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Optimum Allocation of Resources in University Management through Goal Programming

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Abstract

This paper proposes a goal programming model that relives these limitations and offers other features as well; in it goals are faculty instruction loads, staff-to-faculty ratios, faculty distribution by rank, and teaching-assistant -to- faculty ratios. These specified goals are achieved as closely as possible, subject to constraints on the projected budget available in each year of the planning horizon and to faculty-flow constraints. The decision variables are the faculty, staff and teaching-assistant levels in each of several academic units over the planning horizon. The model provides a vehicle for long-range budget planning and resource allocation.

AMS subject classification:
Keywords: Management, Goal Programming, Optimum Allocation.

1. Introduction

Assigning offices to members of staff can be a difficult task in large academic institutions that comprise different faculties with diverse requirements in terms of office space. Space allocation on the basis of historical precedent seemed to be dominant approach in the 1970's and early 1980's. Most institutions now days find it extremely difficult to acquire new buildings since they are under increasing financial pressure. In addition,

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vested interests and variability of rooms in terms of size, layout, proximity to departmental offices, etc make the problem even more complicated. Hence, it has become essential for these institutions to methods for staff office allocation to cope with the increasing requirements under limited resources. Furthermore, it has become necessary to demonstrate for audit and funding purpose an effectiveness of resource allocation.

Various models have been developed for the efficient allocation of academic resources in general (financial resources, equipment, etc). The most popular approach appears to be goal programming (see Ignizio [7] and Romero [11]). Goal programming allows the decision maker to specify targets and then attempts to find a solution that comes as close as possible to meeting these targets. Examples of Goal programming models for academic resources allocation can be found in Lee and Clayton [8], Schroeder [13] and Diminme and Kwak [2].

The specific problem of allocation space in academic establishments has received relatively title attention from operational researches. Ritzman et al. [10]. Presented a goal programming used to reassign 144 offices to 289 staff members at the Ohio State University. Benjamin et al. [1]. Combined goal programming with the analytical Hierarchy Process (AHP) to determine the layout of a new computer laboratory at the university of Missouri-Rolla. The authors specified a set of goals and used AHP to determine how these goals should be priorized. The AHP is a multicriteria decision making technique that employees pair wise comparisons to help decision makers to define priorities and weights to reflect the relative importance of the options in a multicriteria problem. For more details on the AHP see Saaty [12].

2. Data of The Problem

Data were gathered at the JNT University representing there large academic departments over a three year period. Initially, there was an imbalance in resources (faculty and TA levels) relative to desired goal levels. The model redistributed the resource levels over the time horizon to achieve the desired goals as closely as possible with in the specified constraints. The required information is given in the Table I.

With regard to faculty, the initial numbers of professors at each rank in each department were obtained. The goal levels for faculty were specified in terms of total faculty for each department and year. In department A, the desired faculty levels were projected to decrease, in department B an increase in goal level was projected, and department C had a constant faculty goal level. These levels were obtained by projections of student enrollments and faculty workload ratios.

In the situation under consideration, all new faculty hired were at the assistant professor level. In this case, it was not necessary to specify desired faculty distribution goals, since the distribution was fixed by the hiring assumption. Table I also indicates the faculty salaries, loss rates and promotion rates based on historical data: they were approximately constant for all departments. We excluded clerical personnel from this example since a negligible amount of money was involved.

Based on the data in Table I, it was possible to formulate all the constraints of the

Table 1: Distributed resource levels.

	Department		
	A	**B**	**C**
Initial faculty			
Professors	30	24	18
Associate Professors	26	18	12
Assistant Professors	16	1	7
Total	**72**	**43**	**37**
Goal levels			
Total Faculty, Year 1	86	40	35
Total Faculty, Year 2	75	45	35
Total Faculty, Year 3	65	50	35
Desired TA-to-faculty ratio, all years	0.8	0.8	0.9
	Annual Salary (Rs. in Lakhs)	Loss rate (Annual)	Promotion rate (Annual)
Professors	3.5	0.03	-
Associate Professors	2.75	0.06	0.20
Assistant Professors	2.25	0.26	0.20
Teaching Assistant	1.50	-	-
Total Budget for all departments = Rs. 1,37,67,000			

model, a total of 23:

i. Nine faculty goal constraints, one for each year and each department.

ii. Nine TA goal constraints, one for each year and each department.

iii. Three budget constraints, one for each other.

iv. Two upper-bound constraints on faculty, for department B, year 2 and 3.

