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On Cordial Labeling: Gluing of Paths and Quadrilateral Snake Graphs on Cycle Graph

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Abstract

In this paper, have given cordial labeling as an application of communication networks for the graph $C_{4r} \times tP_n, \forall\, r,\ t > 1$ and $n > 1$, that is, gluing of $'t'$ number of path P_n on each vertex of cycle C_{cr} also cycles C_{4rt} passing through vertices of each and every remaining $n - 1$ level of path P_n, and also further shown that cordial labeling for the quadrilateral snake graph $QS_m, \forall\, m$ gluing with each vertex of graphs $QS_m(C_{4r} \times tP_n), \forall\, m$. Finally domination numbers of this graph are analyzed.

AMS subject classification: 05C78.
Keywords: Cordial labeling, Path, Quadrilateral Snake Graph, cycle.

1. Introduction

Let f be a function from the vertices $u,\ v$ of a graph G to binary number of set $\{0, 1\}$ and for each edge uv assign the label $|f(u) - f(v)|$. Call f a cordial labeling of G if the number of vertices labeled / assigned 0 and the number of vertices labeled / assigned 1 differ by at most 1 and the number of edges correspondingly getting label 0 and the number of edges correspondingly getting label 1 differ by at most 1. A graph with cordial labeling is called cordial graph.

The classifications of two main and very important different types of graph labeling of any non-trivial graph are called graceful labeling and harmonious labeling. Graceful labeling was introduced independently by Rosa [5] in 1966 and Golomb [3] in 1972, while harmonious labeling was studied by Graham and Sloane [4] in 1980. I. Cahit shown in his paper that the every tree graph is cordial graph and $K_{m,n}$ is cordial for all m and n and in [6], Sethuraman et al., have shown that the one edge union of shell graphs and one vertex union of complete bipartite graphs are Cordial graphs. More information on graph labeling can be referred in the last version of dynamic survey written by Joseph A Gallian [2]. Let us denote for this binary number/labeling set V_0 and set V_1 respectively define as the number of vertices allotted/matched with the label/digit 0 and the number of vertices allotted/matched the label/digit 1 and let us denote the set E_0 and the set E_1 respectively denote/define as the number of edges getting the label/digit 0 and the number of edges getting the label/digit 1.

Here we had an attempt to introduce new way of construction of graph that is uniquely different from common graph operations like[7–20], one vertex or one edge union of graphs, Cartesian product, Normal and General product and lexicographic product of graph, et..., Here we aimed/tried at deriving new technique/unique construction called attaching of $'t'$ number of path on each vertex of cycle, that is each vertex of the graph $C_{4r} \times tP_n$, $\forall r$, $t \geq 1$ and $n > 1$ attached with Quadrilateral Snake graph QS_m, $\forall m$. We define QS_m, $\forall m$ as follows.

The graph QS_m, $\forall m$ is called Quadrilateral Snake graph. It is defined as series connection of non-adjacent vertices of $'m'$ number of cycle C_4 and in general these vertex set V and edge set E have described below

$$V(QS_m) = \{c_k\}_{k=1}^{m+1} \cup \{u_i\}_{i=1}^{m} \cup \{v_j\}_{j=1}^{m},$$

and

$$E(QS_m) = \{c_k u_k\}_{k=1}^{m} \cup \{c_k\, v_k\}_{k=1}^{m} \cup \{u_k\, c_{k+1}\}_{k=1}^{m} \cup \{v_k\, c_{k+1}\}_{k=1}^{m}.$$

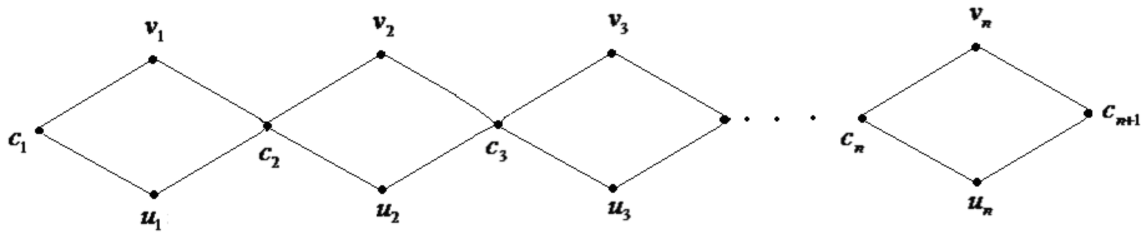
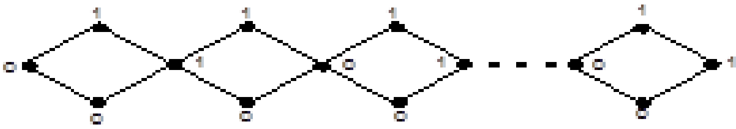


