Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 12, Number 3 (2016), pp. 2635–2650

http://www.ripublication.com/gjpam.htm

Multistage Variational Iteration Method of Temperature Distribution for Annular Fins under Partially Wet-Dry Surface Condition

S. Koonprasert and M. Torvattanabun

Department of Mathematics,
Faculty of Applied Science,
King Mongkut's University of
Technology North Bangkok,
Bangkok 10800, Thailand.

Abstract

This research presents the analytical solutions for the temperature distribution of an annular fin under a partially-wet surface condition. The annular fin is separated into three different regions during the process of dehumidification. The mathematical models for each region are developed based on the conservation of energy principle and the analytical solutions are solved by analytical methods. A Variational Iteration Method and a Multistage Variational Iteration Method for solving analytical solutions by using convergent series are used to obtain the temperature distribution along the fin. The analytical results are validated by comparing with results of symbolic computation in Maple program.

AMS subject classification:
Keywords: Variational Iteration Method, Heat transfer, Air conditioning fin, Multistage Variational Iteration Method.

1. Introduction

Finned tube heat exchangers are widely used in air cooling applications. In real situations for air-conditioning processes, atmospheric air is not completely dry but a mixture of air and water vapor. When the fin surface temperature falls below the dew point temperature of the air being cooled, dehumidification of the air occurs. The boundary conditions for heat exchange in wet-surface heat exchangers are considerably different from the boundary conditions for dry-surface heat exchangers. The problem of the temperature distribution of an annular finned-tube assembly has been studied by many researchers for both wet and dry surface conditions. Coney et al. [1] analyzed the performance of a cooling and dehumidifying vertical rectangular fin. They incorporated thermal resistance into the analysis and solved the resulting equations numerically. Wu and Bong [2] provided analytical solutions for the efficiency of a straight fin under fully wet conditions. They assumed a linear relation between the humidity ratio of the saturated air on the wet surface and the local fin temperature. Kazeminejad [3] numerically analyzed the performance of a cooling and dehumidifying fin assembly with the assumption that the fins were fully wet. However, the results presented showed that the fin surface becomes partially-wet after a certain distance. Salah [4] developed an analytical solution for the performance of rectangular fins in the case that the fin is partially wet. Paisarn [5] obtained the numerical results for the heat transfer characteristics and the temperature distribution of the annular fin. He applied the implicit central finite difference method to determine the temperature distribution along the fin. The purpose of this paper is to develop analytical solutions for the temperature distribution of an annular finned-tube assembly under partially-wet surface condition with the assumption that the humidity ratio of the saturated air on the wet surface varies linearly with the local fin temperature.

The Variational Iteration Method (VIM) was first proposed by Ji-Huan He [6], [7].The idea of the VIM is to construct an iteration method based on a correction functional that includes a generalized Lagrange multiplier. The value of the multiplier is chosen using variational theory, so that each iteration improves the accuracy of the solution. The initial approximation (trial function) usually includes unknown coefficients which can be determined to satisfy any boundary and initial conditions. The VIM has been shown to solve effectively, easily and accurately, a large class of linear or nonlinear problems [12]–[21]. The method gives rapidly convergent successive approximations of the exact solution if such a solution exists, otherwise a few approximations can be used for numerical purposes. In 2007, the Multistage Variational Iteration Method (MVIM) was first introduced by Batiha et al. [21] on a class of nonlinear system of ordinary differential equations. This MVIM offers accurate solutions over a longer time frame (more stable) compared to the VIM. The distinctive strategy grants the iterative algorithm a time-marching scheme which significantly drives forward the convergence of the solutions precisely with great rapidity. Our purpose of this research is to modify and apply the Multistage Variational Iteration Method to solve analytical solutions of the wall tube, wet fin, and dry fin regions which give different models and connected boundary conditions, respectively. We combine all solutions from three regions, so the final solution which gives the power series can determine the temperature distribution along the fin.

2. Temperature distribution model and variational iteration model

2.1. Temperature distribution model

In the below Figure 1, we concentrate on the single annular fin which is attach to the tube wall. Because of the geometrical and thermal symmetry, the heat flows within the wall region A, the wet fin region B, and the dry fin region C. The mathematical models for each region are developed by the conservation of energy principle [4]–[5].

