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Seasonal Constrained Vehicle Routing Problem

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Abstract

There are set of N cities and city 1 is taken as head quarter city. Each city having some raw material in the first season and availability of finished products for the second season. $C(i, j, k)$ mean cost of transporting between i^{th} city to j^{th} city in k^{th} season. $Q(i,1)$ and $Q(i,2)$ are the raw material available and finished products. There is a set of r cities and $M = \{\alpha_1, \alpha_2, ...\alpha_r\} \subset N$. In the set of M, r_0 cities should supply both raw material and availability of finished products. One vehicle having the capacity VA starts from the head quarter city 1 and collects the raw material and returns to the head quarter city or same vehicle with capacity VB starts from the head quarter with finished products and supply to the availability of cities and returns to the head quarter city 1. Both vehicles satisfy r_0 cities of M in their trips. The objective of the problem is to find two schedule i,e for the collection of raw material and supply of the finished products for n_0 cities less than n such that the total cost into two seasons is minimum subjected to the conditions.

Keywords: raw material, finished products, transportation, capacity, availability.

INTRODUCTION

In recent years of organized development, operations research has entered successfully many different areas of research for the military, the government and industry. The main origin of Operations Research was during the Second World- War. At that time, the military management in England called upon a team of scientists to study the strategic and tactical problems related to air and land defense of the country. Since they were having very limited military resources, it was necessary to decide upon the most effective utilization of them, e.g. the efficient ocean transport, effective bombing, etc. Vehicle routing problem is a part of Operations Research.

So many researchers studied in vehicle routing problems. Madhu Mohan Reddy et.al.[8] studied a problem called P-trucks route minimum cost supply to the cities from head quarter city. The objective is to find the minimum total cost of the paths and required capacities are supplied to each city from head quarter city {1} through the P-trucks/P-paths. Each path requirement is not greater than units of capacity.Some of the researchers studied variations in the minimum spanning tree problems. They are Pop [2], Karger [1] found a linear time randomized algorithm based on a combination of Boruvka"s algorithm and the reverse-delete algorithm. The problem can be solved deterministically in linear by Chazelle [5].Suresh Babu et al[7] studied another variation of spanning model. Some of the researchers studied variations in the Minimum Spanning Tree (MST) problems.Pette et al [3,4] have found a probably optimal deterministic comparison-based minimum spanning tree algorithm. SobhanBabu et al [6] studied a variation of spanning models using pattern recognition technique. Let there be N cities to be connected to the headquarter city {1}. There is an individual factor which influences the distances/cost and that factor is represented as a facility K. If the city j_1 and j_2 different cities are connected from city i_1 then k_1 and k_2 should be same.

PROBLEM DESCRIPTION

There are set of N cities. i.e. let $N = \{1,2,3,...n\}$ and city 1 is taken as head quarter city. Each city having some raw material in the first season and availability of finished products for the second season. $C(i, j, k)$ means cost of transporting between i^{th} city to j^{th} city in k^{th} season. $Q(i,1)$ and $Q(i,2)$ are the raw material available and finished products. There is a set of r cities and $M = \{\alpha_1, \alpha_2, ...\alpha_r\} \subset N$. In the set of M, r_0 cities should supply both raw material and availability of finished products. One vehicle having the capacity VA starts from the head quarter city 1 and collects the raw material and returns to the head quarter city another or same vehicle with capacity VB starts from the head quarter with finished products and supply to the availability of cities and returns to the head quarter city 1. Both vehicles satisfy r_0 cities of M in their trips. The objective of the problem is to find two schedule i,e for the collection of raw material and supply of the finished products for n_0 cities less than $n$such that the total cost into two seasons is minimum subjected to the conditions.

