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Linear Reciprocating Generator - Sizing Equations and Mathematical Model

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Abstract

Linear electric machine with reciprocating mover is of great interest now especially as a linear generator with reciprocating mover which is used together with free-piston engine. High power density in such an electric machine is achieved by the use of permanent magnets and fractional slot concentrated armature winding which together provide a big challenge to designer of such machines. In order to make an initial design, perform optimization procedure and evaluate resulting characteristics several mathematical models of different level of complexity were developed. Sizing equations are based on general equations of balanced electric machine, while main mathematical model takes into account real winding diagram, different length of armature winding and moving inductor and even rectified load. This main model is an analytical one, so it can be used for optimization purposes as it provides very fast calculations. The third model of high discretization level is based on FEM analysis and it allows to obtain generator output characteristics with high accuracy in order to exclude or minimize prototyping stage of design process.

Keywords: Linear electric machine, Reciprocating movement, Sizing equation, Mathematical model

Introduction

Electric machine is a very effective energy converter. Electric motor or generator can successfully operate during thousands of hours, while converting electrical energy into mechanical one or vice versa. And high power machines demonstrate efficiency level of more than 95%. This may be one of the reasons why a lot of investments are directed into development of thermodynamic cycle or turbine of power plant and not in electromechanical converter. But all conventional machines are considered to be rotating ones. Its infinite circle provides infinite variation of mutual inductance which leads to infinite electromechanical energy conversion. By contrast, linear electric machines, which are known for many decades, provide energy conversion only on a limited path or time interval and then they must return to initial position or reciprocate. Even infinite reciprocal movement means that twice on a period this energy converter is stopped so the output energy flow is interrupted. That is why linear machines were not so popular for many years.

But in last 30 years the general development of power electronics yields rather cheap and robust devices which allowed to create even new types of machines like switched reluctance drive or breath a new life into old exotic designs. Linear electric machine coupled with semiconductor converter can now produce continuous power flow and is considered as an effective electric generator converting mechanical energy of reciprocating mover.

The growing interest to linear electric machines can be seen in a lot of publications of research papers and PhD theses. A significant amount of such papers was analyzed [1-3].

One of the most debatable fields of application of linear electric machine now is a generator with direct drive from an internal combustion engine stroke [4-12]. Such a generator is expected to obtain advanced efficiency and robustness. Most papers are devoted to tubular design of linear generator, which has permanent magnets on an outer surface of tubular inductor located inside tubular armature. In addition, main attention is paid to inductors with axial magnetization of permanent magnets (with alternating polarity) [13-16].

Very few papers are dedicated to tubular linear generators with double stator which contains two tubular stators with identical armature windings located outer and inner side of single inductor. Particularly such design is analyzed in a paper, which is devoted mainly to investigation of thermodynamics of free-piston Sterling engine[17]. At the same time preliminary analysis predicts that tubular linear generator with double stator may obtain the highest values of specific indexes like ratio of traction force to inductor weight. Unfortunately, this design is very sophisticated and still has problems with inductor stiffness and accuracy and reliability of support assembly.

More simple design with the same advantages implies the use of flat reciprocating inductor with two flat stators located on upper and lower sides of it [18]. This design was selected as a basic design for analysis and prototyping in current research.

Linear reciprocating generator has inductor stroke of 80 mm. It moves with average velocity of 8 m/s and produce reciprocating frequency 50 Hz. Stator winding with 2 periods of initial winding has q = 2/5 and is located in 24 slots. Each slot width is 5 mm while tooth pitch is 10 mm. Pole pitch is equal to 12 mm. Inductor has 13 poles, stator has 20 poles, so inductor length is 156 mm, stator length is 240 mm. Inductor width is 200 mm. Inductor contains permanent magnets of 8.4 mm and magnetic core elements of 3.6 mm. Inductor height is 12.5 mm and its weight is about 3 kg. Each air-gap height is 1 mm.

In order to design and analyze linear reciprocating electric machine several mathematical models were developed which belongs to different levels of abstraction. First level (low level) models are used for design and sizing of electric machine, for calculation of equivalent circuit parameters. Second and third level models are used for more accurate analysis and calculation of characteristics of electric machine. Furthermore, second level model is simple enough to be used in optimization procedures, while third level model is so accurate that it can substitute real physical model until the final optimized design is obtained. First and second level models are analytical. It is realized in MatCAD but can be easily ported to Matlab mathematical environment. Third level model is based on magnetic field analysis and should use any FEM software.

Sizing equations

Initial data for linear reciprocating generator design contains rated power, maximum mover (inductor) velocity, allowed maximum mover weight and the form variation in the velocity of time. Instead of maximum velocity one can use mover stroke, i.e. the maximum mover displacement while reciprocation.

Any generator converts mechanical energy (power) into mechanical one. Mechanical power can be found as a product of average force by average velocity (average at the stroke). Let us find the electromagnetic force acting on a system of wires (generator winding) in the magnetic field

Electromagnetic force

To calculate electromagnetic force let us use a simple model of a thin current layer with harmonic current density distribution instead of real winding and only main harmonic of flux density created by inductor (Figure 1). The current layer is located on a surface of a flat magnetic core (generator's armature).

