Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 12, Number 3 (2016), pp. 2683–2690

http://www.ripublication.com/gjpam.htm

Numerical Integration of Highly Oscillating Functions Using Quadrature Method

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Abstract

In this paper, Numerical evaluation of highly oscillatory integrals of the form

$$\int_0^1 f(x)\sin\left(\frac{\omega}{x^r}\right)$$

and

$$\int_0^1 f(x)\cos\left(\frac{\omega}{x^r}\right)$$

for sufficiently large value of oscillating parameter r are approximated by quadrature method based on Haar wavelets and hybrid functions, to evaluate the typical integrals governed by the proposed method.

AMS subject classification:
Keywords: Numerical integration, Oscillating function, Haar wavelets, Hybrid functions.

1. Introduction

Numerical integration of oscillating functions frequently arise in a large number of engineering applications, in Fourier transform, signal processing and communication, image recognition, fluid dynamics and electrodynamics. Numerical solution of highly oscillatory integrals is challenging task not yields a solution in conventional numerical methods like Simpson, trapezoidal rule, Gauss Legendre quadrature rule, Even if integrals cannot

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be evaluated analytically Highly oscillatory integrals $\int_a^b f(x)e^{i\omega g(x)}dx$ is occurs in wide range of practical problems ranging from nonlinear optics to fluid dynamics, computerized tomography, plasma transport, celestial mechanics, Bose- Einstein condensates, computation of Schrodinger spectra. Evaluation of integrals of oscillating function is difficult when parameter ω is large, In the literature numerical integration of oscillatory integrals using quadrature scheme was first developed by Filon [1], Integral of the form

$$I(x^k) = \int_a^b x^k e^{i\omega g(x)} dx = 0$$

with moment k are efficiently computing numerically based on asymptotic techniques such as steepest descent method, stationary phase method [1]–[4], Events and Chung [5] proposed a method for computing the infinite range irregularly oscillatory integrals, Hascelik [6] evaluating the highly oscillating function numerically by n-point Gauss rule of the three term recurrence relation method. Siraj-ul-Islam, Aziz. and Khan [7]–[9] discuss the numerical study of mild, highly oscillatory and non oscillatory integrals using Haar wavelets and hybrid function. Recently Siraj-ul-islam and Sakhi Zaman [10] evaluate the highly oscillatory integrals with stationary points by modified Levin quadrature method.

In this paper, we evaluate the integral of the form

$$\int_0^1 f(x) \sin\left(\frac{\omega}{x^r}\right)$$

and

$$\int_0^1 f(x) \cos\left(\frac{\omega}{x^r}\right)$$

where $f(x)$ is the smooth function, ω and $r$are positive real number. If the oscillatory parameter r is large value makes the conventional methods ineffective, we first reduce the highly oscillating function into oscillating function by using change of variable method and then evaluated numerically by Haar wavelets and hybrid functions. This method is more accurate and easy to implement for variety of problems arising in different applications. The necessary program has been developed in computer algebra and symbolic computational software MAPLE.

The organized of this paper is as follows. In section 1. The numerical formulas of Haar wavelets and hybrid functions are described for different value of n. In section 3. Numerical evaluation of highly oscillating function using Haar wavelets and hybrid function, Numerical results are reported and compare with Hascelik [6].

2. Numerical Integration using Haar wavelets and hybrid functions

2.1. Numerical Integration formula using Haar functions

Quadrature based on Haar wavelets is given by

$$\int_a^b f(x)dx \approx \frac{(b-a)}{2M}\sum_{i=1}^{2M} f\left(a + \frac{(b-a)(i-0.5)}{2M}\right). \tag{1}$$

We consider $a = 0$ and $b = 1$ in this paper.