3. Goal Programming Model

The Goal programming model can be stated as follows:
Min. $M^{-}Y^{-} + M^{+}Y^{+}$, subject to

$$AX + IY^{-} - IY^{+} = g,\ BX \geq b,\ X \geq 0,\ Y^{-} \geq 0,\ Y^{+} \geq 0 \tag{1}$$

In this formulation, M^{-} and M^{+} are row vectors of goal weights, Y^{+} is a column vector of overachievement of goal levels, and Y^{-} is a column vector of underachievement of goals. A is a matrix of coefficients, X is a column vector of decision variables, I is the identity matrix, and g is a column vector of desired goal levels. The additional constraints defined by BX $\geq$ b be adjoined to the problem.

Suppose there are n goals to be achieved in (1) and that we specialize the objective function for the moment to

$$Min. \sum_{i=1}^{n} M_i (Y_i^+ + Y_i^-) \tag{2}$$

Assume further that the goals are arranged in priority order from 1 to n with goal 1 having the highest priority. Then we require that $M^1 > M^2 > M^3 \cdots > M^n$, Where the $>$ sign indicates an ordinal relation between coefficients M^i.

The Problem can be Stated as Follows:

$$f_{ij}^{t+1} = D_{ij}^t + x_{ij}^{t+1} + p_j^t f_{(i-1)j}^t, \text{ for } i = 1, 2, \ldots, (t-1) \text{ (Faculty Flow)} \tag{3}$$

Faculty at rank i, unit j, period $t+1$ equals those who remain from the previous period (D_{ij}^t, f_{ij}^t), plus those who are hired at the beginning of period $t+1$. (x_{ij}^{t+1}), plus those who are promoted $(P_{ij}^t, f_{i-1,j}^t)$. None that this relation and $f_{ij}^t \geq 0$ imply that faculty cannot be laid off. Reductions in faculty are achieved only by normal attrition.

$$\sum_{i-1}^{m} x_{ij}^t \leq U_j^t \text{ (Maximum Hiring)} \tag{4}$$

We have placed an upper limit on the number of faculty who can be hired in one period, owing to such factors as supply and demand of prospects.

$$z_j^{t+1} \geq r_j^t z_j^t \text{ (Staff Reduction)} \tag{5}$$

Staff cannot be reduced in any one period by a greater amount than natural attrition. This constraint may, of course vary from institution, depending on the work rules.

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^t f_{ij}^t + \sum_{j=1}^{n} a_j^t w_j^t + \sum_{j=1}^{n} a_j^t z_j^t \leq b^t \text{ (Budget Payroll)} \tag{6}$$

The total amount available for payroll (salaries) is limited in each period t.

Goal Constraints

$$\sum_{i-1}^{m} f_{ij}^t + y_{j1}^{t-} + y_{j1}^{t+} = g_j^t \text{ (Teaching Load Goal)} \tag{7}$$

$$w_j^t - d_j^t \sum_{i=1}^{m} f_{ij}^t + y_{j2}^{t-} - y_{j2}^{t+} = 0 \text{ (TA - Ratio Goal)} \tag{8}$$

$$z_j^t - R_j^t \sum_{i=1}^{m} f_{ij}^t + y_{j3}^{t-} - y_{j3}^{t+} = 0 \text{ (Staff - Ratio Goal)} \tag{9}$$

$$f_{ij}^t - b_j^t \sum_{i=1}^{m} f_{ij}^t + y_{j4}^{t-} - y_{j4}^{t+} = 0 \text{ (Faculty - Rank - Distribution Goal)} \tag{10}$$

Objective Function

$$\textbf{Min} \sum_{t=1}^{T} (M_{j1}^{t-} y_{j1}^{t-} + M_{j1}^{t+} y_{j1}^{t+} + M_{j2}^{t-} y_{j2}^{t-} + M_{j2}^{t+} y_{j2}^{t+} + M_{j3}^{t-} y_{j3}^{t-} + M_{j3}^{t+} y_{j3}^{t+})$$
$$+ \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{t=1}^{T} (M_{ij4}^{t-} y_{ij4}^{t-} + M_{ij4}^{t+} y_{ij4}^{t+}), \quad \text{where,}$$

Variables

f_{ij}^t Faculty level in academic unit j, rank i, at the beginning of period t.
x_{ij}^t Number of new faculty hired at the beginning of period t, unit j, rank i.
w_j^t Number of teaching assistance in unit j, at the beginning of period t.
z_j^t Number of staff in unit j, at the beginning of period t.
In all cases, unless otherwise specified, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$, and $t = 1, 2, \ldots, T$; also all variables are non negative.

Constants

c_{ij}^t Salaries per faculty member, unit j, rank i, period t.
g_j^t Faculty goal level desired in unit j, period t (these goals could be based on projected enrollments, desired faculty to- student ratios, or other criteria).
b_{ij}^t Desired proportion of faculty in ranks i, unit j, period t.
D_{ij}^t Proportion of faculty who stay from period t to $t + 1$, rank i, unit j.
P_{ij}^t Proportion of faculty promoted from rank $i - 1$ to rank i, during period t in unit j.
U_j^t Upper bound on the number of faculty who can be hired in period t, unit j.
d_j^t Desired teaching assistant (TA)-to-faculty ratio in unit j, period t.
a_j^t cost per TA during period t, unit j.
R_j^t desired staff to faculty ratio in unit j, period t.
A_j^t Cost per staff member, unit j, during period t.
r_j^t proportion of staff who stay from period t to $t + 1$ (by choice), unit j.
B^t Total budget available during period t.