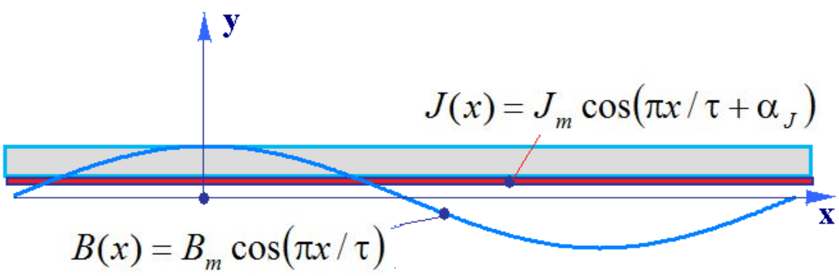

Figure 1: Current layer in a main harmonic magnetic field

For known flux density distribution *B(x)* and current distribution *i(x)* the distribution of electromagnetic force could be found as follows

$f(x) = B(x) \cdot i(x) \cdot l_z$,

while average value of electromagnetic force is equal to

$$F_{av} = \frac{1}{2p\tau} \int_0^{2p\tau} B(x) \cdot i(x) \cdot l_z dx \cdot$$

Here l_z is a model length in direction z orthogonal to the figure plane, τ is a pole pitch, $2p$ is a number of poles.

Instead of current distribution $i(x)$ one can use current density distribution $j(x)$ in current layer of thickness Δ_i, which is an equivalent of linear current density $A(x)=j(x)\Delta_i$. Now the formulae for electromagnetic force can be written as

$$f(x) = B(x) \cdot (j(x)\Delta_i dx) \cdot l_z = l_z \Delta_i \cdot B(x) j(x) dx \cdot$$

Here the product $\Delta_i B(x) j(x)$ means linear density of electromagnetic force per length unit in z direction (i.e. model depth). Then $f(x) = l_z \Delta_i \cdot B(x) j(x) dx$ is an elementary force dF acting on a current layer of dx length, so the total force acting on the whole machine active length is equal to

$$F = l_z \Delta_i \int_0^{2p\tau} B(x) j(x) dx = = l_z \Delta_i \int_0^{2p\tau} B_m \cos\left(\frac{\pi x}{\tau}\right) J_m \cos\left(\frac{\pi x}{\tau} + \alpha_j\right) dx \cdot$$

The maximum value of F can be obtained while $\alpha_j = 0$

$$F_{\max} = l_z \Delta_i B_m J_m \frac{\tau}{\pi} \int_0^{2p\tau} \cos^2\left(\frac{\pi}{\tau} x\right) d\left(\frac{\pi}{\tau} x\right) =$$

$$= l_z \Delta_i B_m J_m \frac{\tau}{\pi} \left[\frac{1}{2} \frac{\pi}{\tau} x \Big|_0^{2p\tau} + \frac{1}{4} \sin\left(\frac{2\pi}{\tau} x\right) \Big|_0^{2p\tau} \right] =$$

$$= l_z \Delta_i B_m J_m \frac{\tau}{\pi} \left[\frac{1}{2} \frac{\pi}{\tau} \cdot 2p\tau + \frac{1}{4} \sin\left(\frac{2\pi}{\tau} \cdot 2p\tau\right) \right] =$$

$$= \frac{1}{2} \Delta_i B_m J_m (2p\tau \cdot l_z) = \frac{1}{2} \Delta_i B_m J_m S_\delta \cdot$$

And finally

$$F_{\max} = \frac{1}{2} B_m A_m S_\delta \cdot$$

So, maximum specific force (per surface unit) is determined by electromagnetic loads which be achieved in machine active zone

$$F_{\max.S} = \frac{1}{2} B_m A_m \cdot$$

In these equations B_m is an amplitude of a main harmonic of a total magnetic field with armature reaction taken into account.

Now for known values of generator output power, its efficiency (approximate value) and mover average velocity one can select electromagnetic loads B_m and A_m and determine necessary square of generator active zone S_δ. For known mover mass (hence, mover volume) one can determine its thickness.

On the next steps one should select pole pitch, stator slots width and height, number of slots and winding diagram. Besides one should select inductor design: PM magnetization direction, relative dimensions of PMs and core elements between PMs (flux concentrators) if any.

Pole pitch

Pole pitch τ determines the number of poles for given inductor and stator length, so it plays significant role in stator winding diagram selection. For some minimal air-gap length lower values of pole pitch corresponds to higher values of flux density main harmonic amplitude. In the absence of core elements between PMs an analytical correspondence between pole pitch and flux density amplitude can be found. While the presence of core elements leads to necessity of numerical analysis [19].

Another note to be taken into account is less value of pole pitch leads to higher value of electromotive force frequency and, hence, current frequency. It means that for the same winding diagram less value of pole pitch leads to higher values of stator winding reactance, for instance, slot leakage reactance.

In general, the pole pitch selection is a multi-factor problem and its evaluation could be made only with a model, which takes into account all these factors or at least most of them.

Initially one should select pole pitch value which provides maximum possible value of air-gap flux density at no-load condition for given air-gap and inductor type.

Winding diagram

In case of low value of pole pitch it is better to use windings with fractional number of slots per phase per pole $q< 1$. Among them one should select windings with higher value of winding factor for a main harmonic like windings with $q = 2/5$ ($k_w = 0.966$ for a single-layer and $k_w = 0.933$ for a double-layer winding), $q = 2/7$ ($k_w = 0.966$) or $q = 3/8$ ($k_w = 0.945$). Relatively small number of slots per phase per pole allows to obtain slots and teeth dimensions which are suitable from technological point of view and decreased slot leakage.