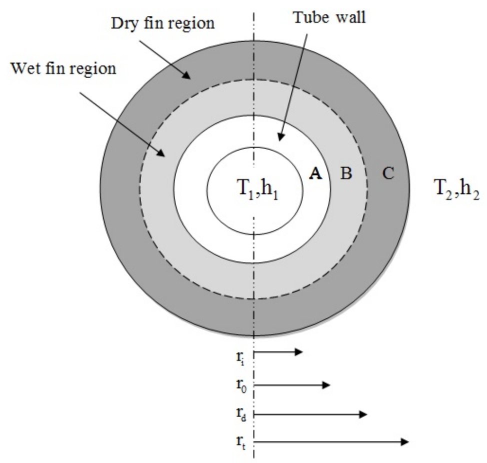

Figure 1: Show three regions of the annular fin.

2.2. Temperature model for the tube wall region

From the heat equation in cylindrical coordinates, thermal conductivity is a constant and under steady-state conditions with no energy generation. The heat flow within the tube wall and the annular fin is assumed to be one dimensional in the radial direction. The temperature distribution of the tube wall region, A, is determined by the differential equation

$$\frac{d^2T_w}{dr^2} + \frac{1}{r}\frac{dT_w}{dr} = 0, \quad r_i < r < r_0, \tag{2.1}$$

where T_w is the temperature within the tube wall region and the tube wall region has an inner radius (r_i), and an outer radius (r_0), then Eq. (2.1) has two boundary conditions. The inner tube wall (r_i) is given by

$$-k_w \frac{dT_w}{dr}\Big|_{r=r_i} = h_1(T_1 - T_w),$$

where T_1 is the cold fluid temperature, k_w is the thermal conductivity of the wall, and h_1 is the heat transfer coefficient at T_1, and interface between the tube wall and the wet fin (r_0) as

$$-k_w p \frac{dT_w}{dr}\Big|_{r=r_0} = -k_f t \frac{dT_{f,w}}{dr}\Big|_{r=r_0} + (p-t)\Big[h_2(T_b - T_2) + h_{fg}h_D(\omega_b - \omega_2)\Big],$$

where $T_{f,w}$ is the temperature within the wet fin region, k_f is the thermal conductivity of the fin, p is the half fin pitch, t is the half fin thickness, h_2 is the heat transfer coefficient at T_2, T_2 is the hot fluid temperature, T_b is the temperature at the base of the fin, h_{fg} is the latent heat of condensation, h_D is the mass transfer coefficient, ω_b is the humidity ratio at the fin base, and ω_2 is the humidity ratio of the moist air.

2.3. Temperature model for annular wet fin region

The difference between the governing equation of the tube wall region and fin region is the term of heat generation. This is because of the condensation of water droplets on the wet surface of the fin region, so the governing equation of the wet fin region, B, is

$$\frac{d^2T_{f,w}}{dr^2} + \frac{1}{r}\frac{dT_{f,w}}{dr} - \frac{1}{k_f t}\Big[h_2(T_{f,w} - T_2) + h_{fg}h_D(\omega_{f,w} - \omega_2)\Big] = 0, \quad r_0 < r < r_d$$

with boundary conditions

$$-k_w p\frac{dT_w}{dr}\Big|_{r=r_o} = -k_f t\frac{dT_{f,w}}{dr}\Big|_{r=r_o} + (p-t)\Big[h_2(T_b - T_2) + h_{fg}h_D(\omega_b - \omega_2)\Big],$$
$$T_{f,w}(r_d) = T_d,$$

where T_d is a dew point temperature.

2.4. Temperature model for annular dry fin region

In dry fin region, the moisture can not condense to the dew point temperature, so the governing equation of the annular dry fin region, C, is given by

$$\frac{d^2T_{f,d}}{dr^2} + \frac{1}{r}\frac{dT_{f,d}}{dr} - \frac{1}{k_f t}h_2(T_{f,w} - T_2) = 0, \quad r_d < r < r_t$$

with boundary conditions

$$T_{f,d}(r_d) = T_b,$$
$$-k_f t\frac{dT_{f,d}}{dr}\Big|_{r=r_t} = h_2(T_{f,d} - T_2).$$