MATHEMATICAL FORMULATION:

Minimize Z(X) = $\sum_{i\in N}\sum_{j\in N}\sum_{k\in K} D(i,j,k)X(i,j,k), \quad k=(1,2)$ … … (1)

Let, $\{i,j/x(i,j,1)=1\}=N_1$

$\{i,j/x(i,j,2)=1\}=N_2$

Subject to the constraints

$N_1 \cap N_2 = \{1,\alpha_i\}$ .....(2)

Here $\alpha_i \in M, and, i=1,2,3,.....m<n=r_0$

$|N_1 \cap N_2| = 2 = r_0.$ ...(3)

$\sum_{i\in N_1} Q(i,1) = VA$ ....(4)

$\sum_{i\in N_2} Q(i,2) = VB$ ...(5)

$X(i,j,k) = 0 or 1.$ ...(6)

The equation (1) represents that the objective function of the problem. i.e., to find total minimum cost to n-1 cities from head quarter city. The equation (2) represents that r_0 cities common in $N_1 and N_2$. The equation (3) represents that the number of common cities in our numerical solution is r_0 =2.The equation (4) represents the total capacity of the vehicle in season 1 is equal to the availability of raw material in all cities. The equation (5) represents the total capacity of the vehicle is equal to the total requirement of the finished products in all cities.

The constraint (6) describes a city is connected in network is 1, otherwise it will be equal to 0.

NUMERICAL EXAMPLE:

The concepts and algorithm developed by illustrated a numerical example for which the cities are N={1,2,3,4,5,6,7,8,} and 1 is head quarter city. Take cities 3,4,8 as one cluster i.e, M={3,4,8}. In the following numerical example, D(i, j, k)'s are taken an non- negative integers, it can be easily seen that this is not a necessary condition. In Table - 1, D(6,3,1)=6 means that the cost of the connecting the city 6 to 3 by using season 1 is 6. The Table-3 represents the available raw material and requirement of finished products of the cities. All the cities are to be connected to the head quarter city through 2-paths, (P=2). Then the cost array is given below, where number of seasons is 2,i.e. k=1, 2.

Table 1: C(I,j,2))

-	1	9	8	8	7	6	8
10	-	13	14	1	15	6	16
1	12	-	29	27	28	7	17
8	7	2	-	7	6	12	2
25	30	24	3	-	14	10	12
19	9	**16**	27	9	-	11	3
20	18	17	1	22	13	-	20
2	8	15	14	25	21	12	-

Table 2: C(I,j,1)

-	8	26	7	20	1	15	22
9	-	18	17	6	16	14	23
1	19	-	19	28	14	11	24
10	12	22	-	11	12	13	25
23	26	21	28	-	16	15	26
24	25	29	30	7	-	5	26
27	14	13	12	10	8	-	4
16	15	4	17	11	14	19	-

Table-3

B =

1	2	3	4	5	6	7	8	9
0	0	1	1	0	0	0	1	0

In the above numerical example given in Table – 3, B (i) = 1 represents that the city i is in cluster M. Otherwise, B (i) = 0. Suppose B (3) =1 means city 3 is in cluster M. r_0 = 1.

Table-4

Q =

	1	2	3	4	5	6	7	8
AV	0	20	20	50	60	30	40	20
RQ	0	50	20	40	50	40	30	50

From the above table-4, Q (j) = α means that the requirement of city j is α. Q (1,2) = 20 and 50 means that the availability of the raw material and requirement of finished products for city 2 is 20 and 50.. Here Q (1 ,1)= 0 means that the city 1 has no availability and requirement . Two trucks having the capacity 250 and 300. Our objective is find total minimum cost supply to the 7 cities from head quarter city {1}.

CONCEPTS AND DEFINITIONS

Definition of a pattern

An indicator two-dimensional array which is associated with the spanning is called a 'pattern'. A pattern is said to be feasible if X is a solution.

$$V(X) = \Sigma\Sigma\, D(i, j,k)\, X(i, j,k)$$

The pattern represented in the **Table-5** is a feasible pattern. The value V(X) gives the total cost/distance of the minimum spanning for the solution represented by X.

Table-5

$$\mathbf{X(i, j,1)} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \mathbf{X(i, j,2)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Feasible solution

Consider an ordered triple set C(1,2,1)+ C(2,5,1)+ C(5,8,1)+C(8,4,1)+C(4,1,1)+ C(1,6,2)+ C(6,7,2)+ C(7,8,2)+C(8,3,2)+C(3,1,2) = 1+1+2+4+2+1+2+3+4+1=21 represents the pattern given in the **Table-5**, which is a feasible solution for minimum spanning network connectivity from cities to the head quarter.

The Figure-1 represents the above feasible solution. The circles represent cities and the value in circles indicates the name of the cities also values at each arc represents distance between the respective two nodes.