Slots number and phase turns number

Slots width and height determine directly the slot leakage permeance and, hence, synchronous reactance components on direct and quadrature axes. These reactance components are also depending on effective winding turns square, while number of wind terms is determined by generator output voltage. So the final selection of active zone dimensions and winding diagram should consider the mutual influence of these factors and should be evaluated by the results of FEM analysis which is an equivalent of physical experiment on prototype.

Basic mathematical model for optimization purposes

Basic mathematical model starts with design stage at which stator and inductor main dimensions are selected, also pole number, air-gap, winding diagram and stator slot

width and height are determined. For selected inductor type the magnitude of magnetic flux density in the air-gap is calculated.
Let us consider synchronous generator phasor diagram, which corresponds to steady state condition of generator with active load (Figure 2).
The phasor diagram could be built in case one knows main harmonic of air-gap flux density and generator parameters r_1, $x_{\sigma 1}$, x_{ad}, x_{aq}. Using this phasor diagram one can easily calculate generator output power and other operating condition data like load angle, current density and electric losses, corresponding load resistance. While comparing these results with required values one can prove the possibility of obtaining rated power and evaluate power indicators and generator performance. In order to select the best design one should vary the effective number of stator winding turns and stator slots height while performing optimization procedure.

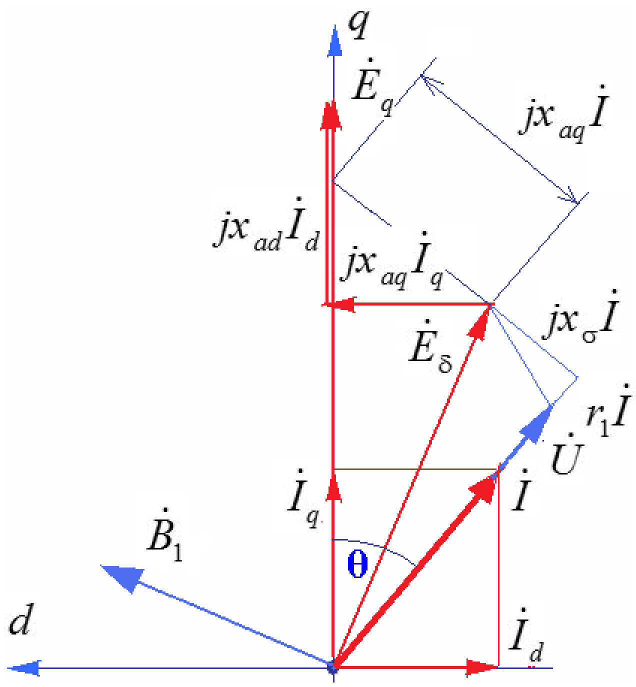

Figure 2: Phasor diagram for cosφ = 1

Most of electric machine parameters could be calculated by conventional calculation methods[20].
It should be noted that stator current main harmonic frequency is rather high:
$f_1 = V_{av} / (2\tau) = 333.3$ Hz.
For a tooth concentrated winding shortening factor is equal to the ratio of tooth pitch to pole pitch:
$\beta = y / \tau = t_z / \tau$.
Then slot leakage specific permeance for rectangular slot can be found as

$$\lambda_{\sigma sl} = \left(\frac{h_i}{3b_s} + \frac{h_2}{b_s} \right) \frac{1+3\beta}{4}$$

and stator winding reactance is

$$x_{\sigma sl} = 4\pi\mu_0 f_1 (w_1)^2 \frac{l_z}{0.5N_{p1}q_1} \lambda_{\sigma sl}.$$

Here b_s is a slot width, h_i is a slot height filled with current, h_2 is the rest part of slot height, N_{p1} is a stator pole number and l_z is an inductor width (in the ortho gonal direction to its movement).

In order to calculate armature reactance components on d and q axes by conventional formulae one should obtain magnetic field shape factors on d and q axes [21]

$$k_d = \frac{B_{ad1m}}{B_{adm}} \text{ and } k_q = \frac{B_{aq1m}}{B_{aqm}}$$

where B_{ad1m} and B_{aq1m} are amplitudes of main harmonics of air-gap flux density on direct and quadrature excitation correspondingly while taking into account real configuration of inductor and stator slotting; B_{adm} and B_{aqm} are the same amplitudes of main harmonics of air-gap flux density on the same excitation while considering minimal air-gap between smooth stator and harmonically excited inductor. Inductor in latter case must be substituted with smooth ferromagnetic core and air-gap height must be multiplied by Carter factor.

To calculate such magnetic field shape factors FEM analysis is necessary with accurate simulation of real stator and inductor shape with permanent magnets substituted by non-magnetic material. Figure 3 shows magnetic field distribution in the model for calculation of magnetic field shape factor on quadrature axis. At this model current density in stator coil is equal to 5 A/mm^2, tooth flux density is not more than 1.3 T.

