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Application of polynomial regression models for prediction of stress state in structural elements

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Abstract

This paper presents the application of linear regression model for processing of stress state data which were collected through drilling into a structural element. The experiment was carried out by means of reflection photoelasticity. The harmonic star method (HSM) established by the authors was used for the collection of final data. The non-commercial software based on the harmonic star method enables us to automate the process of measurement for direct collection of experiment data. Such software enabled us to measure stresses in a certain point of the examined surface and, at the same time, separate these stresses, i.e. determine the magnitude of individual stresses. A data transfer medium, i.e. a camera, was used to transfer the picture of isochromatic fringes directly to a computer.

Keywords: principal normal stresses, harmonic star method, simple linear regression, root mean squared error, mean absolute percentage error, *R*-squared, adjusted *R*-squared, MATLAB.

Introduction

Residual stresses are stresses which occur in a material even if the object is not loaded by external forces. The analysis of residual stresses is very important when determining actual stress state of structural elements. Residual stresses occur as early as in the stage of technological processes and may be of different nature with respect to direction, magnitude, depth, or planar gradient. They cause various failures of structural parts, knots, machines or devices. Clients often require the knowledge of residual stress state in components or structures. It is appropriate to carry out the analysis which is based on the means of reflection photoelasticity, and determine gradient or magnitude of residual stresses [1].

Reflection photoelasticity provides us with complex information on stress or deformation stress in structural elements subjected to loads. However, evaluation of the entire field under analysis is often time-consuming. With the development of the new HSM software application we aimed to shorten the measurement of principal strains, principal normal stresses as well as residual stresses on a photoelastic layer applied to examined objects while using reflection polariscope M030, M040 or LF/Z-2 [2–4]. When determining strains and stresses on the photoelastic layer of the object subjected to loading, measurements are always carried out point after point. When more points are being analyzed, this procedure is lengthy. The new HSM application enables fast and efficient analysis of directions and magnitudes of principal strains and principal normal stresses in individual points over the entire surface with a reflection layer subject to examination [5–8].

It was further necessary to evaluate the experimental data. When analyzing the collected data we found out that polynomial regression models are suitable for data processing.

Determination of residual stresses by means of photoelasticity

In order to specify residual stresses, a reflective optically sensitive layer was bonded with the structural element. This layer exhibits the phenomenon of temporary birefringence. The harmonic star method was used for evaluation of the stress state. This method has enabled us to gain data on principal normal stresses of the examined object [5,9].

For the examination of residual stresses we had to drill a hole through the reflection layer. The hole was drilled to the material of the structural element up to the depth of 5 mm. Parameters of residual stresses in the material were determined from the pattern of colourful isochromatic fringes which occurred on the reflection layer. This pattern occurs when stress is released after drilling and when illuminated with polarized light from the reflection polariscope.

In addition to quantitative values of residual stresses, this method enables visual representation of their distribution around the holes on the reflection layer (Figure 1). Such visual representation allows us to visualize their overall distribution around the holes, and, in this way, enables immediate identification of areas of maximum residual stresses as well as stress gradients [8,10].

Separated values of residual stresses as gained by means of the HSM software were processed with suitable polynomial regression models in the MATLAB environment [11].

