Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 12, Number 3 (2016), pp. 2905–2913

http://www.ripublication.com/gjpam.htm

Mathematical Model for Improper Modulation Based MIMO Transceiver Under Per Antenna Power Constraint and Imperfect CSI

C. Manikandan, Dr. P. Neelamegam,
R. Mathura Janani and P. Noor Mohammed
School of EEE, SASTRA University,
Thanjavur, Tamil Nadu.

Abstract

This article focus on the issue of designing one dimensional improper modulation based Multiple Input and Multiple Output (MIMO) transceivers. Normally the mathematical models developed for the improper modulation based MIMO transceivers are subject to total power constraint (TPC) and perfect Channel State Information (CSI). However, for real time implementation TPC and perfect CSI are not appropriate since they do not take into account the power limitation due to the linear response of individual power amplifiers, transmit antenna correlations, receive antenna correlations and the channel estimation error. To overcome these issues we mathematically modeled an improper modulation based MIMO transceiver utilizing Per Antenna Power Constraint (PAPC) and Imperfect CSI.

AMS subject classification:
Keywords: MIMO, TMSE, Improper Modulation, PAPC, Imperfect CSI, BER.

1. Introduction

In the last seven years there have been several total mean square error (TMSE) based MIMO transceivers designed to get better BER performance. The first conventional transceiver under the minimum total TMSE criterion was designed in [1] to yield a good BER for proper modulation schemes, e.g., M-QAM, and M-PSK. However, while applying the same design to the one dimensional improper modulation schemes, e.g., BPSK and M-ASK, the BER performance degraded significantly. Then TMSE based transceiver design for improper modulation was proposed by designing optimum precoder in [2] to give superior BER performance than the conventional design in [1]. To improve the BER of improper modulation based TMSE transceiver further, the joint optimum precoder and decoder was designed in [3]. All the transceivers were designed in [1–3] using SPC. But, SPC did not take into account the power constraint of individual power amplifier (PA) at each transmit antenna. To overcome this problem for the first time in [4] joint optimum precoder and decoder were designed using per antenna constraint (PAPC). Later, transceiver design with improper modulations using PAPC and perfect CSI was designed [5]. However, no one has designed a transceiver system employing one dimensional improper modulation techniques subject to individual power constraint and imperfect CSI. To fulfill this requirement, in this paper we have developed a mathematical model for improper modulation based SU-MIMO transceiver considering individual power constraint and imperfect CSI.

The rest of the paper is structured as follows: In Section 2 concepts of Imperfect channel state estimation and p-norm based individual power constraint are described. In Section 3, a mathematical model for improper modulation based MIMO transceiver system under per antenna power constraint and imperfect CSI is proposed. Section 4 contains an Iterative procedure for solving optimum precoder and decoder. Section 5 concludes the paper.

2. Related Concept

2.1. Imperfect channel state estimation for MIMO channel

The MIMO channel model from [1]

$$H = R_R^{1/2} H_W R_T^{1/2} \tag{1}$$

where H_W is a matrix whose entries are independent Gaussian components with zero mean and unity variance, T_R and R_R are transmit correlation and receive correlation matrices. From [6] we have

$$R_R, R_T = \begin{bmatrix} 1 & \rho & \rho^4 & \cdots & \rho^{(N-1)^2} \\ \rho & 1 & \rho & \cdots & \vdots \\ \rho^4 & \rho & 1 & \ddots & \rho^4 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{(N-1)^2} & \cdots & \rho^4 & \rho & 1 \end{bmatrix} \tag{2}$$

Where N is equal to N_R (or) N_T and ρ is the fading correlation between two nearby transmit or receive antennas.

For imperfect case, estimated channel matrix $\hat{H}_W$ based on minimumum mean square error estimation is given by [3]

$$\hat{H}_W = R_{e,R} \bar{H}_W \tag{3}$$

Where, $R_{e,R} = [I_{NR} + \sigma_{ce}^2 R_R^{-1}]^{-1}$ and $\bar{H}_W = R_R^{-1/2} Y_{TR} X^{-1}$.

Reference [3] observed $Y_{TR} = HX_{TR} + N_{TR}$ as a received signal in training phase.During the train phase where X_{TR} is a training signal matrix and N_{TR} is a noise signal matrix.