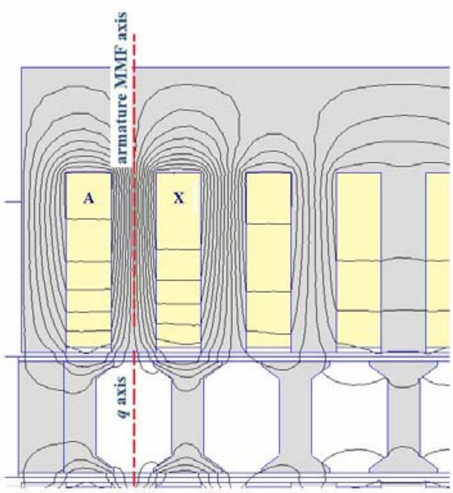

Figure 3: FEM model for field shape factor on quadrature axis calculation

To calculate field shape factor on direct axis inductor must be moved by half of pole pitch. As a result of FEM analysis the following values of field shape factors were obtained: $k_d = 0.382$, $k_q = 0.635$.
Now stator reactance components could be calculated by conventional formulae with field shape factors taking into account real stator slotting:

$$X_{ad} = \frac{4\mu_0}{\pi 0{,}5 N_{p1}} m_1 f_1 (w_1 k_{w1})^2 \lambda_{ad},\ \lambda_{ad} = k_d \frac{\tau l_z}{\delta} \text{ and}$$

$$X_{aq} = \frac{4\mu_0}{\pi 0{,}5 N_{p1}} m_1 f_1 (w_1 k_{w1})^2 \lambda_{aq},\ \lambda_{aq} = k_q \frac{\tau l_z}{\delta}.$$

Phasor diagram analysis

No-load electromotive force E_q one may calculate in several different ways. First one, by main harmonic of air-gap flux density, which could be obtained after Fourier analysis of air-gap flux density curve as a result of FEM simulation of no-load operating condition. Second one, EMF E_q can be obtained after Fourier analysis of phase flux linkage curve as a result of time-domain analysis of generator with continuously moving inductor at no-load operating condition. Second approach is more accurate but it takes a lot of time for magnetic field calculation. While first approach provides almost the same accuracy for our basic design of linear generator as it was proved by several calculations.
Specific feature of linear generator with relatively short inductor means that only part of stator winding being subject of moving inductor magnetic field. The proportion is equal to ratio of inductor to stator pole numbers. So the electromotive force should be calculated as follows:

$$E_q = \pi\sqrt{2} f_1 w_1 k_w \frac{2}{\pi} B_{\delta 1} \tau l_z \frac{N_{p2}}{N_{p1}},$$

where $B_{\delta 1}$ is a main harmonic of air-gap flux density at no-load condition.
Other steps of phasor diagram analysis made by conventional formulae and corresponding notes are skipped here.
Let us select load resistance R_{load} and phase current. For active load stator voltage is definitely determined $U_1 = R_{load} I_1$, so load angle can be obtained as

$$\theta = arctg\left(\frac{x_q}{R_{нгр} + r_1}\right).$$

Now phasor diagram can be fully analyzed:
-stator current components on d and q axes

$I_d = I_1 \sin\theta$ and $I_q = I_1 \cos\theta$;

-stator voltage components on d and q axes

$$U_q = E_q - x_{ad} I_d - x_\sigma I \cdot \sin\theta - r_1 I \cos\theta,$$

$$U_d = x_{aq} I_q + x_\sigma I \cos\theta + r_1 I \sin\theta;$$

-stator voltage RMS value

$$U_1 = \sqrt{U_d^{\ 2} + U_q^{\ 2}} .$$

As it was mentioned above, such analysis is made while stator winding turns number is varied (in fact, it is more convenient to vary number of turns per slot and calculate corresponding number of phase turns for selected winding diagram) and slot height. Such variations could be easily made in MathCAD in order to select appropriate number of turns and slot height which provide stator voltage equal to product of selected R_{load} and I_1.

After phasor diagram analysis one can calculate generator output power (for unit power factor)

$$P_1 = m_1 U_1 I_1$$

and stator winding current density

$$j = \frac{I_1 w_s}{h_i b_s \cdot k_{fill}} ,$$

where w_s is a number of effective turns per slot (we assume that stator winding has no parallel lines), k_{fill} is a slot filling factor.

In case generator output power is less than rated power, load resistance should be changed for another set of variations.

Such a simple model can be used at the first stage of optimization while more sophisticated analytical model taking into account inductor dynamics and even rectifier effects could be useful for quick analysis of various operating conditions even if it is based on a sinusoidal distribution of air-gap flux density.

Analytical model for linear generator analysis

In case we suppose sinusoidal distribution of flux density in the air-gap for each inductor position with certain poles number and stator slots number one can find equation for any stator tooth flux. For selected sort of inductor movement (linear or sinusoidal time variation of inductor velocity) the equation of stator tooth flux could be transformed to time domain. Then for certain stator winding diagram one can get equation of phase electromotive force by time variation of each coil flux-linkages. After that for each load resistance one should solve conventional Kirchhoff equations in order to find currents, electromagnetic force and calculate power losses and current density, specific force and other key factors of generator's efficiency.

The analytical model is applicable for both flat inductor design and tubular design, because it is based on a 2-dimensional model of inductor magnetic field sinusoidally changing along inductor length. In orthogonal direction the air-gap can be linear in a flat design or can make a circle in a tubular design (Figure 4).