2.2. Numerical Integration formula using hybrid functions

Quadrature based on hybrid function is given by:
For $m = 2$

$$\int_a^b f(x)dx \approx \frac{1}{2n}\sum_{i=1}^{n}\left[f\left(\frac{4i-1}{4n}\right) + f\left(\frac{4i-1}{4n}\right)\right] \tag{2}$$

For $m = 3$

$$\int_a^b f(x)dx \approx \frac{1}{8n}\sum_{i=1}^{n}\left[3f\left(\frac{6i-5}{6n}\right) + 2f\left(\frac{6i-3}{6n}\right) + 2f\left(\frac{6i-1}{6n}\right)\right] \tag{3}$$

For $m = 4$

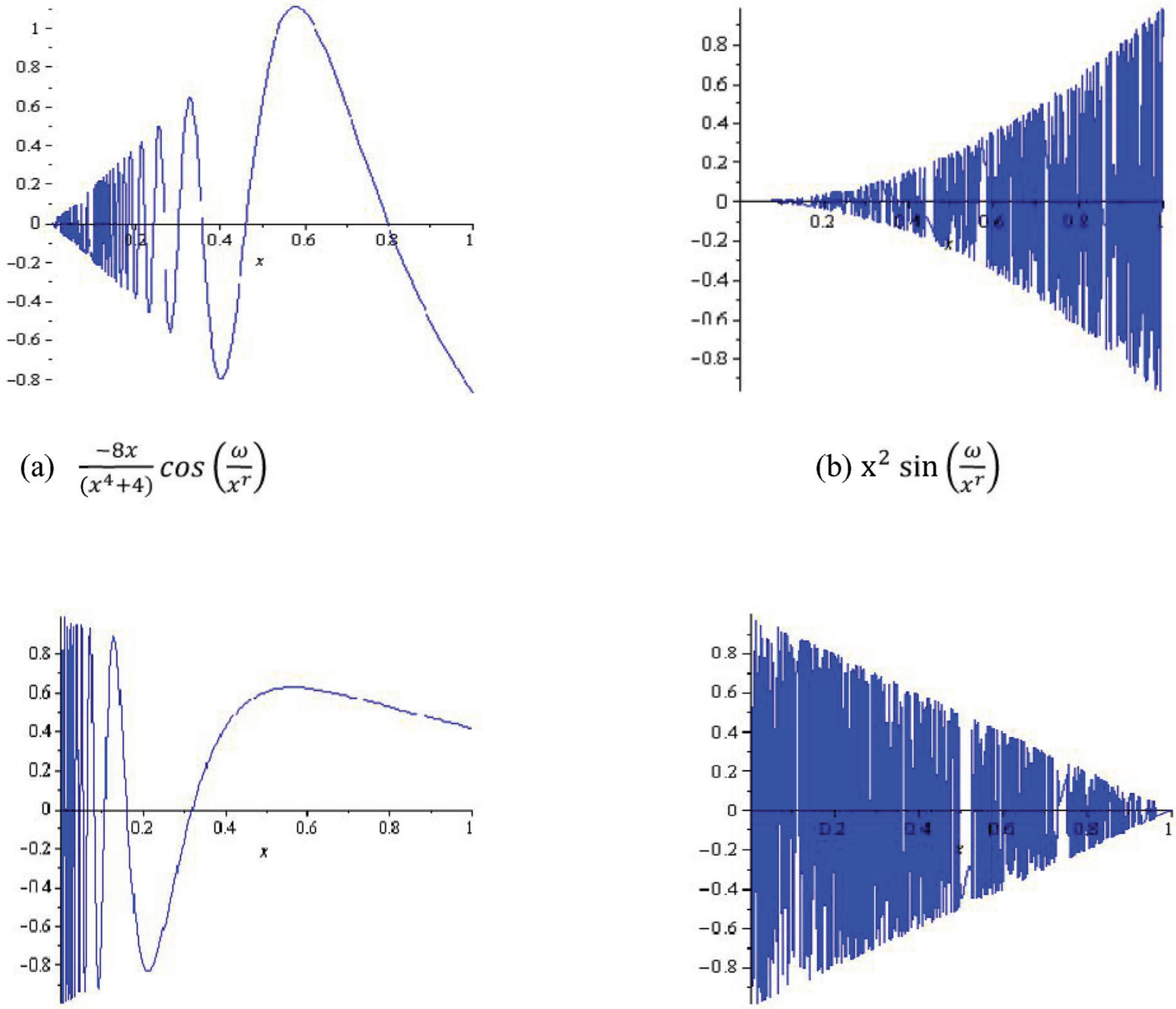
$$\int_a^b f(x)dx \approx \frac{1}{48n}\sum_{i=1}^{n}\left[13f\left(\frac{8i-7}{8n}\right) + 11f\left(\frac{8i-5}{8n}\right) + 11f\left(\frac{8i-3}{8n}\right) + 13f\left(\frac{8i-1}{8n}\right)\right] \tag{4}$$

