the natural convection in the double annuli filled with porous media. Rahman and Alim [45] examined the flow and heat transfer in a lid-driven square enclosure with the presence of a magnetic field, Joule heating and heat conducting horizontal circular cylinder.

Double-diffusion is a type of flow driven mainly by buoyancy forces, induced by the gradients of thermal energy and concentration of a chemical component. In oceanography, the sea can be considered a multi component domain with the presence of salts, and the heating of the surface starts a double-diffusive process which drives the water masses and dictates the circulation. In engineering, double-diffusion has applications in the design of solar power collectors, oil recovery and food processing, to name just a few. Therefore, it is of extreme importance to have a detailed understanding of the physics of this phenomena, and the knowledge of which parameters affects the flow behavior. Computational fluid dynamics can be used to simulate double-diffusive processes aiding the scientific and engineering community in the understanding of this complex phenomenon. There has been many scientific works devoted in the use of CFD in this field. Double-diffusive convection occurs in the sun where temperature and Helium diffusions take place at different rates.

Minkowycz et al. [46] discovered that the discontinuity can be avoided by choosing a non-uniform temperature distribution along the walls (i.e. non-uniformly heated walls) while performing an investigation on mixed convection flow over a vertical plate, which is non-uniformly heated/cooled. Teamah et al. [47] studied the effect of the heater length, Rayleigh number, Prandtl number and buoyancy ratio on both average Nusselt and Sherwood number with uniform heating at left vertical wall. Wan and Kuznetsov [48] investigated the fluid flow in a rectangular cavity whose lid vibrates in one standing wavelength. The effects of uniform and non-uniform heating of wall(s) on double-diffusive natural convection in a lid-driven square enclosure are analyzed by Mahapatra et al. [49]. Recently, Mondal and Sibanda [50] studied effects of buoyancy ratio on unsteady double-diffusive natural convection in a cavity filled with porous medium with non-uniform boundary conditions.

### 4.1. Covection in presence of radiation

#### 4.1.1 Non-Darcian natural convection and radiation in a cavity filled with fluid saturated porous medium

Studies on natural convection have received a great attention in recent years due to its importance in practical applications in various modern systems such as nuclear reactors, electronic cooling, building management and solar energy systems. Natural convection flows are, however, particularly complex as they depend on several important parameters among which the geometry and thermophysical characteristics of the fluid are most important. Heat transfer due to natural convection in a cavity saturated with porous media is a new branch of thermo-fluid mechanics. The heat transfer phenomenon can be described by means of hydrodynamics, convective heat transfer mechanism and electromagnetic field as they have a symbiotic relationship (Vafai [51]; Vadasz [52]). The non-Darcy effects on natural convection in porous media have received a great deal of attention in the recent times due to numerous technical applications associated with it, such as fluid flow in geothermal reservoirs, separation processes in chemical industries, solidification of casting, thermal insulation, petroleum reservoir and so on.

Walker and Homsy [53] and Bejan [54] used the assumption of a constant porosity porous medium based on Darcy's model. In order to account for the transition from Darcy flow to highly viscous flow at high permeability values, Brinkman [55] introduced Brinkman-extended Darcy model. Prasad and Tuntomo [56] examined inertia effects on buoyancy driven flow and heat transfer in a vertical porous cavity using the Forchheimer-extended Darcy equation of motion for flow through porous media. Effects of permeability and different thermal boundary conditions on the natural convection in a square cavity filled with porous media using Darcy-Forchheimer model were investigated by Saied and Pop [57], whereas Marcondes et al. [18] had considered Darcy-Brinkman-Forchheimer model in their study. New dimensions have been added in the study of natural convection flow in a cavity filled with porous medium by considering thermal radiation and internal heat source/sink effects. It is well known that the effect of thermal radiation is important in space technology and high temperature processes. Thermal radiation also plays an important role in controlling heat transfer process in polymer processing industry. The effect of radiation on heat transfer problems have been studied by Hossain and Takhar [58]. Later, Pal and Mondal [59] have investigated radiation effects on combined convection over a vertical flat plate embedded in a porous medium of variable porosity. In critical technological applications like nuclear reactor cooling, the reactor bed can be modelled as a heat generating porous medium, quenched by a convective flow.

Reddy and Narasimhan [60] studied the heat generation effects on natural convection inside a porous annulus. The aim of the present work is to study the influence of thermal radiation and heat on natural convection flow in a square cavity filled with a fluid-saturated porous medium with isothermal vertical walls and adiabatic horizontal walls by considering Darcy-Brinkman-Forchheimer model. The governing equations are discretized using finite-difference method with staggered grid formulation following

MAC method proposed by Harlow and Welch [61]. The Poisson equation for pressure is derived using momentum and continuity equations and solved by Bi-CG-Stab method. The numerical results for streamline, isotherms, velocity, temperature profiles and the heat transfer rate at the heated walls in terms of local Nusselt number and average Nusselt number are presented. With this help of this method Mahapatra et al. [62] examined influence of thermal radiation on non-Darcian natural convection in a square cavity filled with fluid saturated porous medium of uniform porosity. They have computed average Nusselt number for Darcy, non-Darcy and non-porous models for different values of Rayleigh number and thermal radiation parameter (see Table 1). It is observed from this table that the values of average Nusselt number for non-porous model are higher than those of Darcy and non-Darcy models. So it is useful to include thermal radiation effects in the study of natural convection in a square cavity with uniform porosity.