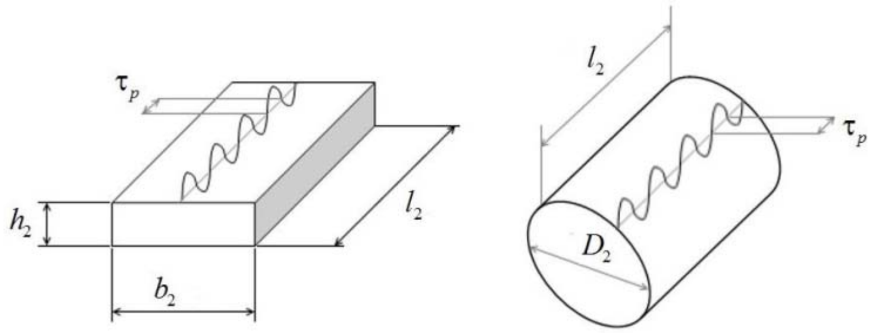

Figure 4: Inductor dimensions for flat and tubular design.

In case of a flat design magnetic flux amplitude is determined by air-gap flux density and one pole pitch square as follows

$$\Phi_m = \frac{2}{\pi} B_{\delta 1m} \tau_p b_2 \cdot$$

In case of tubular design magnetic flux amplitude is equal to

$$\Phi_m = \frac{2}{\pi} B_{\delta 1m} \tau_p \pi D_2 \cdot$$

In order to take into account various sorts of inductor movement let us prepare equations of velocity and coordinate changing. For a uniform inductor movement its velocity is equal to

$$v(t) = \begin{cases} \dfrac{2\Delta l}{T_2} & \text{for} \quad 0 \le t < \dfrac{T_2}{2} \\ -\dfrac{2\Delta l}{T_2} & \text{for} \quad \dfrac{T_2}{2} \le t \le T_2 \end{cases},$$

while inductor position is equal to

$$x(t) = \begin{cases} -\dfrac{\Delta l}{2} + 2\Delta l \dfrac{t}{T_2} & \text{for} \quad 0 \le t < \dfrac{T_2}{2} \\ \dfrac{3\Delta l}{2} - 2\Delta l \dfrac{t}{T_2} & \text{for} \quad \dfrac{T_2}{2} \le t \le T_2 \end{cases}.$$

Here Δl is an inductor stroke and T_2 is a period of inductor reciprocating movement. For a sinusoidal inductor movement its velocity is equal to

$$v(t) = \pi f_2 \Delta l \sin(2\pi f_2 \cdot t).$$

while inductor position is equal to

$$x(t) = -\frac{\Delta l}{2} \cos(2\pi f_2 \cdot t).$$

For a sinusoidal distribution of air-gap flux density with N_{p2} poles along coordinate x (along inductor movement direction) magnetic flux of a tooth located at origin could be found as follows

$$\phi(x)=\begin{cases}\Phi_m \sin\left(\dfrac{\pi x}{\tau_p}\right) & \text{for } -\dfrac{\pi N_{p2}}{2}\le\dfrac{\pi x}{\tau_p}<\dfrac{\pi N_{p2}}{2}\\ 0 \quad \text{for } \left(\dfrac{\pi x}{\tau_p}<-\dfrac{\pi N_{p2}}{2}\right) & \text{or } \left(\dfrac{\pi x}{\tau_p}>\dfrac{\pi N_{p2}}{2}\right)\end{cases}$$

For example, Figure 5 shows tooth flux variation while inductor with N_{p2} = 13 poles uniformly moves relative to this stator tooth.

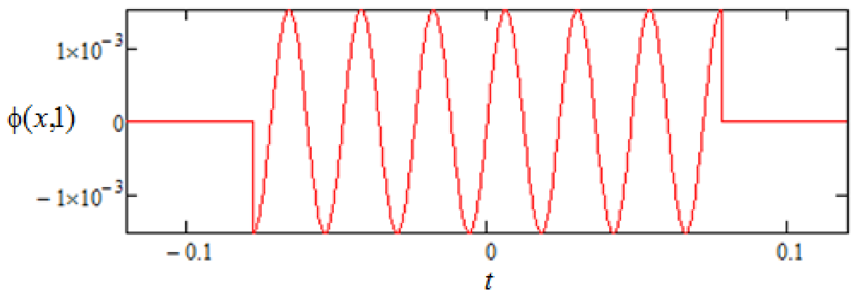

Figure 5: Tooth flux vs inductor position

For any stator tooth k which is shifted from origin by distance $(k-N_z/2)t_z$ tooth flux could be found as follows

$$\phi(x,k)=\begin{cases}\Phi_m \sin\left(\dfrac{\pi}{\tau_p}(x-(k-N_z/2)t_z)\right)\\ \quad \text{for } -\dfrac{\pi N_{p2}}{2}\le\dfrac{\pi}{\tau_p}(x-(k-N_z/2)t_z)\le\dfrac{\pi N_{p2}}{2}\\ 0 \quad \text{for } \left(\dfrac{\pi}{\tau_p}(x-(k-N_z/2)t_z)<-\dfrac{\pi N_{p2}}{2}\right)\\ \quad \text{or } \left(\dfrac{\pi}{\tau_p}(x-(k-N_z/2)t_z)>\dfrac{\pi N_{p2}}{2}\right).\end{cases}$$

This tooth flux is totally linked with stator coil while using tooth concentrated coils (which was assumed for a stator winding with fractional number of slots per pole per phase). Hence, stator tooth coil flux linkage is equal to

$\psi(x,k)=w_k\phi(x,k)$.

where w_k is a number of turns per coil.

Now the whole phase flux linkage can be calculated as a sum of tooth coil flux linkages which are series connected in this phase, while taking into account the direction of each coil.