There are several points to be considered regarding this model. First, it is focused basically on the division of a payroll budget between faculty, staff, and TA's. It is a planning model and it indicates how staff, faculty, and TA levels should be set relative to the costs and priorities of the goals. This is a critical planning problem for university management.

Table 2: Three different sets of goal structures used in the model

	Priorities					
	Faculty			**Teaching Assistants**		
	Dept. A	Dept. B	Dept. C	Dept. A	Dept. B	Dept. C
Case I	1	2	3	4	5	6
Case II	1	1	1	2	2	2
Case III	1	1	1	1	1	1

Equation (3) indicates that faculty flows are explicitly represented by rank in the model. This level of details is used in order to represent cost changes that are due to increasing or decreasing faculty levels. Since a full professor can receive on the order of twice the salary of an assistant professor, a change in the mix of faculty ranks in future years can substantially affect total faculty costs. For example, a decision by the model not to hire any new faculty in a particular unit could result in increasing levels of associate and full professors, as full professors, as those who are left are promoted, with little or no saving in salaries. This type of phenomenon, and others, is represented by the detailed faculty flows.

In using the model, the faculty-flow constraints (3) should be used to eliminate all f_{ij}^t variables. This can be done by substituting the expressions for f_{ij}^t from (3) into (6), (7), (8), (9), and (10). This substitution serves to reduce both the member of variables and the number of constraints.

Since the faculty flow constraints are Markovian in nature, relatively large academic units must be considered for planning purposes to ensure accurate projections. The academic unit can be a department, if something like 50 or more faculty are assigned; it is more likely, however, that the academic unit must be a college of school in order to obtain a large enough group of faculty in a unit. (We use the term academic unit refer to any levels of aggregation that is large enough that Markov flow constraints provide reasonably accurate projections.)

The size of the model as formulated can be quite large. For example, with 20 departments, colleges, or schools, four faculty ranks and five years, the model has 2001 variables (including slacks) and 705 structural constraints (plus 200 upper bound constraints). However the size of the model can be reduced by simplifying assumptions. One possible assumption is that all new faculty members hired are at the assistant professor level which eliminates all the constraints of (10) and reduces the number of constraints from 705 to 305; the number of variables is reduced to 1205. if the model is used in this way, it can easily represent even the largest universities at the college or school level.

4. Analysis

Three different sets of goal structures were used for the model, as shown in the Table II.

Case I used a strict preemptive priority from 1 to 6; in case II, faculty received priority over TAs, but all departments had equal priority; in case III, faculty, TA's and departments all had equal priority. These cases were constructed in order to investigate the sensitivity of the model to goal structures.

A Linear–programming code was used to determine the numbers of new faculty members that should be hired by each department in each year and the desired TA levels. This code was solved once for each of the goal structures.

In Case I, the nature of the optimal solution was easily predicted: it hired faculty to balance shortages against excesses in each department over the three years. It could not meet the faculty goal levels exactly, owing to the faculty-flow constraints, since it was not possible to fire faculty in the model. After faculty goals were satisfied as nearly as possible, the model satisfied.

In case II, the model arrived at the same faculty hiring decisions as in case I. However, the TA's were reallocated, because of all departments now had the same priority on TA's. In Case III, the model changed the TA and faculty decisions completely. All TA goals were met, because TA's were cheap relative to faculty, and TA goals had the same priority as faculty goals. The model satisfied the least expensive goals first in this case. The example illustrated has several features of the model. First, the goal structure does a dramatic effect on hiring decisions, but the resulting decisions can often be explained quite easily in terms of the goals that were specified. Second, it is not always possible to satisfy completely the highest priority goal first, then the second priority goal, and so on down the list until the budget runs out, because of the faculty-flow constraints and the multiyear character of the model. Third, the model handles automatically a fairly complicated set of tradeoffs between goal priorities, unit costs, budget levels, and other model parameters to arrive at optimal faculty and TA allocations. The best allocation is not trial in nature, owing to the interactions within the model itself.

The goal–programming formulation has excellent flexibility to consider alternative goal levels, priorities, and budgets. With parameterization and sensitivity analysis, it is possible to explore a wide variety of assumptions in the resource-allocation process and thus determine a set of pervasive decisions. The model assists our analysis by its ability to handle large amounts of data and by the many tradeoffs that are considered automatically.

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