Table 1: Computed values of $\overline{Nu}_H|_{x=0}$ when $He = 0.2$ for various values of $Ra$ and $N_R$ (source: Adapted from [62]).

| | | $\overline{Nu}_H\|_{x=0} \longrightarrow$ | | |
|---|---|---|---|---|
| $R_a$ | $N_R$ | Darcy model $Da = 10^{-6}, \epsilon = 0.4$ | non-Darcy model $Da = 10^{-6}, \epsilon = 0.4$ | non-porous |
| $10^8$ | 0.5 | 2.2946 | 2.3129 | 34.0726 |
| | 1.0 | 2.5604 | 2.5817 | 34.6553 |
| | 5.0 | 2.9046 | 2.9298 | 35.1909 |
| | 10.0 | 2.9623 | 2.9881 | 35.2642 |
| $10^9$ | 0.5 | 10.5798 | 11.4722 | 60.4263 |
| | 1.0 | 11.2802 | 12.2260 | 60.9234 |
| | 5.0 | 11.9901 | 12.9798 | 61.3493 |
| | 10.0 | 12.0909 | 13.0857 | 61.4045 |

### 4.1.2 Convection-radiation in a Lid-driven Cavity

The problem of natural convection flow in a lid-driven cavity filled with a porous medium has received considerable attention in recent years because of its relevance to the thermal performance of many industrial and technological applications. Some of these include nuclear reactors, heating and cooling of electronic devices, solar collectors, oil extraction and crystal growth. In the past several decades, a number of experimental and numerical studies have been performed to analyze the flow field and heat transfer characteristics of lid-driven cavity flow. In this context, non-Darcy effects on natural convection in porous media has received a great deal of attention in the recent times due to a large number of industrial and technological applications associated with it such as fluid flow in geothermal reservoirs, separation processes in chemical industries, solidification of casting, storage of nuclear waste, thermal-hydraulics of nuclear reactors, thermal insulation, solar power collectors, crude oil production, packed-bed catalytic reactors, separation process in chemical industries and so on (Vafai [63]; Nield and Bejan [64]).

Medeiros et al. [16] studied numerically heat transfer by natural convection in a porous cavity under a non-Darcian approach for uniform porosity using Darcy-Brinkman-Forchheimer model. They compared the numerical results with the works that considered uniform porosity and found to have excellent accuracy. Nithiarasu et al. [17] examined the effect of applied heat transfer on the cold wall of the cavity flow and heat transfer inside the porous medium. When technological processes take place at high temperatures, thermal radiation heat transfer become very important and its effects cannot be neglected. Recent developments in hypersonic flights, missile reentry rocket combustion chambers and gas cooled nuclear reactors have focused attention of researchers on thermal radiation as a mode of energy transfer and emphasized the need for inclusion of radiative transfer in these processes. For keeping these studies in mind, we concentrated on the interaction of radiative flux with thermal convection flows.

Recently, Aldawody and Elbashbeshy [65] investigated the effects of thermal radiation and magnetic field on flow and heat transfer over an unsteady stretching surface in a micropolar fluid. Turkyilmazoglu [66] analyzed the combined influences of viscous dissipation, joule heating, temperature-dependent viscosity on the time- dependent MHD permeable flow having variable viscosity. Rafique et al. [67] presented numerical simulations of natural convection heat transfer along a vertical cylinder for four different geometries. Cheng [68] examined the flow and heat transfer in 2D square cavity where the flow is induced by a shear force resulting from the motion of the upper lid combined with buoyancy force due to bottom heating. Moreover, the presence of the internal heat-generation effect in the model was found to have significant influence on the isotherms and less significant effects on the streamlines for small values of the Richardson number. Then Mahapatra et al. [69] studied the influence of thermal radiation and internal heat generation on natural-convection flow field and temperature distribution in a lid-driven square cavity filled with fluid-saturated porous medium (see Fig. 2).

#### 4.1.3 Non-uniform heating of convective-radiative flow

In previous sections, we analyzed the effect of thermal radiation in non-Darcy natural convection in a square cavity and in a lid-driven cavity with uniform heating of walls. Studies on natural convection in a lid-driven square cavity filled with a porous medium have received a considerable attention in recent years because of its relevance to many industrial and technological applications such as crude oil production, storage of nuclear waste, compact beds for the chemical industry and thermal insulation etc. Extensive literature survey has already been discussed before in this paper.

New dimensions are added in the study of natural convection flow in a cavity filled with porous medium in the presence of thermal radiation and internal heat source/sink. It is well known that the effect of thermal radiation is important in space technology and high temperature processes. Thermal radiation also plays an important role in controlling heat transfer process in polymer processing industry as mention in previous few chapters. The effect of radiation on heat transfer problems have been studied by many authors

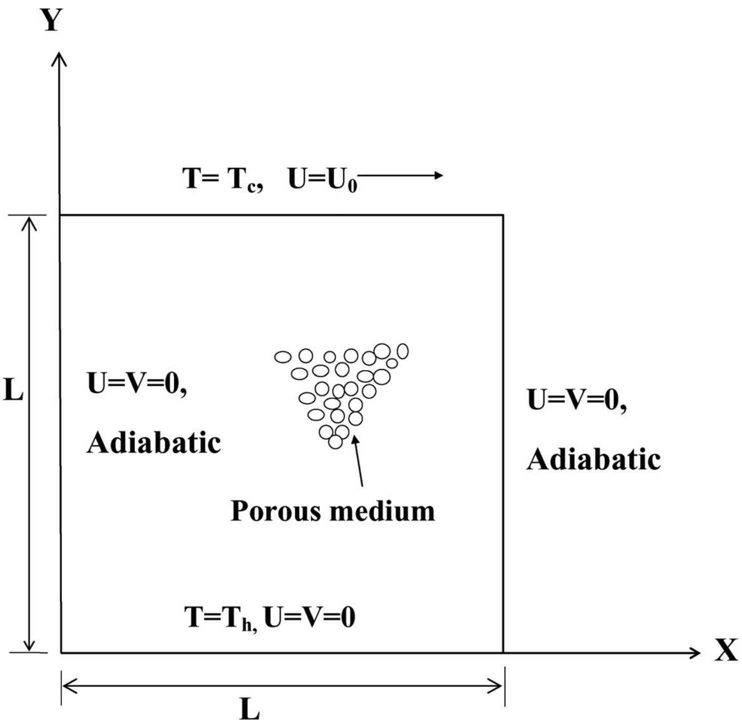


Figure 2: Schematic diagram of the the physical model (source: Adapted from [69]).

(Badruddin et al. [70]; Salman Ahmed et al. [71]). There are few works in the field of mixed convection cavity flow in the presence of thermal radiation. The concept of convecting flow is utilized in critical technology applications, for example in nuclear reactor cooling, the reactor bed can be modelled as a heat generating porous medium are very important from the point of view of convective flows. Recently, Mahapatra et al. [72] studied the influence of thermal radiation and internal heat generation on natural convection in a lid-driven square cavity having fluid-saturated porous medium and non-uniform heating by considering Darcy-Forchheimer-Brinkman model using finite-difference technique with staggered grid distribution (see Fig. 3).