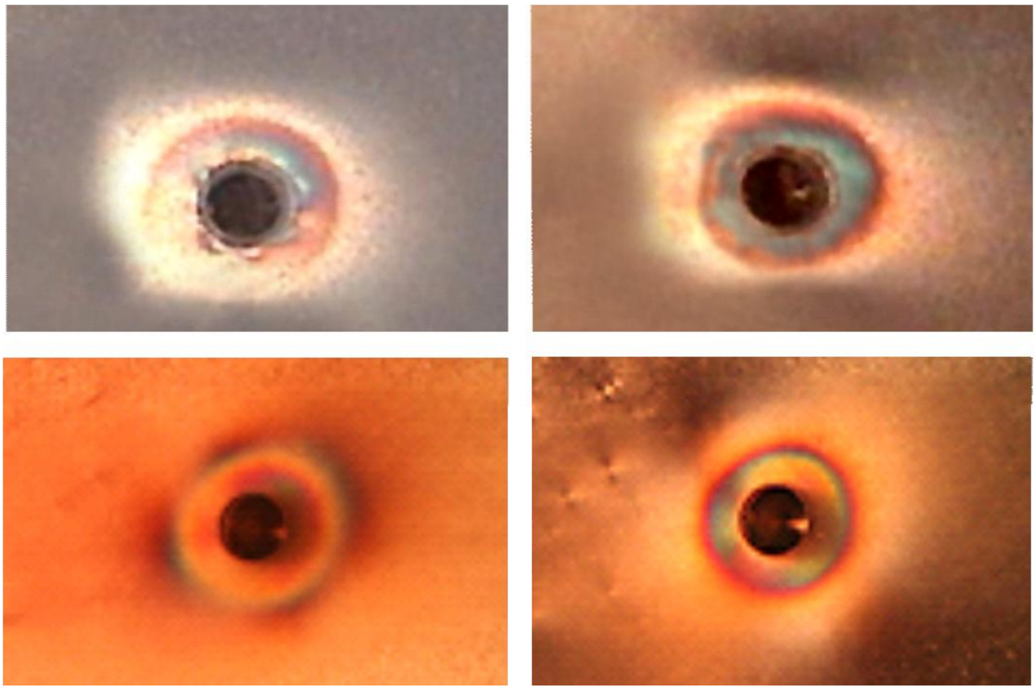

Figure 1. Photographic record of isochromatics

Experimented object, measurement and evaluation sequence for determination of residual stresses

The experiment was carried out on a supporting element constituting a structural component of a lorry trailer extension frame (Figure 2) [12]. The manufacturer of the structure had requested this experiment due to frequent failures of the structure. In addition, the manufacturer requested structural measures which would eliminate these failures. One of the ways in which relevant data on the examined object can be collected is a complex of measurements which aim to define stress state in critical areas of the structure. We found out that residual stresses in the material of the supporting element (bracket) have a significant effect on the failure. The steel bracket is cut by laser technology and shaped by bending. A series of drilling experiments was carried out in predetermined areas of the structural component in order to specify residual stresses [1,2].

For the purposes of the experiment we decided to use reflection photoelasticity which provided us with initial reference information on the stress gradient in the area of residual stresses. At the same time, this method enabled us to determine stress magnitudes. The measurement was carried out directly on the supporting element. Figure 3 depicts the process of drilling as well as drilling equipment RS-200 installed on the supporting element.