The overall channel estimate represented in [3]

$$H_W = \hat{H}_W + R_R^{-1/2} R_{e,R}^{-1/2} E_W \tag{4}$$

Here equation (4) is the sum of the estimated channel matrix and an estimation error matrix.Where the entries of E_W are independent and identically distributed with zero mean and variance σ_{ce}^2. The paramater σ_{ce}^2 reflects the quality of the channel estimate.

Finally the exact overall channel matrix H for the Correlated MIMO channel with the imperfect case can be expressed as:

$$\begin{aligned} H &= R_R^{1/2} \hat{H}_W R_T^{1/2} + R_R^{1/2} [R_R^{-1/2} R_{e,R}^{1/2} E_W] R_T^{1/2} \\ &= \hat{H} + E \end{aligned} \tag{5}$$

2.2. Implementation of Individual power constraint using p-norm

In this section, the concept of p-norm and its relation to individual antenna power constraint are discussed. In linear algebra, the p-norm is given by

$$\|x\|_p = \left(\sum_{i=o}^{n} |x_i|^p \right)^{1/p} \qquad \text{For } p \geqslant 1 \tag{6}$$

For smaller value of p in the range of $p = 1 < p < \infty$, the p-norm constraint can satisfy individual power constraint and give the performance closer to the total power constraint.

3. System Model

One dimensional improper modulation based transceiver designed in [5] using individual power constraint and perfect CSI is no longer optimum. In practice MIMO wireless channels are time varying in nature such that CSI obtained at both transmitter and receiver won′t be perfect. In this section our objective is to develop a one dimensional improper modulation based transceiver by considering individual power constraint, transmit antenna correlations, receive antenna correlations and the channel estimation error to minimize TMSE and give near optimum performance at the output.

In one dimensional improper modulation MIMO transceiver the estimated output signal can be written as,

$$\hat{s} = Re(GHFs + Gn) \tag{7}$$

Generally total MSE can be expressed as,

$$E[\|e\|^2] = E[\|\hat{s} - s\|^2] \tag{8}$$

For imperfect CSI the TMSE can be calculated as follows,

$$
\begin{aligned}
&= E[\| Re(G(\hat{H} + E)Fs + Gn) - s\|^2] \\
&= E[\|(G(\hat{H} + E)Fs + G^*(\hat{H} + E)^* F^* s^*)/2 + (Gn + G^* n^*)/2 - s\|^2] \\
&= Tr\Bigg\{E\Bigg[[0.5(G(\hat{H} + E)Fs + G^*(\hat{H} + E)^* F^* s^*) + 0.5(Gn + G^* n^*) - s] \\
&[0.5(G^H(\hat{H} + E)^H S^H F^H + G^T(\hat{H} + E)^T F^T s^T) + 0.5(G^H n^H + G^T n^T) - s^H]\Bigg]\Bigg\}
\end{aligned}
\tag{9}
$$

First replace $E = R_{e,R}^{1/2} E_W R_T^{1/2}$ in (9) and use the following factors

$$E[ss^T] = E[ss^H] = I_B,\ E[E_W A E_W^H] = \sigma_{ce}^2 tr(A) I_N$$

and

$$E[nn^H] = \sigma_n^2 I_{N_T}$$

$$E[n] = E[nn^T] = E[n^* n^H] = 0,\ E[sn] = E[ns] = E[s^H n] = E[n^H s] = 0$$

$$E[E_W] = E[E_W^H] = E[E_W A E_W^T] = 0$$

Then the equation (9) can be simplified as,

$$
\begin{aligned}
= Tr\Bigg\{&0.25(G\hat{H}FF^H\hat{H}^H G^H + G\hat{H}FF^T\hat{H}^T G^T + GR_{e,R}G^H Tr(R_T FF^H)\sigma_{ce}^2 \\
&+ GG^H\sigma_n^2 + G^*\hat{H}^* F^* F^H \hat{H}^H G^H + G^*\hat{H}^* F^* F^T \hat{H}^T G^T \\
&+ G^* R_{e,R}^* G^T Tr(R_T FF^H)^* \sigma_{ce}^2 + G^* G^T \sigma_n^2 I_n) \\
&- 0.5(G\hat{H}F + G^*\hat{H}^* F^* + F^H \hat{H}^H G^H + F^T \hat{H}^T G^T) + I_B\Bigg\}
\end{aligned}
\tag{10}
$$

The design target is to find a couple of matrices F and G to minimize $E[\|e\|^2]$ subject to individual power constraint is

$$\min_{(F,G)} E[\|e\|^2] \text{ subject to } [Tr(FF^H)^p]^{1/p} \leqslant \alpha$$

Here in the above equation the value of p is calculated based on the SPC(β), PAPC(α) and the number of bit stream (B).