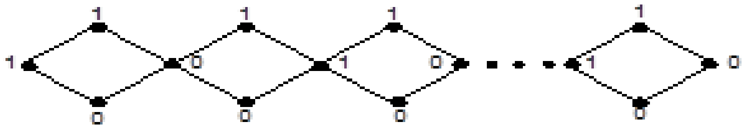
Figure 1: Quadrilateral Snake Graph QS_m, $\forall m$.

Remark 1.1. For the comfortable to use QS_m, $\forall m$ for gluing at all the different vertices $C_{4r} \times tP_n$ as, there are two different possible cases of cordial labeling of QS_m, $\forall m$ as

Case-I



Case-II



From the above two cases, the quadrilateral snake graph $QS_m, \forall m$, by definition of cordial labeling, the vertices set $V_0 = V_1$ and the edges set $E_0 = E_1$. Hence these graphs are called Cordial graph.

2. Cordial labeling of the graph $G = C_{4r} \times tP_n,\ \forall r, t \geq 1$ and $n > 1$

The graph G is a graph generated from by gluing of cycle graph and path graph. In this graph $C_{4r} \times tP_n$, $\forall r, t \geq 1$ and $n > 1$ the base cycle of length $4r$, that is C_{4r} and remaining cycles are of the form $C_{4rt}('t'$ number of path gluing at each vertex) and the base cycle (cycle with length $4r$) have parallel chords (an edges between non-adjacent vertices of C_{4r}), that is, $V^1_{2,0}$, $V^1_{4r,0}$, $V^1_{3,0}$, $V^1_{4r-1,0}, \ldots,$ and $'t'$ number of path P_n start from each vertices of base cycle C_{4r} in G and $n-1$ number of cycle C_{4rt} passes through remaining $n-1$ vertices path P_n. It is denoted by the graph $G = C_{4r} \times tP_n$, $\forall r, t \geq 1$ and $n > 1$, is shown in the following figure 2.1.

The convenience of cordial labeling for the graph $G = C_{4r} \times tP_n$, $\forall r, t \geq 1$ and $n > 1$ the vertices are arranged in a sequence in the following manner/pattern, when n is even.

$$v^n_{1,1},\ v^n_{1,2},\ v^n_{1,3}, \ldots,\ v^n_{1,t},\ v^n_{2,1}, \ldots,\ v^n_{4r,t}$$

$$v^{n-1}_{4r,t},\ v^{n-1}_{1,1},\ v^{n-1}_{1,2}, \ldots,\ v^{n-1}_{4r,t-1}, \ldots,\ v^{n-1}_{4rt(n-1),0}, \ldots,\ v^1_{a,0},$$

where $a = 4r - \left\lfloor \dfrac{n-2}{t} \right\rfloor - 1$ when n is odd

$$v^n_{1,2},\ v^n_{1,3},\ v^n_{1,4}, \ldots,\ v^n_{4r,t},\ v^n_{1,1}$$

$$v^{n-1}_{1,1},\ v^{n-1}_{1,2},\ v^{n-1}_{1,3}, \ldots,\ v^1_{4rt(n-1,0)}, \ldots,\ v^1_{a,0},$$

where $a = 4r - \left\lfloor \dfrac{n-2}{t} \right\rfloor - 1$ $\qquad\cdots\cdots I.$

Theorem 2.1. The graph $C_{4r} \times tP_n$, $\forall r, t \geq 1$ and $n > 1$ is cordial graph.