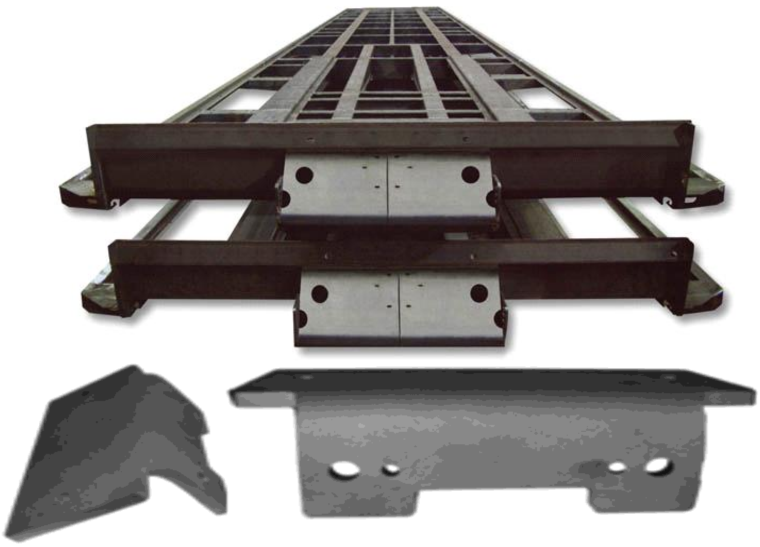

Figure 2: Structure of a trailer frame and bracket subjected to examination

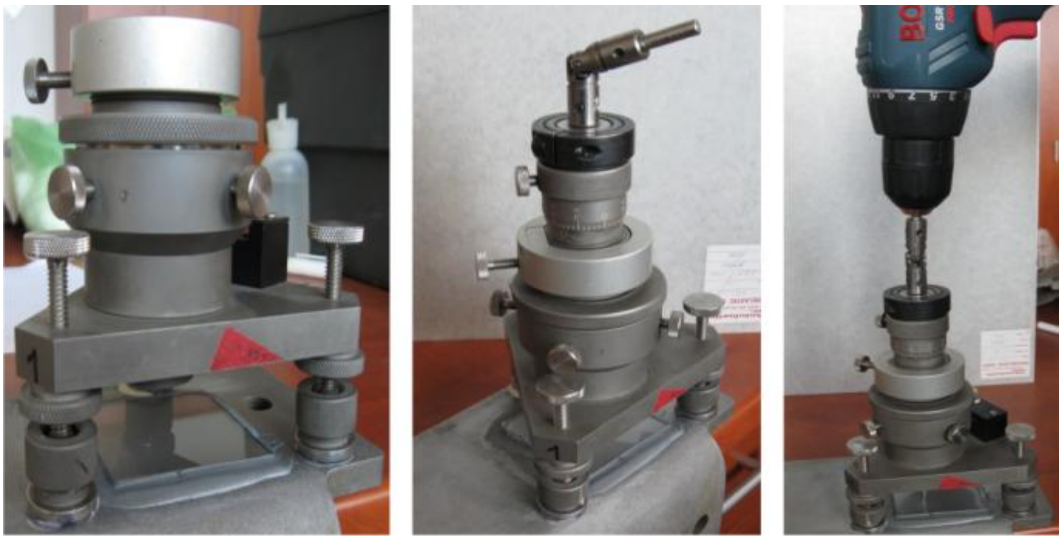

Figure 3: A part of the supporting element with optically sensitive layer and the installation of drilling equipment

The measurement and evaluation chain for determination of residual stresses by means of photoelasticity included drilling device RS-200, an object coated with photoelastic layer in the area of maximum stress concentration, polariscope LF/Z-2, camera CANON D 450 transferring isochromatic fringes from the reflection layer to a computer after drilling, software for subsequent separation of principal stresses by means of the harmonic star method, output device, e.g. printer (Figure 4). The reflection layer was made of a 3.125 mm thick mass with designation PS-1. The manufacturer, company Vishay, presented its fringe constant as $f = 615\ \mu\varepsilon$ [1,2].

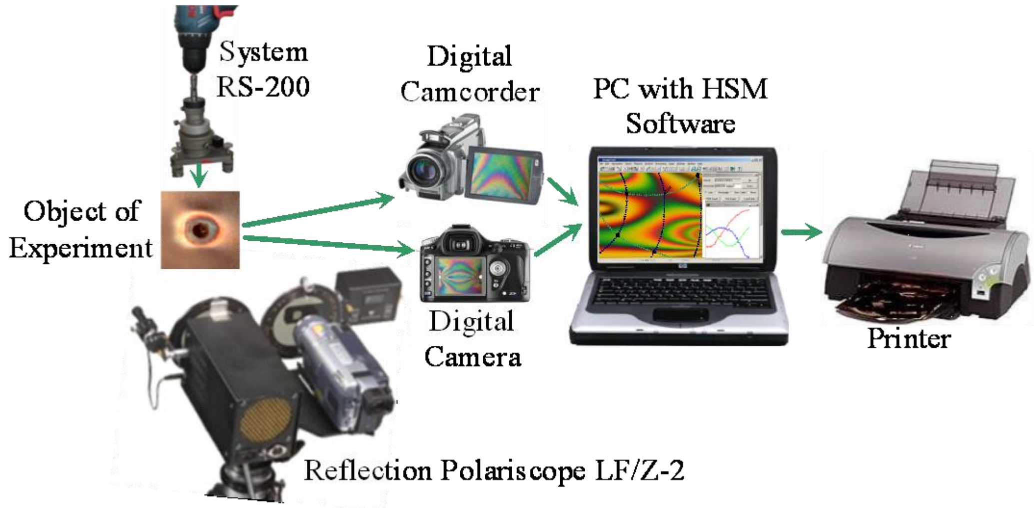

Figure 4: Experimental chain for determination of residual stresses

Execution of the experiment, values of principal stresses identified by HSM software