#### 4.1.4 Convective-radiative flow in an inclined enclosure in presence of magnetic field

The study of the effect of magnetic field on natural convection in fluid-saturated porous medium has received considerable attention in recent years due to its wide variety of applications in engineering and technology such as nuclear reactor insulation, solar energy collection, cooling of electronic devices, furnaces, drying technologies and crystal growth in liquids, etc. As the Lorentz force suppresses the convection currents by reducing the velocities when the fluid is electrically conducting and exposed to a magnetic field, so the presence of an external magnetic field is being used as a control mechanism in material manufacturing industry. Rudraiah et al. [73] studied the effect of magnetic

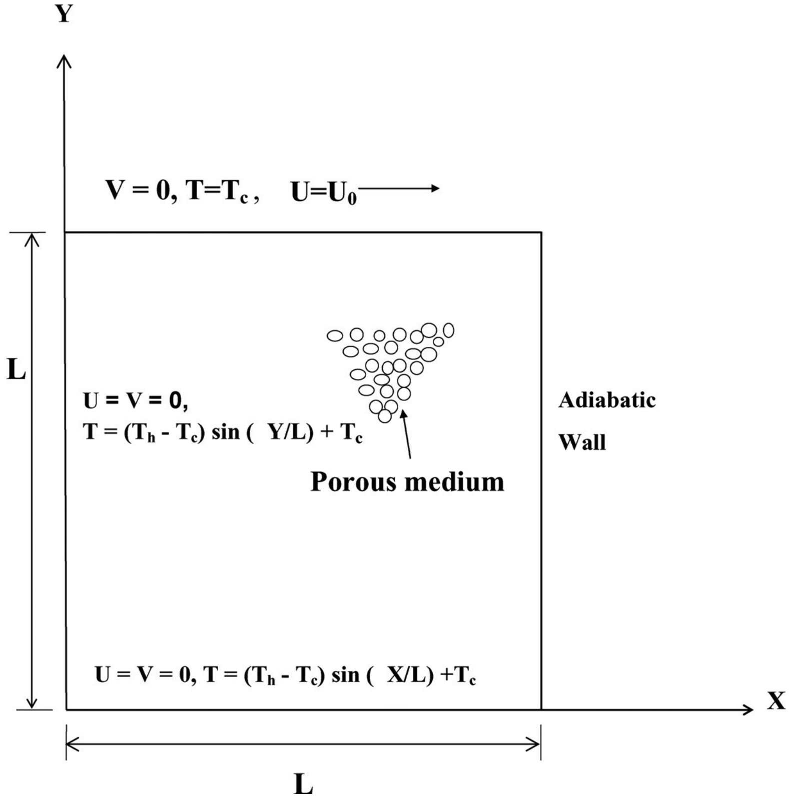


Figure 3: Schematic diagram of the physical system (source: Adapted from [72]).

field on free convection inside a rectangular enclosure. They found that a circular flow was formed with a relatively weak magnetic field, the convection was suppressed and the rate of convective heat transfer decreased when the magnetic field strength was increased. Garandet et al. [74] analyzed the effect of magnetic field on buoyancy driven convection in a rectangular enclosure. Al-Najem et al. [75] showed that the increase in the Hartmann number causes reduction in the heat transfer rate from cavity sidewalls. Sarris et al. [76] examined two-dimensional unsteady simulations of MHD natural convection of a liquid-metal in a laterally and volumetrically heated square cavity. Chamkha [77] investigated on unsteady laminar combined forced and free convection flow and heat transfer of an electrically conducting and heat generating or absorbing fluid in a vertical lid-driven cavity in the presence of a magnetic field.

Mahmud et al. [78] studied combined free and forced convection flows of an electrically conducting and heat-generating/absorbing fluid analytically in a vertical channel with two parallel plates under the action of transverse magnetic field. Ece and Büyük [79] studied numerical solutions for the velocity and temperature fields inside the rectangular enclosure and determined the effect of strength and direction of the magnetic field, the aspect ratio and the inclination of the enclosure on the transport phenomena. Later, Ece and Büyük [80] investigated the steady natural convection flow in an inclined square enclosure with differentially heated adjacent walls under the influence of magnetic field.

Jordan [81] studied the effects of thermal radiation and viscous dissipation on MHD unsteady free-convection flow over a semi-infinite vertical porous plate. He examined the velocity, temperature, local skin-friction and local Nusselt number for various physical parameters like the radiation parameter, Eckert number, magnetic number and suction (or injection). The effect of heat-generation/absorbtion in an enclosure in the presence of magnetic field also plays an important role in convective flows. Grosan et al. [82] discussed the effects of magnetic field and internal heat generation on the free convection in a rectangular cavity filled with a porous medium. Mansour et al. [83] studied the effects of an inclined magnetic field on the unsteady natural convection in an inclined cavity filled with a fluid saturated porous medium considering heat source in the solid phase. After that, Mahapatra et al. [84] studied mixed convection flow in an inclined enclosure under magnetic field with thermal radiation and heat generation (see the mathematical model in Fig. 4).

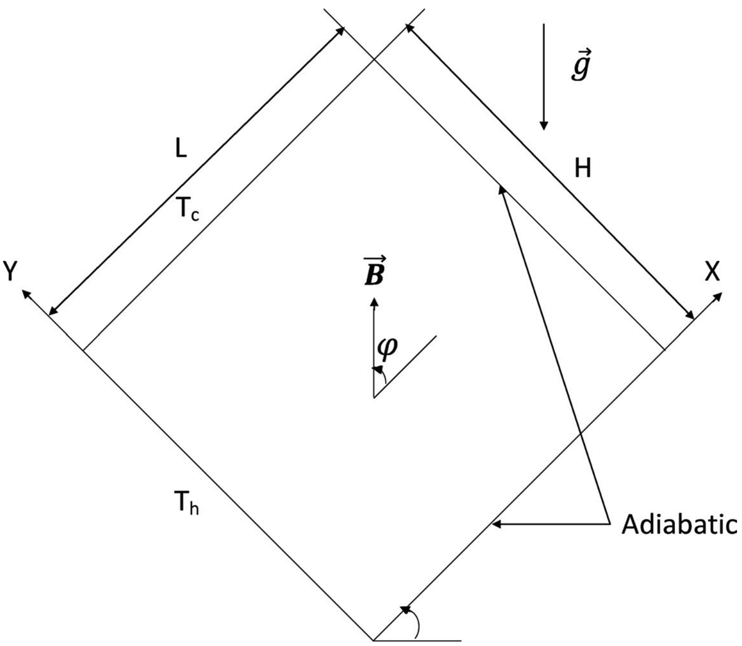


Figure 4: Schematic diagram of the physical system (source: Adapted from [84]).