2.5. Variational iteration method

Consider the following differential equation

$$Lu + Nu = g(t)$$

where L is a linear operator, N is a nonlinear operator and $g(t)$ is a known real function. According to VIM [6]–[12], we can construct a correction functional $u(t)$, as follows:

$$u_{n+1}(t) = u_n(t) + \int_0^t \lambda\big(Lu_n(s) + N\tilde{u}_n(s) - g(s)\big)ds. \; n = 1, 2, \ldots \tag{2.2}$$

where $u_n(t)$ is a correction functional but $\tilde{u}_n(t)$ is considered as a restricted variation [9]–[11], i.e. $\delta u_n(t) = 0$. The subscript n denotes the n^{th}-order approximation. The optimal value of the general Lagrange multipliers λ [11] can be identified using the stationary conditions of the variational theory.

2.6. Multistage Variational Iteration Method

For the correction function in (2.2), the constant t^* will change or in fact increase according to the designated time-step in each iteration computation. This methodology was inspired by various researches [21]:

$$u_{n+1}(t) = u_n(t) + \int_{t^*}^{t} \lambda\big(Lu_n(s) + N\tilde{u}_n(s) - g(s)\big)ds. \;\; n = 1, 2, \ldots \tag{2.3}$$

Notice that this strategy gives a new construction of the correction functional in (2.3) with a variable t^* .

3. Multistage variational method for the temperature distribution of annular fin

The annular fin is divided into three segments along the radial direction, so we have three models that are appropriate for each region. These three models are solved by using a Multistage Variational Iteration Method (MVIM) and determine the temperature distribution along the fin.

3.1. MVIM for the tube wall model

We consider the model of the tube wall region in section 2.2

$$\frac{d^2T_w}{dr^2} + \frac{1}{r}\frac{dT_w}{dr} = 0, \quad r_i \le r < r_0 \tag{3.4}$$

with boundary condition at $r = r_i$

$$-k_w \frac{dT_w}{dr}\Big|_{r=r_i} = h_1(T_1 - T_w). \tag{3.5}$$

We use the Multistage Variational Iteration Method to solve the analytical solution, so the correction variational functionals can be expressed by the following:

$$T_{w,n+1}(r) = T_{w,n}(r) + \int_{r_i}^{r} \lambda\Big(\frac{d^2T_{w,n}(s)}{ds^2} + \frac{1}{s}\frac{dT_{w,n}(s)}{ds}\Big)ds. \tag{3.6}$$

We apply the boundary condition (3.5) to determine Lagrange multiplier λ which can be identified as $\lambda = -1$ and substitute this value of Lagrange multiplier into functional (3.6) which gives the iteration formula as

$$T_{w,n+1}(r) = T_{w,n}(r) - \int_{r_i}^{r} \Big(\frac{d^2T_{w,n}(s)}{ds^2} + \frac{1}{s}\frac{dT_{w,n}(s)}{ds}\Big)ds. \tag{3.7}$$

Consequently, MVIM begins with the given initial approximation as $T_{w,0} = a_1 + a_2 lnr$, where $a_1 = T_w(r_i)$. So we substitute into (3.7) to find the following approximation as

$$\begin{aligned} T_{w,1}(r) &= T_{w,0}(r) - \int_{r_i}^{r} \frac{d^2T_{w,0}(s)}{ds^2} + \frac{1}{s}\frac{dT_{w,0}(s)}{ds} ds \\ &= a_1 + a_2 lnr - \int_{r_i}^{r} \Big(\frac{-a_2}{s^2} + \frac{a_2}{s}\Big) ds \\ &= a_1 + a_2 lnr, \end{aligned} \quad (3.8)$$

Further calculations will confirm that $T_{w,1} = T_{w,2} = \cdots = a_1 + a_2 lnr$, i.e. which gives the exact solution. The coefficient a_2 in (3.8) can be evaluated by the specific boundary condition that is given in (3.5).