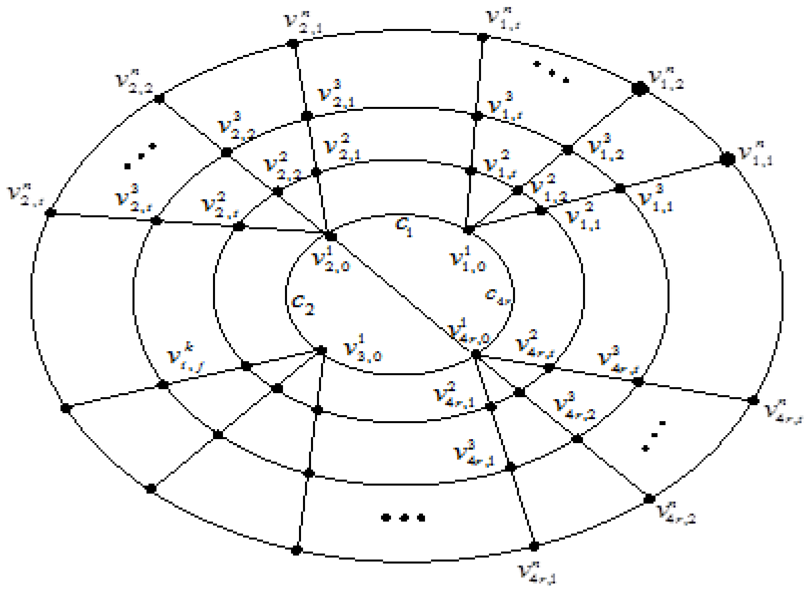


Figure 2: The general graph $C_{4r} \times tP_n$.

Proof. Let G be a graph $C_{4r} \times tP_n$, $\forall r, t \geq 1$ and $n > 1$ has $V = 4r[t(n-1)+1]$ number of vertices and $E = 2r[4t(n-1)+3]-1$ number of edges. For convenience of labeling, the vertices of graph G are arranged as patterns are desribed in (I). In the graph G edges set are divided into three sets as the edges along the path $P_{i,j}^{k}$, $i = 1\, to\, t$, $j = 1\ to\ n-1$, $k = 1\ to\ 4rt$ and the edges along the cycle $C_{i,j}^{k}$, $i = 1\ to\ 4r$, $j = 1\ to\ t$, $k = 1\ to\ n$ and the edges as chords K_i, $i = 1\ to\ 2r-1$.

The Cordial labeling of graph $C_{4r} \times tP_n$ as shown in the table 1.

From table 1, the number of vertices of graph $G = C_{4r} \times tP_n$, $\forall r, t \geq 1$ and $n > 1$ assigned the label/digit 0 are equal to the number of vertices of graph $G = C_{4r} \times tP_n$, $\forall r, t \geq 1$ and $n > 1$ assigned the label/digit 1 and the number of edges of graph $G = C_{4r} \times tP_n$, $\forall r, t \geq 1$ and $n > 1$ getting the label/digit 0 and the number of edges of the graph $G = C_{4r} \times tP_n$, $\forall r, t \geq 1$ and $n > 1$ getting the label/digit 1 differ by at most 1. Hence it is clear that the graph G is Cordial labeled graph or Cordial graph. ∎

Remark 2.2. In graph $G = C_{4r} \times tP_n$, $\forall r, t \geq 1$, Edge labeling for edge between cycle C_{4r} and C_{4rt} when $n = 2$. The Cordial labeling for edges of $P_{i,j}^{k}$, $i = 1\ to\ t$, $k = 1\ to\ 4rt$ and $j = 1$ as 0011 sequence is as follows: When $j = 1$, the edge labels are difference of vertex label of cycle C_{4rt} and cycle C_{4r} with label for vertices of cycle C_{4r} using sequence (0011)r starts with vertex $\left|4r - \left\lfloor \frac{n-2}{t} \right\rfloor\right|$. In this the number of edges with label '0's and '1' s are equal.