The experiment provided us with a large number of information. Therefore we have decided to demonstrate herein only the measurement in one point of the bracket. In the area of highest stress concentration we applied the layer of a photosensitive material while following a prescribed procedure. A hole was drilled through the photoelastic layer on the material of the supporting element progressively with increment of 0.5 mm. A Vishay cooling spray was used during the experiment. Cooling was necessary in order to exclude temperature alteration in the photoelastic layer during drilling. Such alteration of temperature could cause incorrect outcomes of the experiment. For drilling in the reflection layer and the structure in the analyzed point we used drilling equipment RS-200. The material was drilled up to the depth of 8.4 mm. Diameter of the drill was 3.2 mm. Total depth of the hole was 8.4 mm. The depth of drilling was chosen with respect to the thickness of the photoelastic layer (3.125 mm) and the thickness of adhesive layer of cca 0.3 mm. The hole of 5 mm was sufficient to determine residual stresses in the material of the supporting element. Diameter of the hole was 3.2 mm which corresponds with the diameter of the drill [13,14].

After each drilling, visible isochromatic fringes on the examined surface were recorded with the camera. These fringes represent differences of principal normal stresses $\sigma_1 - \sigma_2$. Using the camera the recorded colourful fringes were transferred to the computer for additional identification by means of the HSM software [5]. Stresses were separated, i.e. individual extreme components $\sigma_1 = \sigma_{\max}$ and $\sigma_2 = \sigma_{\min}$ of normal stresses were specified from the difference of $\sigma_1 - \sigma_2$. Individual separated values of these stresses during every drilling step are listed in Tab. 1.

Table 1: Separated values of extreme normal stresses

Drilling depth [mm]	$\sigma_{\mathbf{min}}$ [MPa]	$\sigma_{\mathbf{max}}$ [MPa]
0.50	79.40	269.12
1.00	46.33	197.89
1.50	44.43	170.19
2.00	44.85	163.33
2.50	48.54	164.83
3.00	60.97	180.04
3.50	65.78	183.21
4.00	69.23	187.15
4.50	71.62	192.99
5.00	74.89	201.20

Measurements provided us with basic information on residual stresses in the supporting element. At the same time, it was necessary to predict stresses in dependence on the depth of the hole in the supporting element. For this purpose we decided to use the means of regression analysis.

Mathematical and statistical determination of extreme normal stresses tendency on the sample

Basic task of statistical analysis of data in Tab. 1 was to estimate the dependence between extreme normal stresses σ_{min} or σ_{max} (variable dependent) and the depth of drilling h (variable independent) while using a suitable regression function, as well as to determine the level of intensity on which the given dependence occurs between various secondary interfering factors. Statistical analysis of measurement values was done with MATLAB software.

The first step was to roughly assess the art and intensity of dependence between the analyzed quantitative attributes. For this purpose we used a correlation diagram in which every data pair is represented graphically by one point in a plane. The art of dependence is estimated by means of a curve which fits the outcome values. In both cases we have chosen the 2$^{\mathrm{nd}}$ up to the 4$^{\mathrm{th}}$ degree of polynomial regression model (see Figure 5 and Figure 6).