Mathematically p is defined as follows,

$$\frac{\alpha B}{B^{1/p}} = \beta,\ \beta^{1/p} = \frac{\alpha B}{\beta},\ \ln \beta^{1/p} = \ln \frac{\alpha B}{\beta},\ p = \frac{\ln \beta}{\ln \frac{\alpha B}{\beta}}$$

For p in the interval $1 < p < \infty$, p-norm constraint sufficiently meets both the SPC and PAPC.

To obtain the solution of the above problem, the Lagrangian form,

$$\eta = [E(\|e\|^2 + \mu[Tr(FF^H)^p]^{1/p}) - \alpha] \tag{11}$$

μ is the Lagrange multiplier.

Simplifying Lagrangian form after substituting (10) in (11),

$$\begin{aligned}\eta = E(Tr\Big\{&0.25(G\hat{H}FF^H\hat{H}^HG^H + G\hat{H}FF^T\hat{H}^TG^T + GR_{e,R}G^HTr(R_TFF^H)\sigma_{ce}^2\\
&+ GG^H\sigma_n^2 + G^*\hat{H}^*F^*F^H\hat{H}^HG^H + G^*\hat{H}^*F^*F^T\hat{H}^TG^T\\
&+ G^*R_{e,R}^*G^TTr(R_TFF^H)^*\sigma_{ce}^2 + G^*G^T\sigma_n^2)\\
&- 0.5(G\hat{H}F + G^*\hat{H}^*F^* + F^H\hat{H}^HG^H + F^T\hat{H}^TG^T) + I_B\Big\}\\
&+ \mu[Tr(FF^H)^p]^{1/p}) - \alpha\end{aligned} \tag{12}$$

Taking the derivatives of η with respect to F and G [5], the associated Karush-Kuhn-Tucker (KKT) conditions can be derived by using the cyclic property of the trace function. It is shown in the following

$$\frac{\partial \eta}{\partial G} = 0 \quad \text{in (12)}$$

$$\begin{aligned}0.25\Bigg[&\frac{\partial Tr(G\hat{H}FF^H\hat{H}^HG^H)}{\partial G} + \frac{\partial Tr(G\hat{H}FF^TH^TG^T)}{\partial G}\\
&+ \frac{\partial Tr(GR_{e,R}G^HTr(R_TFF^H)\sigma_{ce}^2}{\partial G} + \frac{\partial Tr(GG^H\sigma_n^2)}{\partial G}\\
&+ \frac{\partial Tr(G^*\hat{H}^*F^*F^H\hat{H}^HG^H)}{\partial G} + \frac{\partial Tr(G^*\hat{H}^*F^*F^T\hat{H}^TG^T)}{\partial G}\\
&+ \frac{\partial Tr(G^*R_{e,R}^*G^TTr(R_TFF^H)^*\sigma_{ce}^2)}{\partial G} + \frac{\partial Tr(G^*G^T\sigma_n^2)}{\partial G}\Bigg]\\
&- 0.5\Bigg[\frac{\partial Tr(G\hat{H}F)}{\partial G} + \frac{\partial Tr(G^*\hat{H}^*F^*)}{\partial G}\\
&+ \frac{\partial Tr(F^H\hat{H}^HG^H)}{\partial G} + \frac{\partial Tr(F^T\hat{H}^TG^T)}{\partial G}\Bigg]\\
&+ \mu\frac{\partial[Tr(FF^H)^p]^{1/p}}{\partial G} + \frac{\partial(\alpha)}{\partial G} = 0\end{aligned} \tag{13}$$