### 4.1.5 Radiative heat and mass transfer in cavity

In our study we have found very few studies on double diffusive convection in presence of thermal radiation. Among those we have tried to focus some literature in our study. Shanker et al. [85] studied Radiation and mass transfer effects on unsteady MHD free convection fluid flow embedded in a porous medium with heat generation/absorption. The effect of thermal modulation on the onset of double-diffusion natural convection in a horizontal fluid layer was studied analytically and numerically using a linear stability analysis by Malashetty et al. [86]. The effects of combined thermal and solutal buoy-

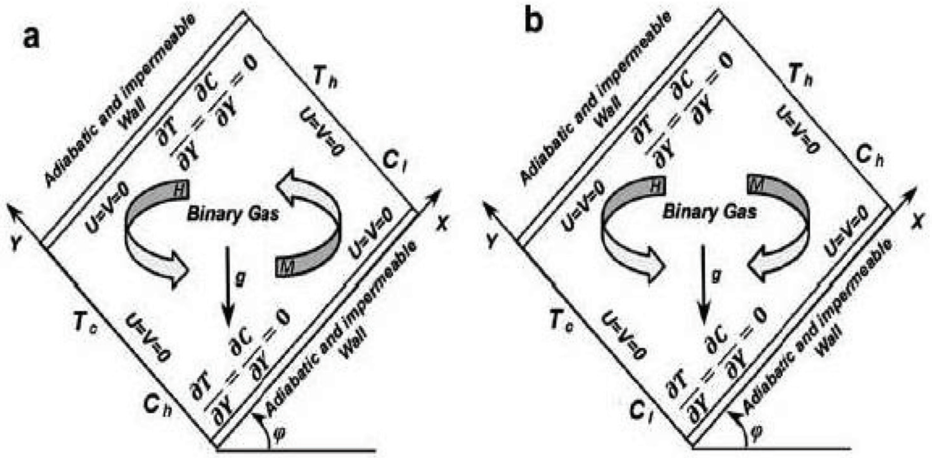


Figure 5: Schematic geometry of the problem studied: (a) aiding flow, (b) opposing flow (source: Adapted from [88]).

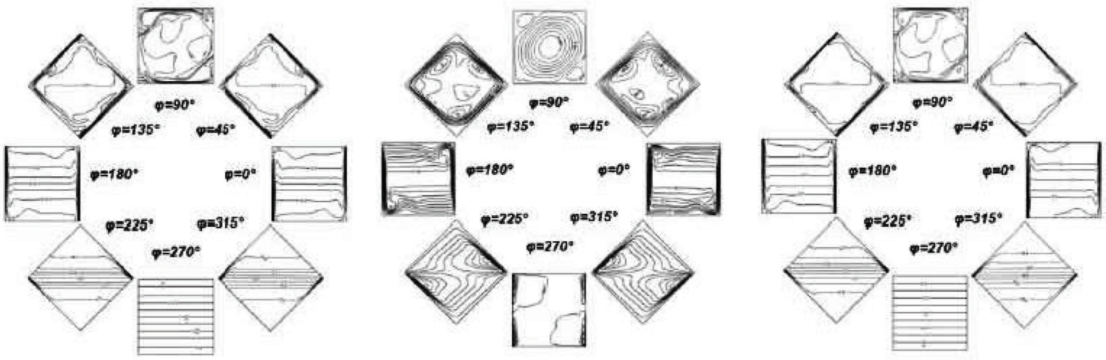


Figure 6: Isotherms, streamlines and iso-concentrations for the angle from 0° to 360° (source: Adapted from [88]).

ancy induced by temperature and concentration gradients have, however, not been widely studied. Mezrhab et al. [87] studied double-diffusion convection coupled to radiation in a square cavity filled with a participating grey gas. In there paper presents numerical solutions for the coupled radiation and natural convection heat transfer by double diffusion in a square cavity. Moufekkir et al. [88] studied double-diffusive natural convection and radiation in an inclined cavity using lattice Boltzmann method (see the mathematical model in Fig. 5). This study deals with the presentation of a numerical investigation of coupled double diffusive convection and volumetric radiation in a tilted and differentially heated square enclosure filled with a gray fluid participating in absorption, emission and non scattering. It is seen from this figure that the isotherms, iso-concentrations and streamlines present mirror symmetry against the cooperating flow in absence of thermal radiation (see Fig. 6). Teamah et al. [89] studied the effects for a wide range of thermal

Grashof number and aspect ratio coupled with the inclination angle. The obtained results for average Nusselt and Sherwood numbers were correlated.

## 5. Computational analysis of convective flow with chemical reaction in cavity

The presence of chemical reaction can have a impact on double-diffusive flow and heat and mass fluxes, mainly when the heat equation is complemented with internal heat generation which may occur as a result of a chemical reaction. We can get detail of the property, application about the effect of the chemical reaction in [90]. After that, Lee and Lee [91] first studied unsteady convective diffusion problems with chemical reaction in a rectangular, numerically. By seeing so many application in real life like combustion in hybrid rockets and fires over structures and liquid pools the researcher start their research works on convection in cavity after the long time. Bortoli [92] examined the laminar flows inside the cavity caused by temperature increase due to chemical reaction (see the model Fig. 7). It is seen from this study the average temperature inside the cavity for $t \leq 0.01$ (see Fig. 8). Recently, Rashad et al. [93] examined the unsteady heat and mass transfer in a square porous cavity in presence of chemical reaction and thermal radiation.