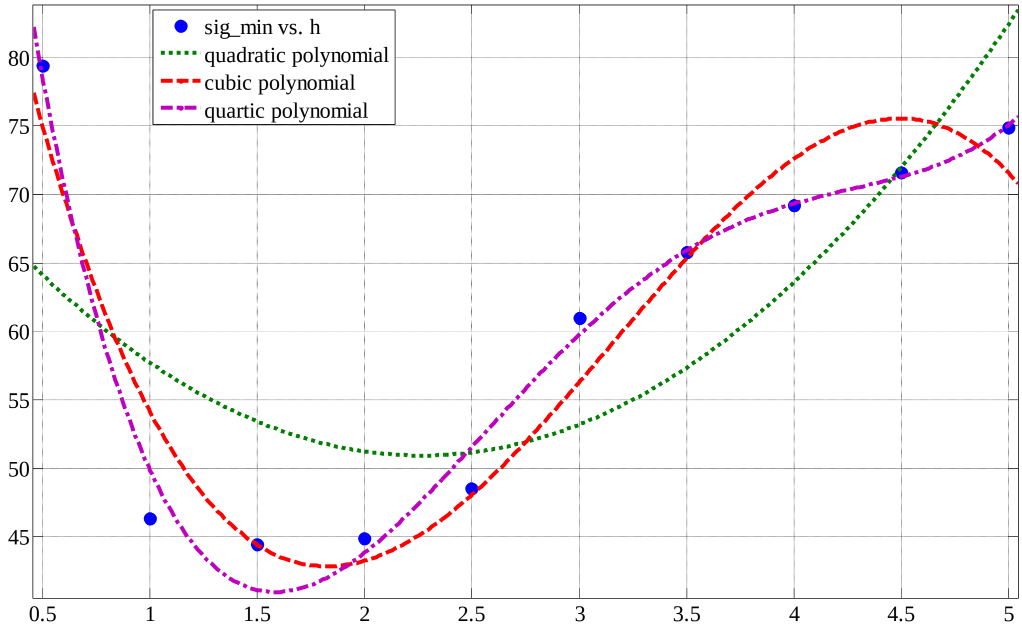

Figure 5: Comparison of three polynomial regression models with measured data – for $\sigma_{\min}$ vs. h

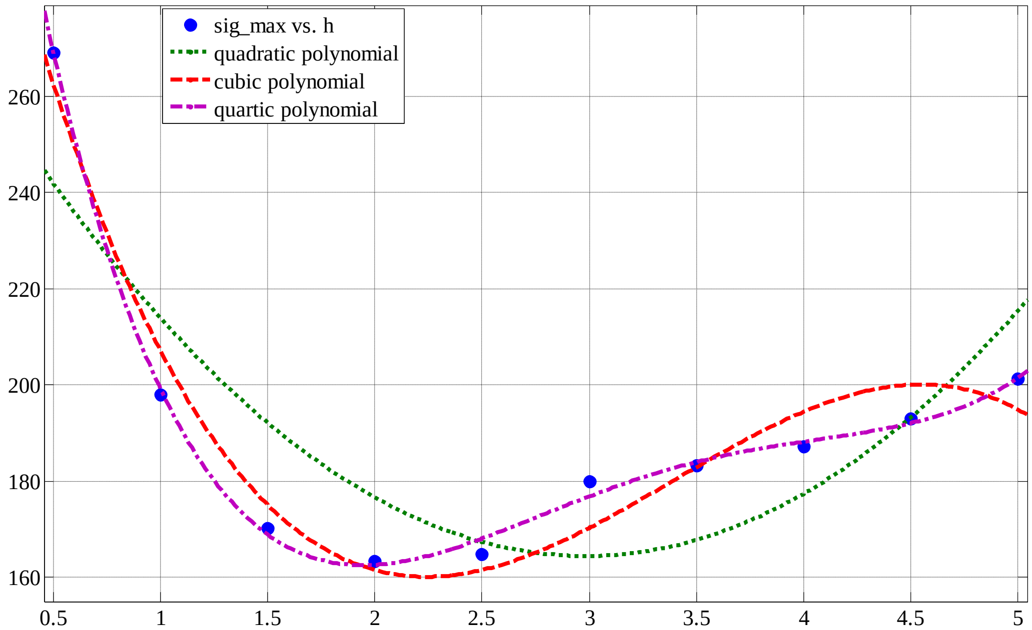

Figure 6: Comparison of three polynomial regression models with measured data – for $\sigma_{\max}$ vs. h

Neither methodology, relations of the least square method, nor residual characteristics and statistical tests are discussed in this paper. A comprehensive description of these aspects can be found in relevant literature [11,15].