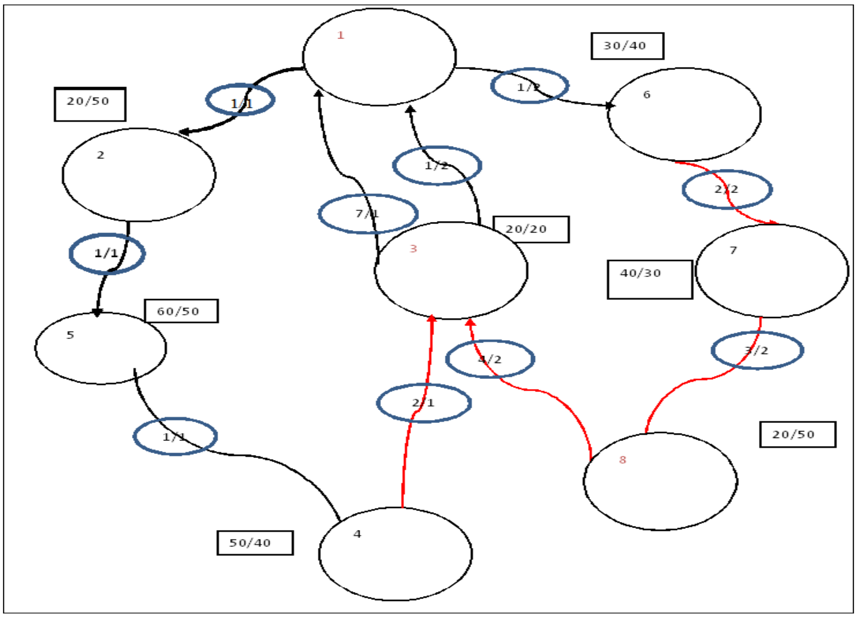

Figure- 1

By using Lexi- Search algorithm we can find the optimal solution. The following solution is called feasible solution. The circles represent cities and the value in circles indicates the name of the cities also values at each arc represents cost between the respective two nodes.

From the above figure, feasible solution = C(1,2,1)+ C(2,5,1)+ C(5,8,1)+C(8,4,1)+ C(1,6,2)+ C(6,7,2)+ C(7,8,2)+C(8,3,2)+C(3,1,2) = 1+1+2+4+2+1+2+3+4+1=23.

Alphabet Table:
The following table is called alphabet table and it is increasing order of the matrix.

S.N.	**C**	**CC**	**R**	**C**	**K**
1	1	1	1	2	1
2	1	2	2	5	1
3	1	3	7	4	1
4	**1**	**4**	**1**	**6**	**2**
5	1	5	3	1	2
6	1	6	3	1	1
7	2	8	8	1	1
8	2	10	4	3	1
9	2	12	4	8	1
10	3	15	5	4	1
11	3	18	6	8	1
12	4	22	7	8	2
13	4	26	8	3	2
14	5	31	6	7	2
15	6	37	1	7	1
16	6	43	4	6	1
17	6	49	2	5	2
18	7	56	1	6	1
19	7	63	3	7	1
20	7	70	4	2	1

Let us consider 4€SN. It represents the ordered pair (R (4), C (4),k(4)) = (1, 6, 2).

Algorithms

Algorithm-1:(algorithm for feasible checking) :

STEP2 : IX=0 GO TO 4

STEP4 : IS[MX(RA)= =1]IF YES GOTO 12

IF NO GOTO6

STEP6 : IS[IR(RA)=1]IF YES GOTO END

IF NO GOTO8

STEP8: IS[MX(CA)=1]IF YES GOTO 18

IF NO GOTO10

STEP10: IS[IC(CA)=1]IF YES GO To End.

IF NO GOTO 22

STEP12: IS[IR(RA)=2]IF YES GO To End.