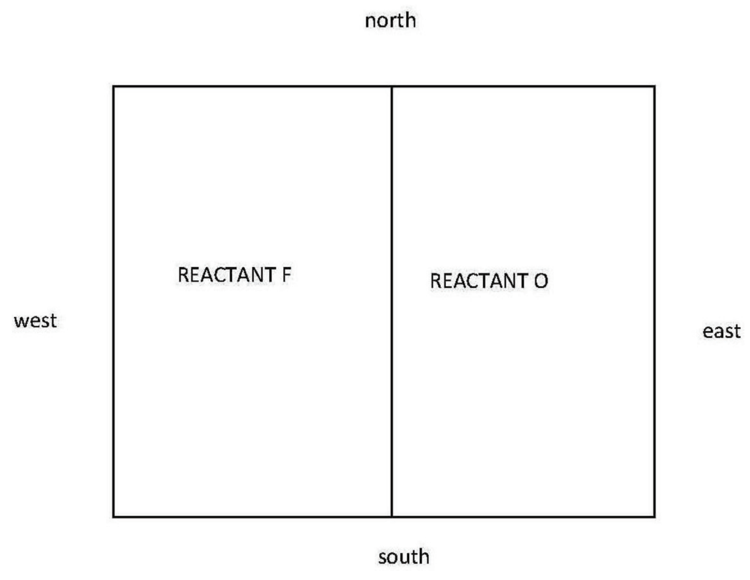


Figure 7: Indication of cavity boundaries (source: Adapted from [92]).

## 6. Effects of buoyancy ratio on an unsteady double diffusive convection in an inclined rectangular enclosure

The study of double diffusive natural convection in fluid-saturated cavity has received a great attention due to its importance in engineering and technology in past years. Ece and

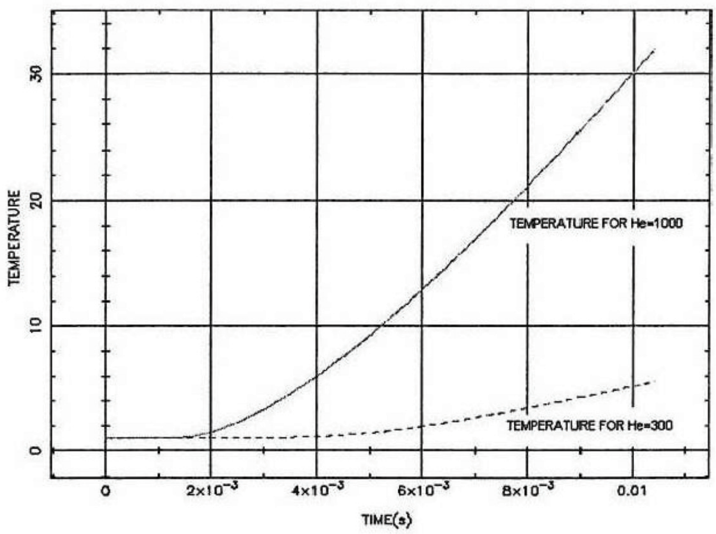


Figure 8: Temperature for square cavity mixing (laminar) flow including chemical reaction for $Da = 300$, $Z = 5$, $He = 300$ and 1000, $Gr = 10$, $Re^2 = 1000$ (source: Adapted from [92]).

Büyük [94] studied the steady natural convection flow in an inclined square enclosure with differentially heated adjacent walls under the influence of magnetic field. Mahapatra et al. [84] studied mixed convection flow in an inclined enclosure under magnetic field with thermal radiation and heat generation. Here, we analyze the effects of buoyancy ratio on unsteady double-diffusive natural convection in an inclined rectangular enclosure with a fixed magnetic field angle 45° angle.

## 6.1. Mathematical Model

Consider an unsteady-state two-dimensional rectangular cavity of height $H$ and length $L$ as shown in Fig. 9. It is assumed that the top wall is considered is cooled by means of a constant temperature. The bottom wall and right vertical wall are to be adiabatic and left vertical wall is uniformly heated and concentrated. The thermophysical properties of the fluid are assumed to be constant except the density variation in the buoyancy force, which is approximated according to the Boussinesq approximation. The density of the fluid can be described by the following equation:

$$\rho = \rho_0[1 - \beta_T(T - T_c) - \beta_C(C - C_c)] \tag{15}$$

where $\beta_T$ and $\beta_C$ are the thermal and concentration expansion coefficients, respectively. The angle of inclination of the enclosure with the horizontal line in the counter-clockwise direction is denoted by $\phi$. The magnetic field strength $B_0$ is applied at an angle $\varphi$ with respect to the coordinate system. The magnetic Reynolds number is assumed to be small and the induced magnetic field due to the motion of the electrically conducting fluid is neglected. The Joule heating of the fluid and the effect of viscous dissipation are also negligible.

### 6.2. Results and discussions

Staggered grid formulation with finite-difference technique by MAC (marker-cell) method is used here to solve the governing equations (see MAC cell in Fig. 10). The working fluid in this study was chosen to be air. The inclination angle $\phi$ of the enclosure, magnetic field angle in the enclosure and buoyancy ratio are such that $45° \leq \phi \leq 135°$, $\varphi = 45°$ and $-20 \leq N \leq 20$ respectively. Numerical results for the streamline, isotherm and iso-concentration contours inside the inclined rectangular cavity for different values of buoyancy ratio ($N$) and inclination angle ($\phi$) have been examined and are presented graphically in Figures 11–13.