Table 2.1

Labeling sequence		V_0	V_1	$V_0 - V_1$	$E_0 - E_1$
Vertices	Edges				
$V=(0011)^{r[t(n-1)+1]}$	$P^k_{i,j}=\begin{cases}\text{Remark 1.1}\\ \text{For each } k=1 \text{ to } 4rt\\ j=1 \text{ and } i=1 \text{ to } t\end{cases}$ $P^k_{i,j}=\begin{cases}(10)^{\frac{n-2}{2}} & \begin{cases}n \text{ is even and}\\ i \text{ is odd},\\ j=2 \text{ to } n-1, \forall k\end{cases}\\ (10)^{\frac{n-2}{2}} & \begin{cases}n \text{ is even and}\\ i \text{ is even},\\ j=2 \text{ to } n-1, \forall k\end{cases}\\ (10)^{\frac{n-3}{2}}1 & \begin{cases}n \text{ is odd and}\\ i \text{ is odd},\\ j=2 \text{ to } n-1, \forall k\end{cases}\\ (01)^{\frac{n-3}{2}}0 & \begin{cases}n \text{ is odd and}\\ i \text{ is even},\\ j=2 \text{ to } n-1, \forall k\end{cases}\end{cases}$ $C_{i,j}=\begin{cases}(01)^{2rt}, \text{for } k \text{ odd } \forall i,j\\ (10)^{2rt}, \text{for } k \text{ even } \forall i,j\end{cases}$ $K_i=\{1(01)^{r-1}, \text{for } i=1 \text{ to } 2r+1$	$\frac{V}{2}$	$\frac{V}{2}$	0	1

3. Cordial labeling of gluing of quadrilateral snake graph QS_m, $\forall m$ on each vertex of $C_{4r} \times tP_n$, $\forall r, t \geq 1$ and $n > 1$ graph

The beginning vertices of graph QS_m, $\forall m$ gluing at each vertex of graph $C_{4r} \times tP_n$, $\forall r, t \geq 1$ and $n > 1$ are cordial graph because as mentioned in remark 1.1, cordial labeling of QS_m, $\forall m$ has number of vertices with label 0 is equal to number of vertices with label 1 and number of edges with label 0 is equal to number of edges with label 1 in both the cases and with the above table 2.1, the graph G satisfies the definition of Cordial Labeling and domination number of G is

$$\gamma(G)=\begin{cases}\left\lceil \frac{4r}{3} \right\rceil & \text{if n=1}\\ 2r & \text{if } \quad n=2, \ t=1\\ 4r & \text{if } \quad n=2, \ t>1\\ 4r\left\lceil \frac{n}{3} \right\rceil & \text{if } \quad n>2, \ t=1\\ 4r+(n-2)\left\lceil \frac{4rt}{3} \right\rceil & n>2, \ t>1\end{cases}$$

4. Conclusion

We proved in this paper that the graph $G = C_{4r} \times tP_n$, $\forall r, t \geq 1$ is Cordial graph also gluing of QS_m, $\forall m$ graph with G is again Cordial graph. We pose the following open problem: Is there any other labeling will satisfy this graphs and also gluing of QS_m, $\forall m$ graph with this graphs.

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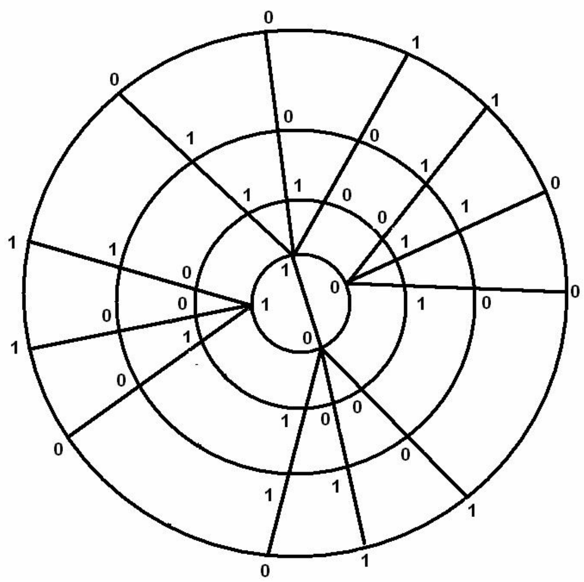
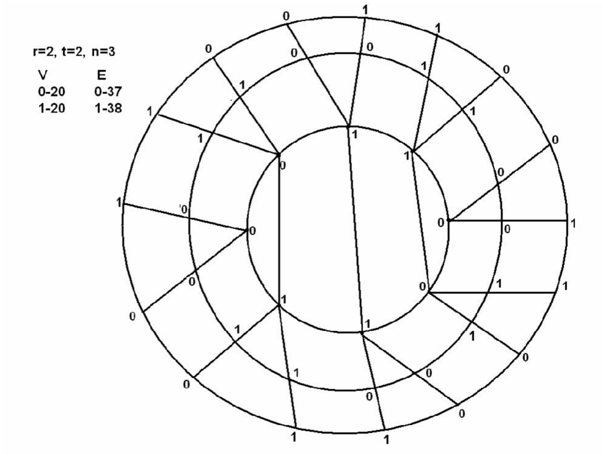
Illustrative Examples for $C_{4r} \times tP_n$