IF NO GOTO 14

STEP14: IS[MX(CA)=1]IF YES GOTO 20

IF NO GOTO 16

STEP16: IC[IC(CA)=1]IF YESGOTO END
IF NO GOTO 22

STEP18: IC[IC(CA)=2]IF YES GOTO ENDIF NO GOTO 22,

STEP20: IS[IC(CA)=2] IF YES GOTO END,IF NO GOTO 22

STEP22 : IS(KA=1)IF YES GOTO 24,IF NO GOTO 28

STEP24: RC=RAL(i-1)+RP(RA)+FP(CA) GOTO26

STEP26: IS(RC≤RF) IF YES GOTO 30

IF NO GOTO End

STEP28: FC=FPL(i-1+)+FP(RA)+FP(CA)

IS(FC≤PF) IF YES GOTO 30 IF NO GOTO End

STEP30: W=SW(CA)

IS(W=0)IF YES IX=1GOTO ENDIF NO GOTO 32

STEP 32: IS(RA=W) IF YES GOTO ENDIF NO W=SW(W) GOTO 34

STEP34: IS[SW(W)=0] IF YES IX=1 GOTO END

IF NO GOTO 36

STEP36: IS[IR(KA,1)=1] IF YES IX=1 GOTO END

IF NO GOTO END

Algorithm-2: (Lexi - Search algorithm)

STEP A : (Initialization)

The arrays SN, D, DC, R, C and T the value N are made available IR, IK, L, V, LB are initialized to zero. The values I=1, J=0, VT=999, NZ=M*N*K-1 MAX=NZ-1

STEP B: J=J+1

IS (J>MAX) IF YES GOTO 11

IF NO GOTO 3

STEP C: L (I) = J

RA=R (J)

CA=C (J)

KA=T (J) GOTO 4

STEP 1 : V (I) =V (I-1) +D (J)

LB (I) =V (I) +DC (J+N+2)-DC (J) GOTO 5

STEP 2 : IS (LB (I)$\geq$VT) IF YES GOTO 10

IF NO GOTO 6

STEP 3 : CHECK THE FEASIBILITY OF L (USING ALGORITHM-1)

IS (IX=1**)** IF YES GOTO 3A

IF NO GOTO End

STEP 3A : IS (I=N+2) IF YES GOTO 3B

IF NO GOTO 4

STEP 3B : L (I) = J

L (I) IS FULL LENGTH WORD AND IS FEASIBLE.

VT=V (I), RECORD L, VT GOTO 23

STEP4**:** IR(RA)=IR(RA)+1 GOTO 6
IC(CA)=IC(CA)+1, RA,CA€M

STEP6: IS[MX(RA)=1] IF YES GOTO 8

IF NO GOTO 10

STEP8: SW(RA)=CA
TX(RA)=KA

STEP10: SW(KA,RA)=CA
MM(RA)=MM(RA)+1
IS[(KA)=1] IF YES GOTO 12

IF NO GOTO 14

STEP12: RP[I]= RC GOTO 14
STEP14: FP[I]=FC
NXA=NX(I-1)+1
IS(MX(RA)=1) IF YES GOTO 16

IF NO NXA=NXA+1
STEP16: IS(MX(CA)=1) IF YES GOTO END

IF NO NXA=NXA+1

GOTO 18

STEP18: NX(I)=NXA
STEP20: IS(MM(RA)=2) IF YES GOTO 22

IF NO GOTO END

STEP 22: MN^1=MN(I-1)+1

MN(I)= MN^1

GOTO 2

STEP 23 : I=I-1 GO TO 25

STEP24 : J=L (I)

TR = R (J)

TC = T (J)

TK = T (J)

IS (TR, TC) € M IF YES

{IR (TR) =IR(TR)-1

IC (TC) =IC(TC)-1 }

IF NO

{IR (TR) =0

SW (TR) = 0}

GOTO 2

STEP 25 : IS IR(I) = 1 IF YES GOTO 26

IF NO GOTO 23

STEP26 : STOP & END.

By using the alphabet table, Lexi-Search algorithm and search table we find the optimal solution.

Search Table:

SN	1	2	3	4	5	6	7	8	9	10	V	LB	R	C	K	REMARKS
1	1										1	14	1	2	1	A
2		2									2	14	2	5	1	A
3			3								1	14	7	4	1	R>150
4			4								3	14	1	6	2	A
5				5							4	15	3	1	2	A
6					6						5	15	3	1	1	A
7						7					6(2d)		8	1*	1	R
8						8					7		4	3	1	A
9							9				9		4*	8	1	R
10							10				10(3d)		5	4	1	A
11								11			.		6	8	1*	R(k)
12								12			14(4d)		7	8	2	A
13									13		18		8	3	2	A
14										14	23(5d)		6	7	2	A,VT=23
15									14		19	26*	6	7	2	R>VT
16								13			14	26*	8	3	2	R>VT
17							11				10	23*	6	8	1	R=VT
18						9					7	21	4	8	1	A
19							10				10	21	5	4	1	A
20								11			13	21	6	8	1	A
21									1		17	21	7	8	2	R