Apply the following and simplify the equation (13)

$$\frac{\partial Tr(GA)}{\partial G} = A^T, \frac{\partial Tr(GAG^T)}{\partial G} = GA^T + GA,$$

$$\frac{\partial Tr(GG^H)}{\partial G} = G^*, \frac{\partial Tr(A)}{\partial G} = 0$$

$$\begin{aligned} &= 0.25[G^*(\hat{H}FF^H\hat{H}^H)^T + G\hat{H}FF^T\hat{H}^T + G^*R_{e,R}^*Tr(R_TFF^H)^*\sigma_{ce}^2 \\ &\quad + G^*(\hat{H}FF^H\hat{H}^H)^T + G\hat{H}FF^T\hat{H}^T + G^*R_{e,R}Tr(R_TFF^H)\sigma_{ce}^2] \\ &\quad - 0.5[F^T\hat{H}^T + F^T\hat{H}^T] + 0.25\sigma_n^2(G^* + G^*) = 0 \end{aligned} \tag{14}$$

$$\begin{aligned} 0 &= -F^T\hat{H}^T + 0.5G^*(\hat{H}FF^H\hat{H}^H)^T + 0.5G\hat{H}FF^T\hat{H}^T + 0.5\sigma_n^2G^* \\ &\quad + 0.5G^*R_{e,R}Tr(R_TFF^H)\sigma_{ce}^2 \end{aligned} \tag{15}$$

$$\begin{aligned} F^T\hat{H}^T &= 0.5G^*(\hat{H}FF^H\hat{H}^H)^T + 0.5G\hat{H}FF^T\hat{H}^T + 0.5\sigma_n^2G^* \\ &\quad + 0.5G^*R_{e,R}Tr(R_TFF^H)\sigma_{ce}^2 \\ 2F^T\hat{H}^T &= G^*(\hat{H}FF^H\hat{H}^H)^T + G\hat{H}FF^T\hat{H}^T \\ &\quad + \sigma_n^2G^* + G^*R_{e,R}Tr(R_TFF^H)\sigma_{ce}^2 \end{aligned}$$

Taking complex conjugate,

$$G(\hat{H}FF^H\hat{H}^H + R_{e,R}Tr(R_TFF^H)\sigma_{ce}^2) + G^*\hat{H}^*F^*F^H\hat{H}^H + \sigma_n^2G = 2F^H\hat{H}^H \tag{16}$$

Similarly now setting $\frac{\partial \eta}{\partial F} = 0$ in (12),

$$\begin{aligned} &0.25\left[\frac{\partial Tr(G\hat{H}FF^H\hat{H}^HG^H)}{\partial F} + \frac{\partial Tr(G\hat{H}FF^TH^TG^T)}{\partial F}\right. \\ &\quad + \frac{\partial Tr(GR_{e,R}G^HTr(R_TFF^H)\sigma_{ce}^2}{\partial F} + \frac{\partial Tr(GG^H\sigma_n^2)}{\partial F} + \frac{\partial Tr(G^*\hat{H}^*F^*F^H\hat{H}^HG^H)}{\partial F} \\ &\quad \left. + \frac{\partial Tr(G^*\hat{H}^*F^*F^T\hat{H}^TG^T)}{\partial F} + \frac{\partial Tr(G^*R_{e,R}^*G^TTr(R_TFF^H)^*\sigma_{ce}^2)}{\partial F} + \frac{\partial Tr(G^*G^T\sigma_n^2)}{\partial F}\right] \\ &\quad - 0.5\left[\frac{\partial Tr(G\hat{H}F)}{\partial F} + \frac{\partial Tr(G^*\hat{H}^*F^*)}{\partial F} + \frac{\partial Tr(F^H\hat{H}^HG^H)}{\partial F} + \frac{\partial Tr(F^T\hat{H}^TG^T)}{\partial F}\right] \\ &\quad + \mu\frac{\partial[Tr(FF^H)^p]^{1/p}}{\partial F} + \frac{\partial(\alpha)}{\partial F} = 0 \end{aligned} \tag{17}$$

Applying the following and simply the equation (17)

$$\frac{\partial Tr(FA)}{\partial F} = A^T, \frac{\partial Tr(FAF^T)}{\partial F} = FA^T + FA,$$

$$\frac{\partial Tr(FF^H)}{\partial F} = F^*, \frac{\partial Tr(A)}{\partial F} = 0, \frac{\partial Tr(F^T A)}{\partial F} = A$$

where the partial derivative of $[Tr(FF^H)^p]^{1/p}$ with respect to F is obtained using the Chain rule of Matrix differentiation and the following properties.