In order to test statistical relevance of individual regression coefficients of polynomial regression functions $\sigma_{\min} = \beta_0 + \beta_1 h + \cdots + \beta_m h^m$ or $\sigma_{\max} = \beta_0 + \beta_1 h + \cdots + \beta_m h^m$ for $m = 2, 3, 4$ we used t-tests within which the null hypothesis H_0: $\beta_j = 0$ was tested in relation to the alternative hypothesis H_1: $\beta_j \neq 0$, $j = 0, 1, \ldots, m$.

We used the level of significance $\alpha = 0.05$. If p-value is lower than the significance level, a particular regression coefficient is considered statistically relevant. Point estimations $\hat{\beta}_0, \hat{\beta}_1, \ldots, \hat{\beta}_m$ of regression coefficients $\beta_0, \beta_1, \ldots, \beta_m$ which were gained by means of the least square method, as well as the resulting p-values for examined regression coefficients are listed in Tab. 2. − Tab. 5.

Table 2: Least squares parameter estimates for $\sigma_{\min}$ vs. h

Type of polynomial	Parameter estimates
Quadratic	$\hat{\boldsymbol{\beta}} = (72.5862, -19.1022, 4.2129)^T$
Cubic	$\hat{\boldsymbol{\beta}} = (109.4107, -84.4249, 32.5394, -3.4335)^T$
Quartic	$\hat{\boldsymbol{\beta}} = (137.0692, -155.3442, 84.4394, -17.6174, 1.2894)^T$

Table 3: P-values of t-tests for $\sigma_{\min}$ vs. h

Type of polynomial	***P*-values**
Quadratic	[0.0005; 0.0952; 0.0475]
Cubic	[$2.539 \cdot 10^{-5}$; 0.0010; 0.0014; 0.0027]
Quartic	[$2.0644 \cdot 10^{-5}$; 0.0006; 0.0017; 0.0051; 0.0120]

Table 4: Least squares parameter estimates for $\sigma_{\max}$ vs. h

Type of polynomial	Parameter estimates
Quadratic	$\hat{\boldsymbol{\beta}} = (276.1293, -74.8267, 12.5339)^T$
Cubic	$\hat{\boldsymbol{\beta}} = (345.7367, -198.3027, 66.0780, -6.4902)^T$
Quartic	$\hat{\boldsymbol{\beta}} = (397.6817, -331.4950, 163.5506, -33.1287, 2.4217)^T$

Table 5: P-values of t-tests for $\sigma_{\max}$ vs. h

Type of polynomial	***P*-values**
Quadratic	[$4.6263 \cdot 10^{-6}$; 0.0047; 0.0062]
Cubic	[$6.2653 \cdot 10^{-7}$; 0.0002; 0.0005; 0.0015]
Quartic	[$5.1824 \cdot 10^{-8}$; $7.3088 \cdot 10^{-6}$; $3.8766 \cdot 10^{-5}$; 0.0001; 0.0004]

After the estimation of parameters of regression functions it is appropriate to evaluate relevance of the selection according to specified criteria. The following residual characteristics were used as criteria for the estimation of regression function relevance: root mean squared error *RMSE*, mean absolute percentage error *MAPE*, coefficient of determination R^2 or modified coefficient of determination R^{*2}.

The *RMSE* statistics is a point estimation of standard deviation σ of a random component ε. The *MAPE* statistics is used as an accuracy indicator of individual predictions as opposed to reality. There is no general scale in analytical practice which would specify acceptable *MAPE* values. If the calculated value of $MAPE < 10$ %, it is interpreted as an excellent prediction; values between 10 – 20 % are interpreted as a good prediction, values between 20 – 50 % are interpreted as an acceptable prediction, and values above 50 % are interpreted as an inaccurate prediction [16–18]. The coefficient (index) of determination (R-squared) R^2 specifies the part of the total variability of values under examination which can be explained by the particular regression model. It is hence an important feature when estimating the appropriateness of the selected regression model. Values close to zero indicate that the selected regression function is not appropriate. Contrary to the above mentioned, values close to 1 indicate that the regression function is very appropriate for extrapolation. The R-squared is a point estimation of the coefficient of determination ρ^2 of the basic group. If the sample size is small, in this case such estimation is biased and overestimates the appropriateness for the regression model. A modified coefficient of determination (adjusted R-squared) R^{*2} provides us with estimation without bias. This coefficient was modified with respect to the number of model parameters and sample size [15,19].