									2							
22									13		17	22	8	3	2	A
23										14	22	22	6	7	2	22=VT
24									14		18	26*	6	7	2	R>VT
25								12			14	23*	7	8	2	R>VT
26							11				10	23*	6	8	1	R>VT
27						10					8	24*	5	4	1	R>VT
28					7						6	20	8	1	1	A
29						8					8	20	4	3	1*	R
30						9					8	22*	4	8	1	R=VT
31					8						6	22*	4	3	1	R=VT
32				6							4	20	3	1	1	A
33					7						6	20	3	1	2	A
34						8					8	20	4	3	1	A
35							9				10	20	4*	8	1	R
36							10				11	22*	5	4	1	R=VT
37						9					8	22*	4	8	1	R=VT
38					8						8	24*	4	3	1	R>VT
39				7							5	23*	8	1	1	R=VT
40			5								3	20	3	1	2	A
41				6							4	20	3	1	1	A
42					7						6	20	8	1	1	R
43					8						6	22*	4	3	1	R=VT
44				7							5	23*	8	1	1	R>VT
45			6								3	23*	3	1	1	R>VT
46		3									2	17	7	4	1	A
47			4								3	17	1	6	2	A

48				5							4	17	3	1	2	A
49					6						5	17	3	1	1	A
50						7					7	17	8	1	1	R
51						8					7	19	4	3	1	A
52							9				9	19	4*	8	1	R
53							10				10	21	5	4*	1	R
54							11				10	23*	6	8	1	R>VT
55						9					7	21	4	8	1	R>150
56					7						6	20	8	1	1	R>150
57				6							4	21	3	1	1	A
58					7						6	21	8	1*	1	R
59					8						6	22*	4	3	1	A
60				7							5	23*	8	1	1	R>VT
61			5								3	20	3	1	2	A
62				6							4	20	3	1	1	A
63					7						6	20	8	1	1	R
64					8						6	22*	4	3	1	R>VT
65				7							5	23*	8	1	1	R>VT
66			6								3	23*	3	1	1	R>VT
67		4									2	20	1	6	2	A
68			5								3	20	3	1	2	A
69				6							4	20	3	1	1	A
70					7						6	20	8	1*	1	R
71					8						6	22*	4	3	1	R=VT
72				7							5	23*	8	1	1	R>VT
73			6								3	23*	3	1	1	R>VT
74		5									2	23*	3	1	2	R>VT
75	2										1	17	2	5	1	A
76		3									2	17	7	4	1	R>150
77		4									2	18	1	6	2	A

78			5								3	18	3	1	2	A
79				6							4	18	3	1	1	A
80					7						5	18	8	1 *	1	R
81					8						6	22 *	4	3	1	R=VT
82				7							5	23 *	8	1	1	R>VT
83			6								3	23 *	3	1	1	R>VT
84		5									2	23 *	3	1	2	R>VT
85	3										1	20	7	4	1	A
86		4									2	20	1	6	2	A
87			5								3	20	3	1	2	A
88				6							4	20	3	1	1	A
89					7						6	20	8	1	1	A
90						8					8	20	4	3	1 *	R
91						9					8	22 *	4	8	1	R=VT
92					8						6	22 *	4	3	1	R=VT
93				7							5	23 *	8	1	1	R=VT
94			6								3	23 *	3	1	1	R>VT
95		5									2	23 *	3	1	2	R>VT
96	4										1	23 *	1	6	2	R>VT

The all possible ordered pairs from the alphabet table is searched in the search table and it is end in 96th row of the search table. At the end of the search the current value of VT is 22 and it is the value of the optimal feasible word L_{10} = (1, 2, 4, 5, 6, 18,19,20, 22, 23). It is given in the 23rd row of the search table.

Infigure-2, circles represent cities; the values in the circle represent respective cities. Values in the rectangle shape, like numerator represents the availability of the raw material and denominator represents the requirement of the finished products of cities. Values in the rounded rectangle shape, like numerator represents the distance between the corresponding cities and denominator represents the used facility between the corresponding cities. In path-1,the truck starts from head quarter it reaches the head quarter city with capacity of 150 units and come back to the head quarter collects the raw material of quantity 140 units by using facility of 1.In path-2,the same truck starts from head quarter it reaches the head quarter city with capacity of 150 units and come back to the head quarter to supply finished products of quantity 140 units by using facility of 2.In cluster cities, 8 is common to both the paths.The truck capacity is 150.So, it is satisfied all the constraints of the objective function and it is optimal feasible solution.