Let us consider stator winding with 10 pole pairs and 24 teeth (with a number of slots per pole per phase q = 2/5). Winding diagram is presented at Figure 6.

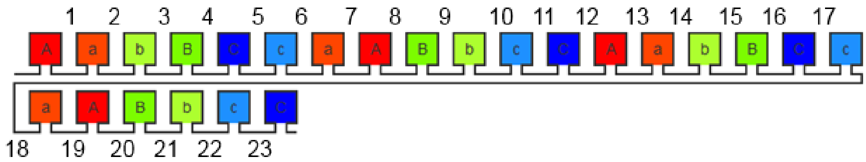

Figure 6: Stator winding diagram with $N_{p1} = 20$ and $N_z = 24$

For this winding phase flux linkages could be calculated as follows

$$\psi_A(x) = \psi(x,1) - \psi(x,7) + \psi(x,13) - \psi(x,19) =$$
$$= w_k\left(\phi(x,1) - \phi(x,7) + \phi(x,13) - \phi(x,19)\right);$$
$$\psi_B(x) = w_k\left(-\phi(x,3) + \phi(x,9) - \phi(x,15) + \phi(x,21)\right);$$
$$\psi_C(x) = w_k\left(\phi(x,5) - \phi(x,11) + \phi(x,17) - \phi(x,23)\right).$$

Phase EMF can be evaluated as a derivative of phase flux linkage

$$e_A(t) = k_w \frac{d\psi_A}{dt} = k_w \frac{d\psi_A(x(t))}{dt} =$$
$$= k_w \frac{\partial \psi_A}{\partial x}\frac{dx}{dt} = k_w w_k \frac{\partial \phi_A}{\partial x}\frac{dx}{dt}.$$

where k_w is a main harmonic winding factor.

For selected winding diagram phase A EMF is equal to

$$e_A(t) =$$
$$= k_w w_k \left(\frac{\partial \phi(x,1)}{\partial x} - \frac{\partial \phi(x,7)}{\partial x} + \frac{\partial \phi(x,13)}{\partial x} - \frac{\partial \phi(x,19)}{\partial x}\right)\frac{dx}{dt}.$$

Supposing uniform movement of inductor one can get time domain phase EMFs as shown at Figure 7.

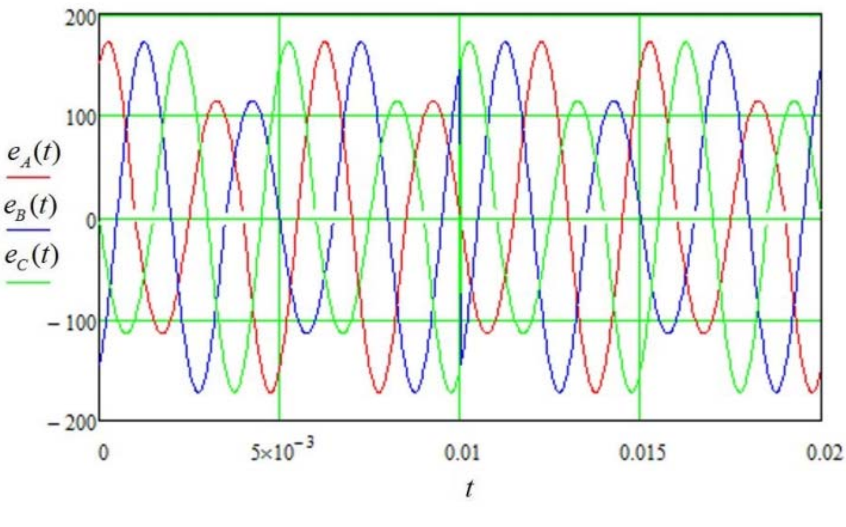

Figure 7: Phase EMFs at uniform movement of inductor

Similar time domain phase EMFs for sinusoidal change of inductor velocity at reciprocating movement are shown at Figure 8.

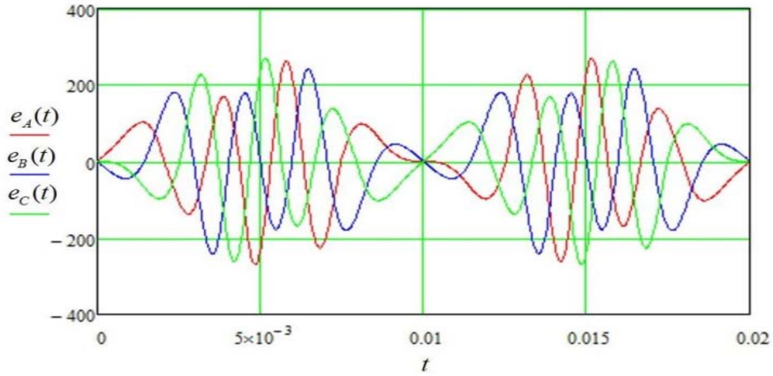

Figure 8: Phase EMFs at sinusoidal change of inductor velocity.