Fig. 11 shows the effects of $N$ on the streamlines, isotherms and as well as on the iso-concentrations for wide range of variations in the buoyancy ratio ($N$) when $\phi = 45°$ for $P_r = 0.7$, $Gr_T = 10^4$, $Ha = 10^2$, $\delta = 2$, $Le = 1.0$ and $He = 1.0$. As expected due to a hot vertical wall, fluids rise up along the side of the hot vertical wall and flow down along the cold vertical wall forming a roll with clockwise and anticlockwise rotations inside the cavity with forming three different eddies when $N = 20$. Among them two of the eddies are rotated in clockwise direction. These three different eddies are form due to the large value of $N$. As the value of $N$ decreases from 20 to 1 the values of stream functions decrease, because of the two buoyancies are equal to and opposite each other. Here, we can see that the eddy (among the three eddies) with the flow circulation in the anticlockwise directions is covered more area in the cavity then the previous figure when $N = 20$. Again, when the value of $N$ is further decreased from 1 to $-20$ then the values

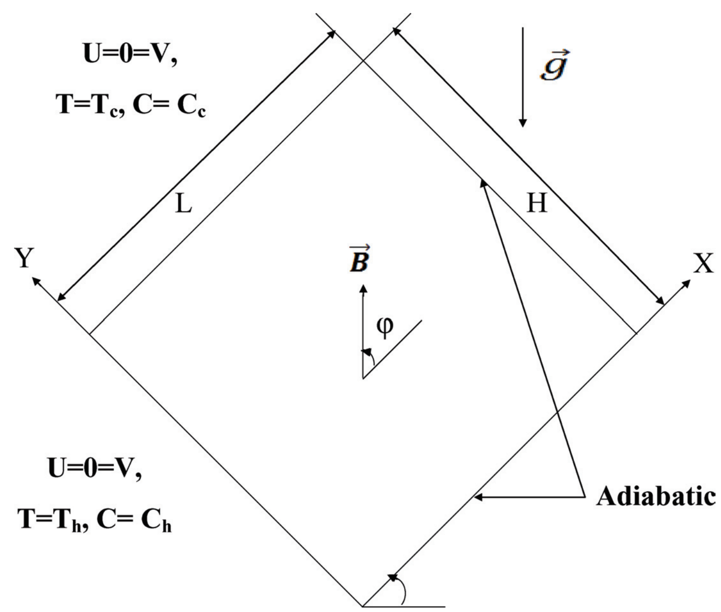


Figure 9: A sketch of the physical problem.

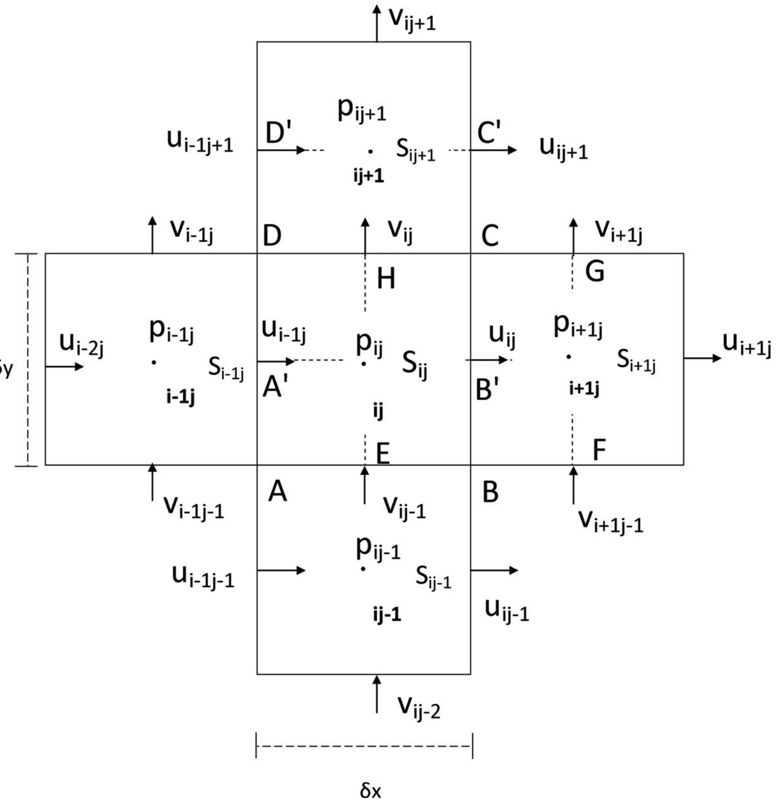


Figure 10: Control volume for $u$-momentum, $v$-momentum, temperature and concentration equations.

of stream function increases. Because of the buoyancy forces that drive the fluid motion are mainly due to the gradients of temperature when $N = -20$. In this figure it is clear that when $N = 20$, the contours of the isotherm are dispersed in the whole cavity, but as $N$ decreases from 20 to 1 the maximum isotherms are concentrated towards the cold wall. Again, for further decreasing of the value $N$ from 1 to $-20$, the isotherms are concentrated towards the cold wall indicating that most of the heat transfer rate is carried out by heat conduction. Again, in this figure we can see that iso-concentration contours are concentrated near the hot wall. But, as the value of $N$ decreases from 20 to $-20$ the iso-concentration contours are dispersed towards the adiabatic wall this is due to higher mass transfer rate.

Fig. 12 depicts the effect of $N$ on the streamlines, isotherms and as well as on the iso-concentrations for wide range of variations in the buoyancy ratio ($N$) when $\phi = 90°$ for $P_r = 0.7$, $Gr_T = 10^4$, $Ha = 10^2$, $\delta = 2$, $Le = 1.0$ and $He = 1.0$. As the inclination angle increases the streamlines forms a single eddy near the uniformly heated and concentration wall with anticlockwise directions when $N = 20$. The center of the single eddy is slightly shifted away from the heated wall towards the adiabatic wall as $N$ decreases from 20 to 1. Again, it interesting to note that for $N = -20$, the effect of solutal buoyancy force is in the opposite direction of thermal buoyancy force.