For $m = 5$

$$\int_a^b f(x)dx \approx \frac{1}{1152n}\sum_{i=1}^{n}\left[275f\left(\frac{10i-9}{10n}\right) + 100f\left(\frac{10i-7}{10n}\right) + 402f\left(\frac{10i-5}{10n}\right) + 100f\left(\frac{10i-3}{10n}\right) + 275f\left(\frac{10i-1}{10n}\right)\right] \tag{5}$$

For $m = 7$

$$\int_a^b f(x)dx \approx \frac{1}{1280n}\sum_{i=1}^{n}\left[247f\left(\frac{12i-11}{12n}\right) + 139f\left(\frac{12i-9}{12n}\right) + 254f\left(\frac{12i-7}{12n}\right) + 254f\left(\frac{12i-5}{12n}\right) + 139f\left(\frac{12i-3}{12n}\right) + 247f\left(\frac{12i-1}{12n}\right)\right] \tag{6}$$



(a) $\frac{-8x}{(x^4+4)} cos\left(\frac{\omega}{x^r}\right)$ (b) $x^2 \sin\left(\frac{\omega}{x^r}\right)$

(c) $\frac{1}{(x+1)} \sin\left(\frac{\omega}{x^r}\right)$ (d) $(x-1)\cos\left(\frac{\omega}{x^r}\right)$

Figure 1: Integrand of oscillating function with (a) $\omega = 1$ and $r = 2$ (b) $\omega = 1$ and $r = 200$ (c) $\omega = 1$ and $r = 1$ (d) $\omega = 2$ and $r = 100$.

2.3. Numerical examples

Example 1.

$$I_1 = \int_0^1 \frac{-8x}{(x^4+4)} \cos\left(\frac{\omega}{x^r}\right) dx$$

If $\omega = 1$ and $r = 2$ then

$$I_1 = \int_0^1 \frac{-8x}{(x^4+4)} \cos\left(\frac{1}{x^2}\right) dx$$

$$= \int_0^1 \frac{-4}{(x^4+4)} \cos\left(\frac{1}{x}\right) dx = 0.094652806418 \text{ (Hascelik [6])}$$

Example 2.

$$
\begin{aligned}
I_2 &= \int_0^1 \frac{10\omega}{(x^2+\omega^2)} \sin\left(\frac{\omega}{x^r}\right) dx \\
&\text{If } \omega = 2 \text{ and } r = 1 \text{ then} \\
I_2 &= \int_0^1 \frac{20}{(x^2+4)} \sin\left(\frac{2}{x}\right) dx = 0.193783272144 \text{ (Hascelik [6])}
\end{aligned}
$$

Example 3.

$$
\begin{aligned}
I_3 &= \int_0^1 x^2 \sin\left(\frac{\omega}{x^r}\right) dx \\
&\text{If } \omega = 1 \text{ and } r = 200 \text{ then} \\
I_3 &= \int_0^1 x^2 \sin\left(\frac{1}{x^{200}}\right) dx \\
&= \int_0^1 \frac{1}{200} x^{\frac{-197}{200}} \sin\left(\frac{1}{x}\right) dx = 0.003117835926
\end{aligned}
$$

Example 4.

$$
\begin{aligned}
I_4 &= \int_0^1 (x-1) \cos\left(\frac{\omega}{x^r}\right) dx \\
&\text{If } \omega = 2 \text{ and } r = 100 \text{ then} \\
I_4 &= \int_0^1 (x-1) \cos\left(\frac{2}{x^{200}}\right) dx \\
&= \int_0^1 \frac{1}{100} \left(x^{\frac{-99}{100}}\right) \left(x^{\dfrac{1}{100}} - 1\right) \cos\left(\frac{2}{x}\right) dx = 0.000006560827
\end{aligned}
$$

Example 5.