$$\frac{\partial g(U)}{\partial F} = Tr\left[\left(\frac{\partial g(U)}{\partial U}\right)^T \frac{\partial (U)}{\partial F}\right],$$

$$Tr\left(\frac{\partial g(U)}{\partial F}\right) = \frac{\partial Tr(g(U))}{\partial F}, \frac{\partial Tr(F^p)}{\partial F} = p(F^T)^{p-1}$$

$$\begin{aligned} &0.25[2(\hat{H}^H G^H G \hat{H} F)^* + 2\hat{H}^T G^T G \hat{H} F + 2F^* R_T Tr(R_{e,R} G G^H)^* \sigma_{ce}^2] \\ &- 0.5[\hat{H}^T G^T + \hat{H}^T G^T] + \mu[[Tr(FF^H)^p]^{(1/p)-1}((FF^H)^T)^{p-1} F^*] = 0 \end{aligned} \quad (18)$$

Again, by applying complex conjugates on both sides, we get

$$\begin{aligned} &\hat{H}^H G^H G \hat{H} F + \hat{H}^H G^H G^* \hat{H}^* F^* + F R_T Tr(R_{e,R} G G^H) \sigma_{ce}^2 \\ &+2\mu[[Tr(FF^H)^p]^{(1/p)-1}((FF^H)^T)^{p-1} F^*]^* = 2\hat{H}^H G^H \end{aligned} \quad (19)$$

Post multiplying (16) by G^H

$$\begin{aligned} &GG^H(\hat{H} F F^H \hat{H}^H + R_{e,R} Tr(R_T F F^H) \sigma_{ce}^2) + G^H G^* \hat{H}^* F^* F^H \hat{H}^H \\ &+ \sigma_n^2 G G^H = 2F^H \hat{H}^H G^H \end{aligned} \quad (20)$$

Next, by pre multiplying (19) by F^H

$$\begin{aligned} &\hat{H}^H G^H G \hat{H} F F^H + \hat{H}^H G^H G^* \hat{H}^* F^* F^H + F F^H R_T Tr(R_{e,R} G G^H) \sigma_{ce}^2 \\ &+2\mu[[Tr(FF^H)^p]^{(1/p)-1}((FF^H)^T)^{p-1} F^*]^* F^H = 2F^H \hat{H}^H G^H \end{aligned} \quad (21)$$

Equating the equations (20) and (21), we get

$$\begin{aligned} &\sigma_n^2 G G^H + \sigma_{ce}^2 R_{e,R} Tr(R_T F F^H) G G^H = \sigma_{ce}^2 F F^H Tr(G G^H R_{e,R}) R_T \\ &+ 2\mu[Tr(FF^H)^p]^{(1/p)-1}(F^H F) F F^H \end{aligned}$$

Applying trace on both sides and using $Tr(CA) = CTr(A)$

$$\mu = \frac{\sigma_n^2 Tr(GG^H)}{2\alpha} \quad \text{where, } \alpha = [Tr(FF^H)^p]^{1/p} \quad (22)$$

An iterative procedure is developed to find the solutions, $G = G_{Re} + jG_{Im}$ and $G^* = G_{Re} - jG_{Im}$,

$$\hat{H} F F^H \hat{H}^H + R_{e,R} Tr(R_T F F^H) \sigma_{ce}^2 = A_{Re} + jA_{Im},$$

$$\hat{H}^* F^* F^H \hat{H}^H = B_{Re} + jB_{Im}, \;\; 2F^H \hat{H}^H = C_{Re} + jC_{Im}$$

A,B,C represents the terms in the equation

$$(C_{Re} \quad C_{Im}) = (G_{Re} \quad G_{Im}) \begin{pmatrix} A_{Re} + B_{Re} + \sigma_n^2 I_{NR} & A_{Im} + B_{Im} \\ B_{Im} - A_{Im} & A_{Re} - B_{Re} + \sigma_n^2 I_{NR} \end{pmatrix} \quad (23)$$

The above equation can be rewritten as,

$$(G_{Re} \quad G_{Im}) = (C_{Re} \quad C_{Im}) \begin{pmatrix} A_{Re} + B_{Re} + \sigma_n^2 I_{NR} & A_{Im} + B_{Im} \\ B_{Im} - A_{Im} & A_{Re} - B_{Re} + \sigma_n^2 I_{NR} \end{pmatrix}^{-1} \quad (24)$$