RMS value of phase EMF can be calculated as follows

$$E_A = \sqrt{\frac{1}{T_2 \int_0^{T_2} e_A^2(t)}}\,.$$

Linear reciprocating generator is working on a rectified load. Conventional 3-phase 6-pulse bridge rectifier will produce average rectified voltage as follows

$$U_d = \begin{cases} |e_A| + |e_B| \\ \quad \text{for} \quad (|e_A| + |e_B| \geq |e_B| + |e_C|) \text{ and } (|e_A| + |e_B| \geq |e_C| + |e_A|) \\ |e_B| + |e_C| \\ \quad \text{for} \quad (|e_B| + |e_C| \geq |e_C| + |e_A|) \text{ and } (|e_B| + |e_C| \geq |e_A| + |e_B|) \\ |e_C| + |e_A| \\ \quad \text{for} \quad (|e_C| + |e_A| \geq |e_A| + |e_B|) \text{ and } (|e_C| + |e_A| \geq |e_B| + |e_C|) \end{cases}$$

The time domain rectified voltage together with phase EMFs for sinusoidal change of inductor velocity at its reciprocating movement are shown at Figure 9. Average value of rectified voltage is shown by dash line.

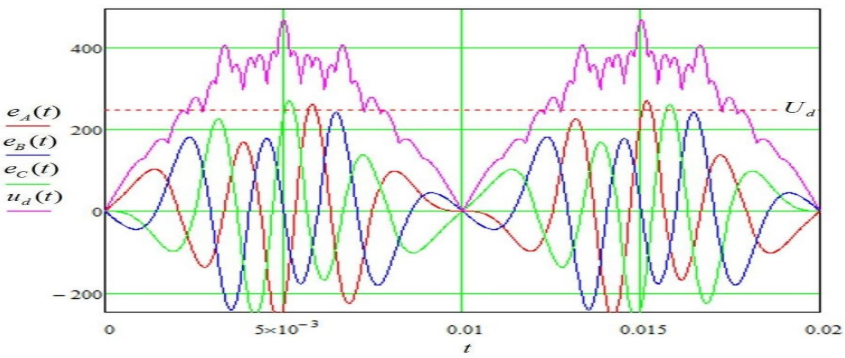

Figure 9: Rectified voltage and phase EMFs at sinusoidal change of inductor velocity

With this analytical model one can select load resistance and obtain rectified current, phase current and electromagnetic force produced by generator under certain load condition.

As it was mentioned above the maximum value of electromagnetic force is determined by electromagnetic loads which can be produced by linear generator

$$F_{\max} = \frac{1}{2} B_m A_m S_\delta,$$

where S_δ is a square of linear generator active zone between stator and inductor.

Rated value of stator current can be calculated as follows

$$I_1 = \frac{P_1}{3U_1 \eta \cos\varphi}.$$

Current linkage for known number of turns per slot u_s and tooth pitch t_z is equal to

$$A_m = \frac{\sqrt{2} I_1 u_s}{t_z}.$$

Current linkage also could be calculated by maximum value of phase current density j_{1m} and slot dimensions (width b_s and height of current in the slot h_{si})

$$A_m = \frac{j_{1m} b_s h_{si}}{t_z}.$$

Parameters of stator winding are calculated by conventional formulae. Also power losses in stator winding and magnetic core losses also could be calculated by conventional means.

The analytical model described provides very fast calculation so it can be used in optimization procedures while selecting the best generator design. To compare efficiency of different generator designs it is convenient to use specific values of the following quantities:

- maximum specific electromagnetic force per active volume $F_{\max}/V_a$, N/m^3;
- maximum specific electromagnetic force per square of active zone $F_{\max}/S_\delta$, N/m^2;
- maximum specific electromagnetic force per inductor mass $F_{\max}/M_2$, N/kg;
- specific power per active volume P_1/V_a, W/m^3;
- specific power per square of active zone P_1/S_δ, W/m^2;
- specific power per inductor mass P_1/M_2, W/kg.

FEM model of linear generator [for operational point analysis]

Previous models which were used for sizing equations and optimization of stator turns number and slot height are very approximate models as they do not take into account influence of the higher harmonics of the air-gap flux density, edge effects of linear machine and magnetic core saturation. To evaluate the selected design accurately one need to perform checking calculation of FEM model while rendering time-domain processes taking into account operating condition effects and material properties.

Figure 10 and Fig 11 show time domain phase EMFs and phase currents correspondingly which were obtained at checking calculation of linear generator with

flat double sided design. FEM simulation was performed while moving inductor on a full stroke with sinusoidal variation of inductor velocity.
This FEM model takes into account real sizing, magnetic core material properties and PM properties, different saturation of magnetic core parts, armature reaction for a certain load. The EMF was calculated by phase flux linkage change while taking into account real field distribution with all harmonic distortions and direct edge effects.

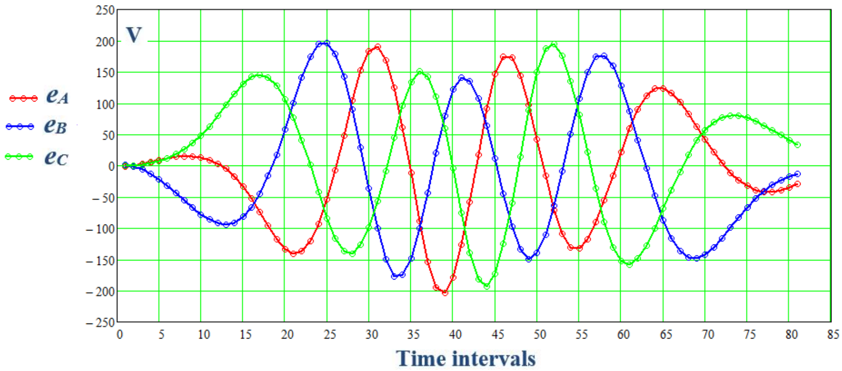

Figure 10: Phase EMFs of linear generator obtained on time intervals of inductor stroke.