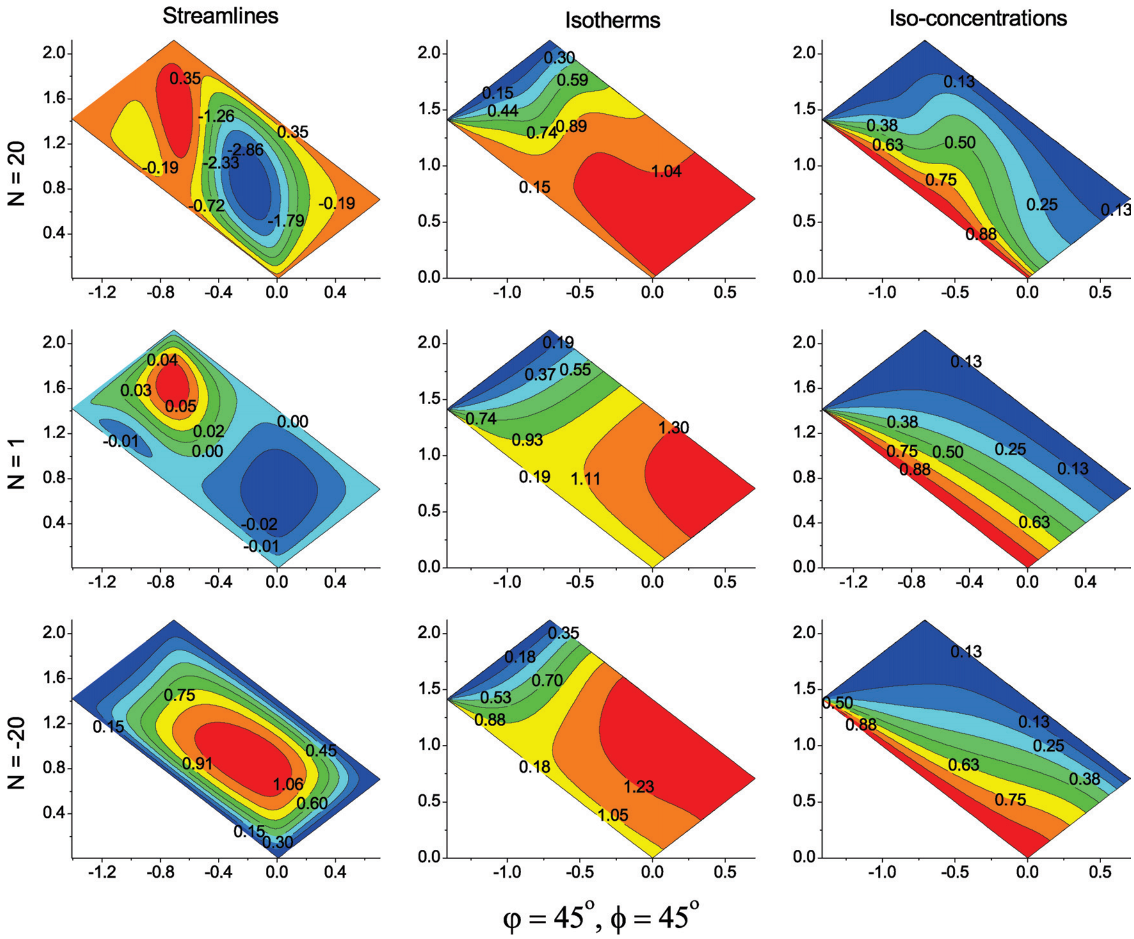


Figure 11: Contour plots for $P_r = 0.7$, $Gr_T = 10^4$, $Ha = 10^2$, $\delta = 2$, $Le = 1.0$, $He = 1.0$ and $\phi = 45^\circ$ for different values of $N$.

Therefore, it is noticed that the magnitude of the thermal buoyancy force is very small compared to the solutal buoyancy force. For which reason we can see the streamlines are in clockwise direction which is different from other two figures of streamlines. As $N$ decreases the isotherms are dispersed towards the adiabatic walls form the cold wall by dividing into two parts due to stronger heat transfer rate from the heated wall. But as $N$ decreases the iso-concentrations are concentrated to the hot wall. Streamline, isotherm and iso-concentration contours for different values of $N$ when $P_r = 0.7$, $Gr_T = 10^4$, $Ha = 10^2$, $\delta = 2$, $Le = 1.0$, $He = 1.0$ and $\phi = 135^\circ$ with uniformly heated and concentrated wall are displayed in Fig. 4. Comparing it can be said by compairing Figs. 12 and 13 that the patterns of streamlines, isotherms and iso-concentrations are almost similar for uniformly heated and concentrated cases.

The concentration field is influenced by the change of flow structure due to large value of buoyancy ratio. The fluid flow inside the cavity is strongly affected by counter-clockwise rotated multiple eddies.