Similarly, we define

$$F = F_{Re} + jF_{Im} \text{ and } F^* = F_{Re} - jF_{Im}, \;\; \hat{H}^H G^H G \hat{H} + R_T Tr(R_{e,R} G G^H) \sigma_{ce}^2 = P_{Re} + jP_{Im},$$

$$2\hat{H}^H G^H = R_{Re} + R_{Im}, \hat{H}^H G^H G^* \hat{H}^* = Q_{Re} + jQ_{Im}, \hat{H}^H G^H G^* \hat{H}^* = Q_{Re} + jQ_{Im}$$

Assuming

$$k = [[Tr(FF^H)^p]^{(1/p)-1}((FF^H)^T)^{p-1}]^*$$

P,Q,R represents the terms in the equation.

$$\begin{pmatrix} R_{Re} \\ R_{Im} \end{pmatrix} = \begin{pmatrix} P_{Re} + Q_{Re} + 2\mu k I_{NT} & P_{Im} - Q_{Im} \\ P_{Im} + Q_{Im} & P_{Re} - Q_{Re} + 2\mu k I_{NT} \end{pmatrix} \begin{pmatrix} F_{Re} \\ F_{Im} \end{pmatrix} \quad (25)$$

The above equation can be rewritten as

$$\begin{pmatrix} F_{Re} \\ F_{Im} \end{pmatrix} = \begin{pmatrix} P_{Re} + Q_{Re} + 2\mu k I_{NT} & P_{Im} - Q_{Im} \\ P_{Im} + Q_{Im} & P_{Re} - Q_{Re} + 2\mu k I_{NT} \end{pmatrix}^{-1} \begin{pmatrix} R_{Re} \\ R_{Im} \end{pmatrix} \quad (26)$$

Based on the above expressions, An iterative approach is obtained for the precoder matrix F and decoder matrix G using per antenna power allocation.

4. Iterative Algorithm

Step 1: Initialize $F = F_0$ the upper matrix of F_0 is chosen to be scaled identity, while the remaining entries of F_0 are set to zero.
Step 2: Update G using (24).
Step 3: Update μ using (22).
Step 4: Update F using (26), if $[Tr(FF^H)^p]^{1/p} > \alpha$. Scale such that $[Tr(FF^H)^p]^{1/p} = \alpha$.
Step 5: If $[Tr((F_i - F_{i-1})(F_i - F_{i-1})^H)^p]^{1/p}$ is sufficiently small(less than 10^{-4}),stop.otherwise go back to Step 2. Here F_i, (F_{i-1}) denotes F in the i-th,(i-1)-th iteration.

5. Conclusion

In this paper one dimensional improper modulation based SU-MIMO transceiver is mathematically modeled using per antenna power constraint for the imperfect channel state condition. In the range of p=1$<$ p$<\infty$ the proposed power allocation scheme will give the BER performance result closer to the optimal total power constraint by considering power restricted by the linear response of the individual power amplifier. The mathematical model developed in this article can be analyzed through simulation and can also be further extended to Multi user MIMO system.

References

[1] Ding, M., and Blostein, S, MIMO minimum total MSE transceiver design with imperfect CSI at both ends, IEEE Transactions on Signal Processing, 57(3)2009, 1141–1150.

[2] Xiao, P., and Sellathurai, M, Improved linear transmit processing for single-user and multi-user MIMO communications systems, IEEE Transactions on Signal Processing, 58(3)2010, 1768–1779.

[3] M. Raja, P. Muthuchidambaranathan, Ha H. Nguyen, Transceiver De-sign for MIMO Systems with Improper Modulations, Wireless Personnal Communications, Springer Science+Business Media, LLC. 68(2) 2011, 265–280.

[4] A. Merline, S. J. Thiruvengadam, Design of Optimal Linear Pre-coder and Decoder for MIMO Channels with Per Antenna PowerConstraint and Imperfect CSI, Wireless Personnal Communications,Springer Science+Business Media New York, 75(2) 2013, 1251–1263.

[5] C. Manikandan, P. Neelamegam, A. Srivishnu and B. Sabari Ganesh, Transceiver Design for SU-MIMO System with Improper Modulations Using per Antenna Power Constraint and Imperfect CSI, International Journal of Mathematical Analysis, 9(31) 2015, 1519–1528.

[6] Ke Lin-Du and M.N.S. Swamy, Wireless Communication Systems: From RF Subsystems to 4G Enabling Technologies 1st Edition, Cambridge University Press, 2010.