When comparing more regression functions, the most appropriate seems to be the regression model in which R^2 or R^{*2} reaches higher values and *RMSE* as well as *MAPE* reach lower values [11,15,20].

Basic statistical outcomes for three selected polynomial regression models and both examined principles are listed in Tab. 6 and 7.

Table 6: Basic regression statistics for $\sigma_{\min}$ vs. h

Statistics	**Polynomial model**		
	quadratic	**cubic**	**quartic**
RMSE	10.0855	4.8829	2.6857
MAPE [%]	12.7878	5.0160	2.7759
$\boldsymbol{R^2}$	0.5656	0.9127	0.9780
$\boldsymbol{R^{*2}}$	0.4415	0.8691	0.9604

The least square method provides us with point estimations of linear regression models without deviations provided that certain preconditions of random errors distribution probability are fulfilled in the model.

Table 7: Basic regression statistics for σ_{max} vs. h

Statistics	Polynomial model		
	quadratic	cubic	quartic
RMSE	18.6607	8.2120	2.3339
MAPE [%]	7.0209	2.9358	0.7247
R^2	0.7076	0.9515	0.9967
R^{*2}	0.6241	0.9272	0.9941

It is assumed that random errors ε are normally distributed with zero mean value and constant dispersion (homoscedasticity). It is also assumed that these errors are uncorrelated. Fulfilment of these preconditions cannot be validated earlier than after regression model is fitted, since residuals are not known before this selection. As long as the model does not fulfil any of the given conditions, it cannot be used for given data even if it is better than the alternating models, e.g. according to *RMSE* or R^{*2} values [11,15].
Preconditions of the model are mainly verified by means of simple diagrams or well-known statistical tests. In the event of standardized residuals diagram versus theoretical values it is true that the model is appropriate as long as cca 95 % of residuals is in the interval of $(-2, 2)$ [15].

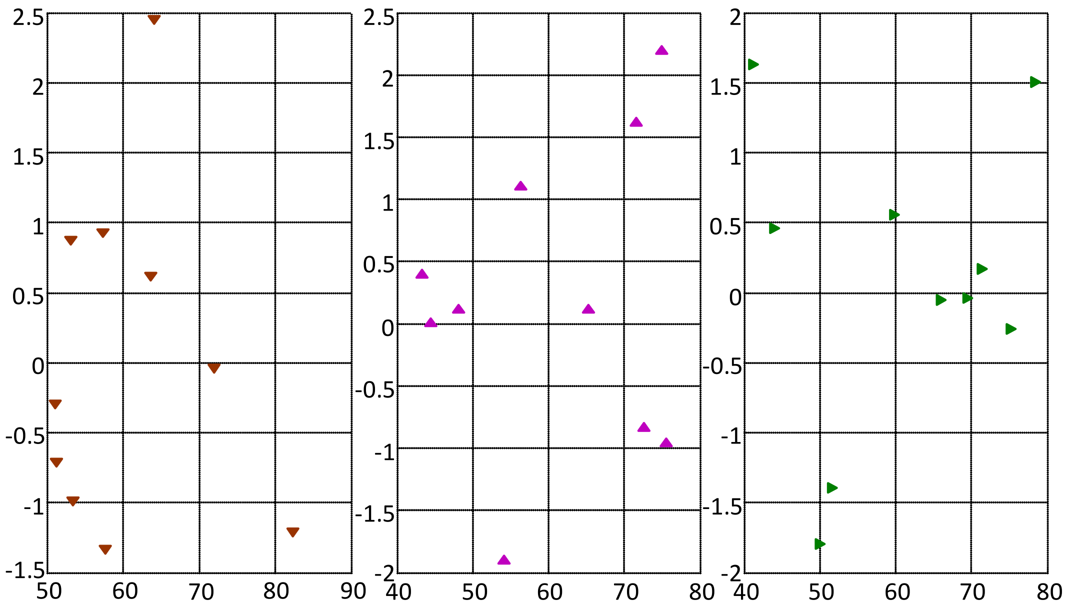

Figure 7: Scatter plots of standardized residuals vs. fitted values of three regression polynomials for σ_{min} vs. h