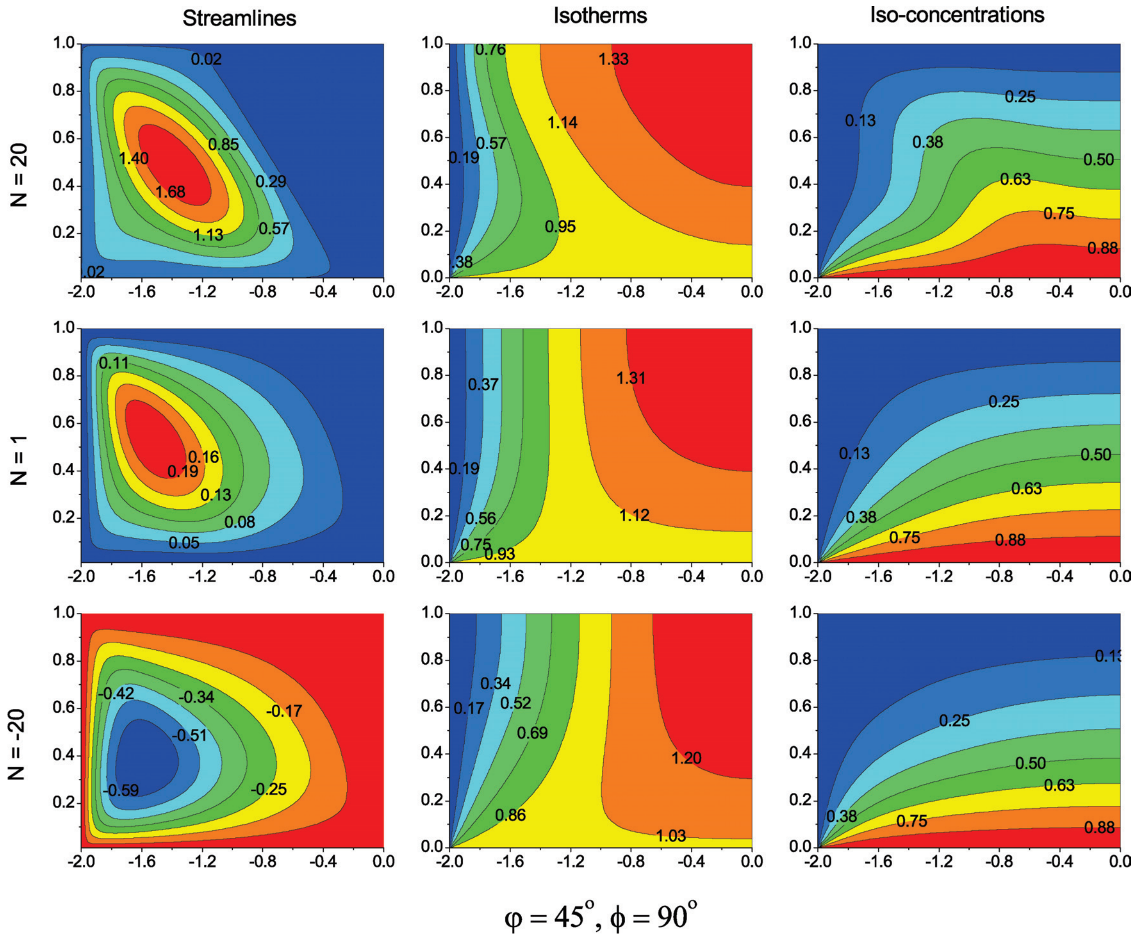


Figure 12: Contour plots for $P_r = 0.7$, $Gr_T = 10^4$, $Ha = 10^2$, $\delta = 2$, $Le = 1.0$, $He = 1.0$ and $\phi = 90°$ for different values of $N$ when all other parameters are fixed.

## 7. Conclusion

In this review paper, a comprehensive outlook on the research progress made on natural convection or mixed convection in different type of enclosures with different models is presented and reviewed, considering both newtonian and non-newtonian fluids as working fluid with chemical reaction and thermal radiation. From this study we have observed that the effects of chemical reaction and thermal radiation in different type of cavities filled with porous or non-porous media are increasing now a day due to the significance importance in science and technology. There are few studies that have been made on mathematical modelling of the cavity flow problem in presence of chemical reaction. In future work we plan to develop mathematical models for different types of cavity flow problems with a chemical reaction.

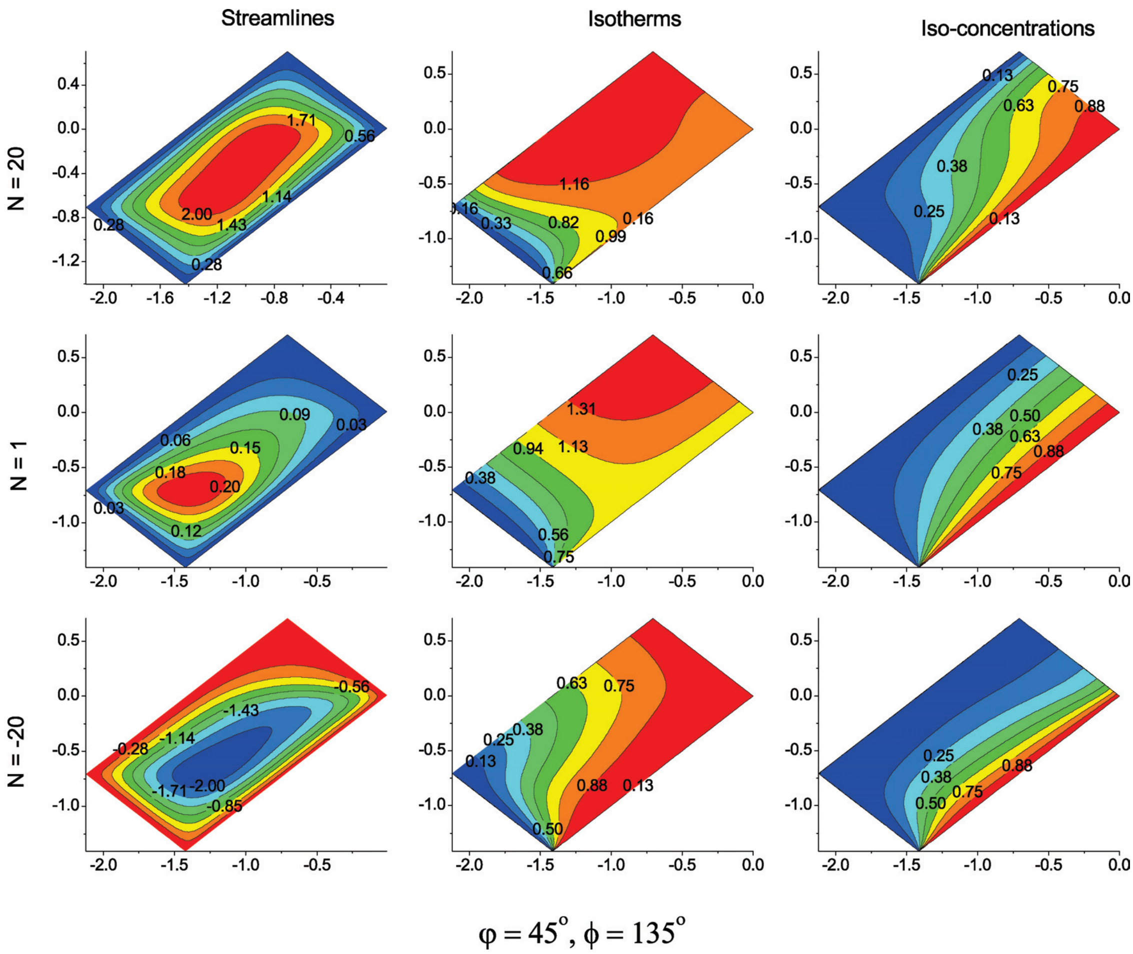


Figure 13: Contour plots for $P_r = 0.7$, $Gr_T = 10^4$, $Ha = 10^2$, $\delta = 2$, $Le = 1.0$, $He = 1.0$ and $\phi = 135°$ for different values of $N$ when all other parameters are fixed.

**Acknowledgement**

Authors are thankful to the University of KwaZulu-Natal, South Africa for the support. The authors wish to thank the referees for helpful comments and suggestions which improved the quality of the paper considerably. The authors wish to thank Dr. Vijay Kumar for his valuable suggestions.

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