Thus $a_2 = -\dfrac{r_i h_1(T_1 - a_1)}{k_f}$, so the solution that satisfies the wall tube region model is in the form

$$T_w(r) = T_w(r_i) - \frac{r_i h_1(T_1 - a_1)}{k_f} lnr, \quad r_1 \le r \le r_0 \quad (3.9)$$

3.2. MVIM for the wet fin model

From section (2.3), we consider the wet fin model that is given by

$$\frac{d^2T_{f,w}}{dr^2} + \frac{1}{r}\frac{dT_{f,w}}{dr} - \frac{h_2}{k_f t} T_{f,w} + b = 0, \quad (3.10)$$

where the constant b is given by $b = \dfrac{1}{k_f t}\big[h_2 T_2 - h_{fg} h_D(\omega_{f,w} - \omega_2)\big]$ with boundary condition at $r = r_0$

$$-k_w p \frac{dT_{f,w}}{dr}\Big|_{r=r_o} = -k_f \frac{dT_w}{dr}\Big|_{r=r_o} + (p - t)(h_2(T_b - T_2) + h_{fg} h_D(\omega_b - \omega_2)). \quad (3.11)$$

By using MVIM with Lagrange multiplier $\lambda = -1$, the iteration formula is

$$T_{f,w,n+1}(r) = T_{f,w,n}(r) - \int_{r_o}^{r} \Big(\frac{d^2T_{f,w,n}}{ds^2} + \frac{1}{s}\frac{dT_{f,w,n}}{ds} - \frac{h_2}{k_f t} T_{f,w,n} + b\Big) ds. \quad (3.12)$$

We start with giving the initial approximation as

$$\begin{aligned} T_{f,w,0}(r) &= ba_1 + a_2 lnr_o - \Big(\frac{h_1}{k_w}(T_1 - a_1)) - (p - t)(h_2(T_b - T_2) \\ &\qquad + h_{fg} h_D(\omega_b - \omega_2))\Big)(r - r_0) \\ &= b_1 + b_2 r, \end{aligned} \quad (3.13)$$

where

$$b_1 = ba_1 + a_2 lnr_o + \Big(\frac{h_1}{k_w}(T_1 - a_1)) - (p-t)(h_2(T_b - T_2) + h_{fg}h_D(\omega_b - \omega_2))\Big) r_o$$
$$b_2 = \frac{h_1}{k_w}(T_1 - a_1)) - (p-t)(h_2(T_b - T_2) + h_{fg}h_D(\omega_b - \omega_2)). \tag{3.14}$$

Next, we compute $T_{f,w,1}(r)$ by substituting $T_{f,w,0}$ into (3.12) as the following approximations

$$\begin{aligned} T_{f,w,1}(r) &= T_{f,w,0}(r) - \int_{r_0}^{r} \Big(\frac{d^2T_{f,w,0}}{ds^2} + \frac{1}{s}\frac{dT_{f,w,0}}{ds} - \frac{h_2}{k_f t}T_{f,w,0} + b\Big) ds, \\ &= b_1 + b_2 r - \int_{r_o}^{r} \Big(\frac{b_2}{s} - \frac{h_2}{k_f t}(b_1 + b_2 s) + b\Big) ds \\ &= b_3 + b_4 r + b_5 r^2 + b_6 lnr, \end{aligned} \tag{3.15}$$

where

$$\begin{aligned} b_3 &= b_1 - b_2 lnr_o - \frac{r_o h_2 b_1}{k_f t} - \frac{r_o^2 h_2 b_2}{2k_f t} + r_o b \\ b_4 &= b_2 + \frac{h_2 b_1}{k_f t} - b, \quad b_5 = \frac{h_2 b_2}{2k_f t} \\ b_6 &= -b_2. \end{aligned}$$

Next step, by substituting $T_{f,w,1}(r)$ into (3.12), we can compute the approximation function $T_{f,w,2}(r)$ as

$$\begin{aligned} T_{f,w,2}(r) &= b_3 + b_4 r + b_5 r^2 - b_6 lnr - \int_{r_0}^{r} \Big[2b_5 - \frac{b_6}{s^2} + \frac{1}{s}\big(b_4 + 2b_5 s + \frac{b_6}{s} \\ &\quad - \frac{h_2}{k_f t}\big(b_3 + b_4 s + b_5 s^2 + b_6 ln(s) + b\big)\Big] ds \\ &= b_7 + b_8 r + b_9 r^2 + b_{10} r^3 + b_{11} lnr + b_{12} r lnr \end{aligned} \tag{3.16}$$