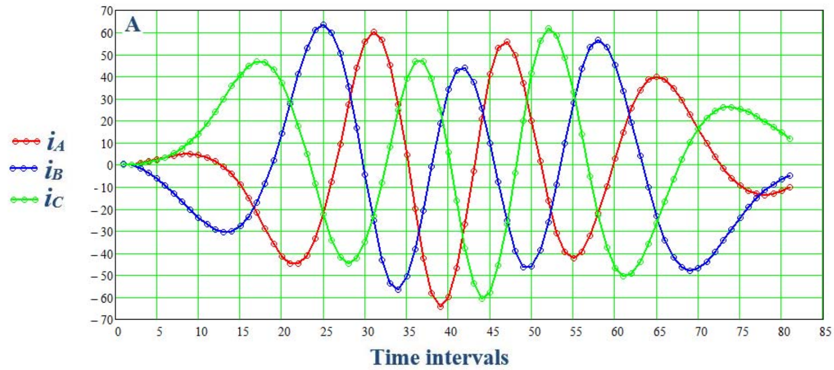

Figure 11: Phase currents of linear generator with active 3-phase resistive load.

The accuracy of time domain analysis was controlled by error of Kirchhoff equation for each phase at each time interval. At high load operating conditions (at low load resistance) the new value of phase current for next time interval was evaluated by relaxation algorithm with 8 to 10 successive approximations at each time interval. Because of each approximation was made by FEM analysis the total computation time was very long. To calculate presented EMF and current curves at one stroke with acceptable accuracy about 800 field calculations were made.
Such an accurate FEM model should be used at a final stage of design process to predict generator output without making prototype for physical experiment. Besides,

this FEM model reproduces accurately the real shape and relative position of magnetic cores. So, this model can be used for evaluation of deviation effects like sizing deviations or position deviation while making or assembling the prototype.
One of the most important deviations to be analyzed with accurate FEM model is inaccurate inductor position while two air gaps between inductor and upper and lower stators are different. In such a case the whole magnetic circuit became unbalanced so the FEM model should reproduce the whole cross-section of double-sided flat linear generator which significantly increases computational resources especially for time domain analysis.
Figure 12 shows magnetic field distribution at one of the time intervals during linear generator operating condition analysis while inductor is shifted by 0.5 mm towards upper stator (with regular air-gap length δ = 1 mm). As a result of FEM analysis it was found that such inductor displacement leads to unbalanced magnetic field distribution among upper and lower stators and, hence, to unbalanced distribution of core losses.

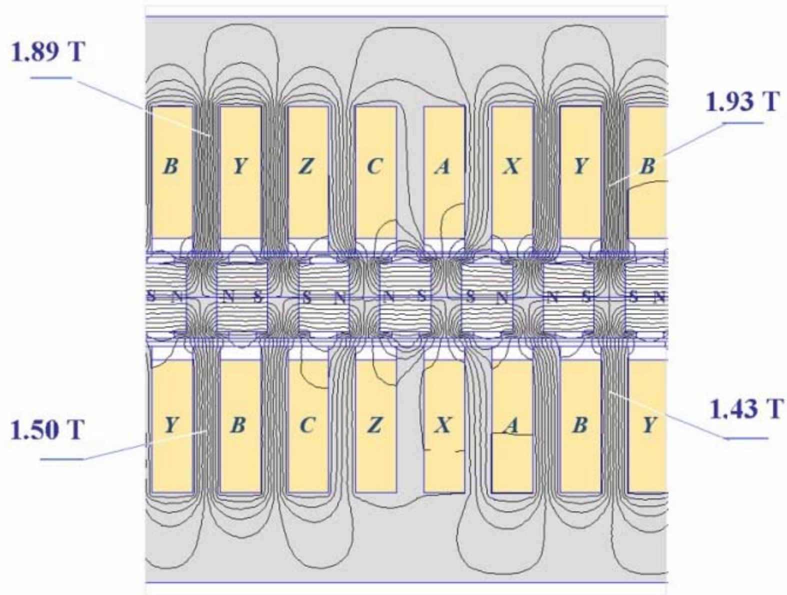

Figure 12: Magnetic field distribution in longitudinal cross-section of linear generator with inductor displaced towards upper stator by half of regular air-gap length.

Such kind of inductor displacement leads also to one-side magnetic attraction, i.e. to additional loads at bearings. For a flat double-sided linear generator, a set of numerical experiments was made which resulted in two important conclusions: first, one-side magnetic attraction force which occurs due to inductor displacement towards one of stator cores depends mainly on PM magneto-motive force and at load operating condition it does not differ from one-side magnetic attraction force at no-load; second, the value of one-side magnetic attraction force is proportional to the value of inductor displacement.

FEM model analysis allows to obtain rather simple dependence of one-side magnetic attraction force on inductor displacement (which can be called eccentricity like same effect in rotating machines). This leads to a simple method of evaluation of deviation effects like pitch and roll of moving inductor.

Conclusion

Any electric machine can be described by mathematical models of different level of complexity. And all these models are necessary for design process of a new electric machine. Simple and fast models are used at first steps for machine sizing and optimization, while accurate but sophisticated models are used for output characteristics calculation and evaluation of technological deviations effects instead of building several prototypes.

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