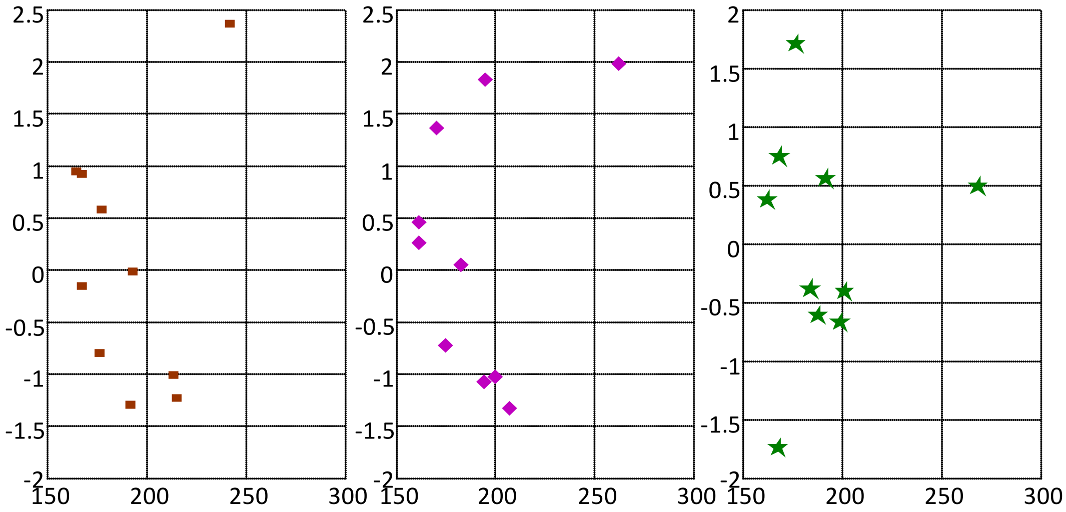

Figure 8: Scatter plots of standardized residuals vs. fitted values of three regression polynomials for $\sigma_{\max}$ vs. h

Based on Tab. 3 it can be stated that in the event of $\sigma_{\min}$ vs. h the quadratic regression polynomial is not satisfactory as its coefficients β_1 and β_2 are of no statistical relevance. As regards $\sigma_{\max}$ vs. h (see Tab. 5), all coefficients of the three examined types of regression polynomials are statistically relevant.
Based on the statistical outcomes in Tab. 6 and 7, for both examined cases the most appropriate degree of polynomial model is the 4$^{\text{th}}$ degree.
Figure 7 and 8 indicate that all three polynomial regression models are appropriate for the residual analysis. The best outcome for both examined cases relates to the 4$^{\text{th}}$ degree of a polynomial model. In this case all residuals are defined in the interval of $(-2, 2)$.

Conclusion

Based on the statistical analysis, in both cases we have chosen polynomial regression models of the 4$^{\text{th}}$ degree for modelling of dependence between extreme normal stresses and the depth of drilling. Estimated regression coefficients for both examined cases are listed in Tab. 2 and 4.
By merging experimental method of photoelasticity and mathematical statistical methods we have specified the tendency of extreme normal stresses on the bracket. The selected regression models can as well be used in the prediction of stresses in dependence of drilling depth.
Through measurements we found values of reduced stress according to von Mises yield criterion. These values reached up to 78 % of yield strength of the bracket material. These stresses are relatively high if compared to yield strength of the material. It was hence recommended that the manufacturer of the trailer frame should

take a series of measures. These measures were related to technology of forming and heat-processing of the bracket.

In comparison to resistance tensiometry, the photoelastic drilling method enables us, in addition to visualization of stress gradient, to specify stresses directly in the edge area of the hole. Tensiometric method enables to find only average stress values in the area close to the hole. These values, however, depend on the length of tensiometer's base. The method of photoelasticity, as well as tensiometric method, can be used directly on a real structure. This paper demonstrated verification of the selected method. Such procedure can be used for future verification of similar tasks.

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