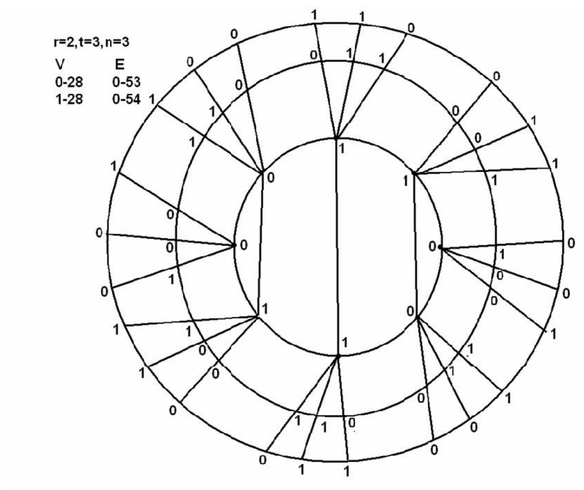
The graph of $C_{4r} \times 3P_4$, (and) $C_8 \times 2P_3$

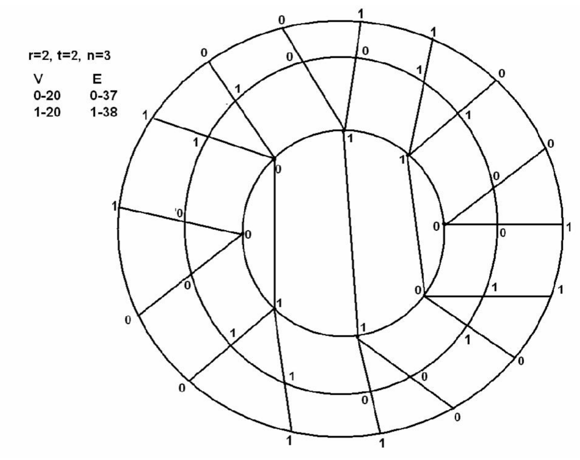
V	$0 - 20$	$1 - 20$
E	$0 - 38$	$1 - 39$

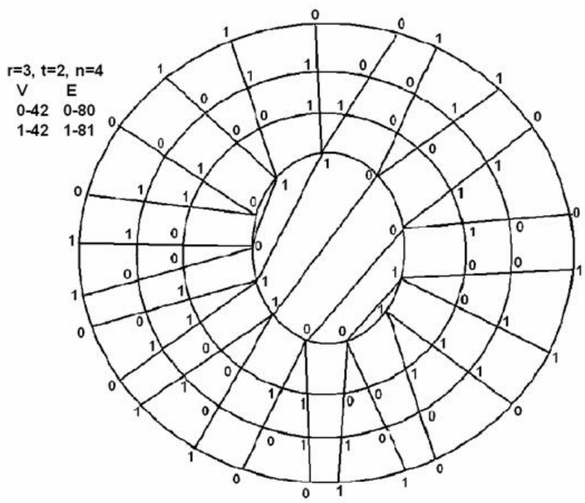




The graph of $C_8 \times 3P_3$ and the graph of $C_12 \times 2P_4$ given below:




r=2, t=2, n=3
V E
0-20 0-37
1-20 1-38


r=3, t=2, n=4
V E
0-42 0-80
1-42 1-81