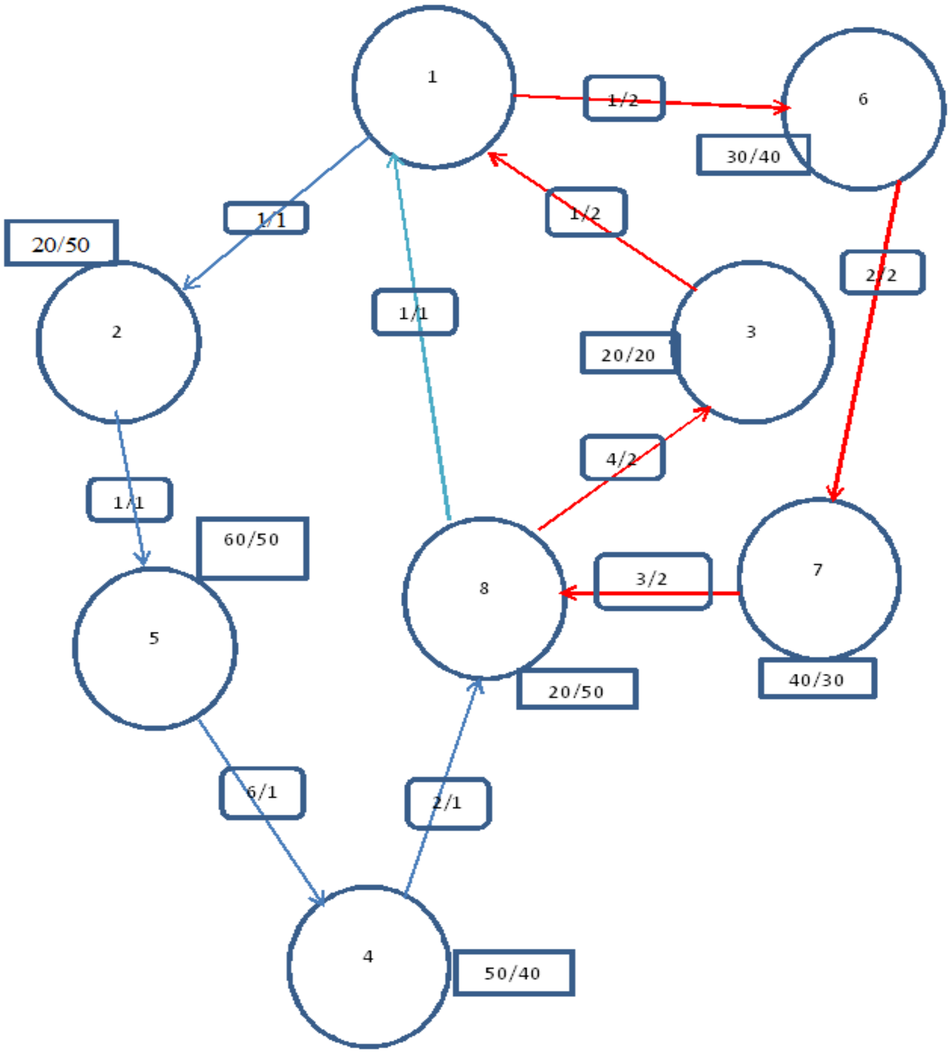

Figure-2

From the above figure, optimal solution = C(1,2,1)+ C(2,5,1)+ C(5,4,1)+C(4,8,1)+C(8,1,1) C(1,6,2)+ C(6,7,2)+ C(7,8,2)+C(8,3,2)+C(3,1,2) = 1+1+2+4+2+1+2+3+4+1=22.

CONCLUSION

In this paper, we have studied a model namely "**Seasonal Constrained Vehicle Routing Problem.** We have developed a newalgorithm which is efficient, accurate and easy to understandthan the algorithm of lexi-search approach using pattern recognition technique. First the model is formulated in to azero one programming problem. The problem is discussed in
detail with help of numerical illustration.

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