$$
\begin{aligned}
I_5 &= \int_0^1 \frac{1}{(x+1)} \sin\left(\frac{\omega}{x^r}\right) dx \\
&\text{If } \omega = 1 \text{ and } r = 1 \text{ then} \\
I_5 &= \int_0^1 \frac{1}{(x+1)} \sin\left(\frac{1}{x}\right) dx = 0.2874906179
\end{aligned}
$$

Example 6.

$$I_6 = \int_0^1 \frac{1}{x}\sin\left(\frac{\omega}{x^r}\right)dx$$

If $\omega = 5$ and $r = 100$ then

$$I_6 = \int_0^1 \frac{1}{x}\sin\left(\frac{5}{x^{100}}\right)dx = \int_0^1 \frac{1}{100}\frac{1}{x}\sin\left(\frac{5}{x}\right)dx = 0.00020865082$$

3. Numerical results

We have compared the Numerical results obtained using the proposed method with that of the exact value of various order of n and m are tabulated in Table 1 and 2.

Table 1: Numerical integration of highly oscillating function using Haar wavelet

Exact value	Haar wavelet		Error
$I_1 = 0.0946528064$	$N = 10$	0.0957008931	1.04808358e-3
$I_2 = 0.1937832721$	$N = 9$	0.1905765120	3.20676014e-3
$I_3 = 0.0031178359$	$N = 25$	0.0038738712	7.56035328e-4
$I_4 = 0.0000065608$	$N = 76$	0.0000194920	1.29312184e-5
$I_5 = 0.28749061795$	$N = 7$	0.2871244423	3.66175607e-4
$I_6 = 0.0002086508$	$N = 28$	0.0004626463	2.53995505e-4

4. Conclusions

In this paper, numerical integration of the form

$$\int_0^1 f(x)\sin\left(\frac{\omega}{x^r}\right)dx$$

and

$$\int_0^1 f(x)\cos\left(\frac{\omega}{x^r}\right)dx$$

are evaluated numerically with different values of Ï‰ and r. We have applied Haar wavelet and Hybrid function to evaluate the typical integration of highly oscillating functions governed by the proposed method. The results obtained are in excellent agreement with the exact values, the present approach can be extended to multi dimensional integrals.

Table 2: Numerical integration of highly oscillating function using Hybrid function

Exact value	Hybrid function		Error
$I_1 = 0.0946528064$	$m = 3, n = 19$	0.0980821124	3.42930602e-3
	$m = 4, n = 8$	0.0947872205	1.34414162e-4
	$m = 5, n = 25$	0.0961576983	1.50489193e-3
	$m = 6, n = 24$	0.0953673868	7.14580462e-4
$I_2 = 0.1937832721$	$m = 3, n = 22$	0.1943815391	5.98266956e-4
	$m = 4, n = 66$	0.1926898097	1.09346244e-3
	$m = 5, n = 89$	0.1912903085	2.49296364e-3
	$m = 6, n = 19$	0.1945508222	7.67550054e-4
$I_3 = 0.0031178359$	$m = 3, n = 21$	0.0035132475	3.95411642e-4
	$m = 4, n = 33$	0.0037681906	6.50354707e-4
	$m = 5, n = 22$	0.0033916911	2.73855209e-4
	$m = 6, n = 16$	0.0035814019	4.63566047e-4
$I_4 = 0.0000065608$	$m = 3, n = 186$	0.0000223410	1.57802388e-5
	$m = 4, n = 39$	0.0000036279	2.93287621e-6
	$m = 5, n = 135$	0.0000025083	4.05248229e-6
	$m = 6, n = 115$	0.0000084564	1.89560307e-6
$I_5 = 0.2874906179$	$m = 3, n = 15$	0.2876215756	1.309577e-4
	$m = 4, n = 15$	0.2870844421	4.061758e-4
	$m = 5, n = 87$	0.2873746599	1.159588e-4
	$m = 6, n = 38$	0.2875781653	8.754741e-5
$I_6 = 0.0002086508$	$m = 3, n = 129$	0.0008807198	6.72069020e-4
	$m = 4, n = 78$	0.0004067063	1.98055499e-4
	$m = 5, n = 211$	0.0005858825	3.77231734e-4
	$m = 6, n = 29$	0.0001767938	3.18569734e-5

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