where the constants $b_7, b_8, b_9, b_{10}, b_{11}$ and b_{12} are given by

$$\begin{aligned} b_7 &= b_3 + b_4 r + b_5 r^2 - b_6 lnr - 4r_o b_5 + r_o b + b_4 lnr_o - \frac{h_2 r_o}{k_f t} b_3 - \frac{r_o^2 h_2}{2k_f t} b_4 \\ &\quad - \frac{r_o^3 h_2}{3k_f t} b_5 - \frac{r_o h_2 lnr_o}{k_f t} + \frac{r_o h_2}{k_f t} b_6, \\ b_8 &= -4b_5 - b + \frac{h_2 b_3}{k_f t} - \frac{h_2 b_6}{k_f t}, \quad b_9 = \frac{h_2 b_4}{2k_f t}, \quad b_{10} = \frac{h_2 b_5}{3k_f t}, \quad b_{11} = -b_4, \quad b_{12} = \frac{h_2 b_6}{k_f t}. \end{aligned} \tag{3.17}$$

At stage n=3, we substitute $T_{f,w,2}(r)$ into (3.11), so the approximation function, $T_{f,w,3}$ at n=3 is given by

$$\begin{aligned} T_{f,w,3} &= b_7 + b_8 r + b_9 r^2 + b_{10} r^3 + b_{11} lnr + b_{12} rlnr \\ &- \int_{r_o}^{r} \Big[2b_9 + 6b_{10}s - \frac{b_{11}}{s^2} + \frac{b_{12}}{s} + \frac{1}{s}\big(b_8 + 2b_9 s + 3b_{10}s^2 + \frac{b_{11}}{b_{12}} \\ &+ b_{12} lns\big) - \frac{h_2}{k_f t}(b_7 + b_8 s + b_9 s^2 + b_{10}s^3 + b_{11} lns + b_{12} slns) + b \Big] ds \\ &= b_{13} + b_{14} r + b_{15} r^2 + b_{16} r^3 + b_{17} r^4 + b_{18} lnr + b_{19} lnr^2 \\ &+ b_{20} rlnr + b_{21} r^2 lnr \end{aligned} \tag{3.18}$$

where the constants $b_{13}, b_{14}, \ldots, b_{20}$ and b_{21} are given by

$$\begin{aligned} b_{13} &= r_o b - \frac{r_o h_2 lnr_o}{k_f t} b_{11} - \frac{r_o^2 h_2 lnr_o}{2k_f t} b_{12} + 2r_o b_9 + \frac{r_o h_2}{k_f t} b_{11} + \frac{r_o^3}{2} b_{10} - \frac{r_o^2 h_2}{2k_f t} b_8 \\ &- \frac{r_o^3 h_2}{3k_f t} b_9 - \frac{r_o^4 h_2}{4k_f t} b_{10} + b_7 + 2lnr_o b_{12} + lnr_o b_8 + \frac{lnr_o^2}{2} b_{12} - \frac{r_o h_2}{k_f t} b_7 + \frac{r_o^2 h_2}{4k_f t} b_{12}, \\ b_{14} &= b_8 - \frac{h_2 b_{11}}{k_f t} + \frac{h_2 b_7}{k_f t} - b - 4b_9, \;\; b_{15} = \frac{h_2 b_8}{2k_f t} - \frac{h_2 b_{12}}{4k_f t} + b_9 - \frac{9b_{10}}{2}, \\ b_{16} &= b_{10} + \frac{h_2 b_9}{3k_f t}, \;\; b_{17} = \frac{h_2 b_{10}}{4k_f t}, \;\; b_{18} = b_{11} - b_8 - 2b_{12}, \\ b_{19} &= -\frac{b_{12}}{2}, \;\; b_{20} = b_{12} + \frac{h_2 b_{11}}{k_f t}, \;\; b_{21} = \frac{h_2 b_{12}}{2k_f t}. \end{aligned}$$

The same manner is repeated in the next stage and so on. The rest of the components of the iteration formula (3.12) are obtained by MVIM source code in Maple program.

3.3. Variational iteration method for Dry fin model

From section (2.4) we consider the dry fin model as

$$\frac{d^2 T_{f,d}}{dr^2} + \frac{1}{r}\frac{dT_{f,d}}{dr} - \frac{h_2}{k_f t}(T_{f,d} - T_2) = 0, \quad r_d < r < r_t$$

with boundary condition

$$\left.\frac{dT_{f,d}}{dr}\right|_{r=r_t} = -\frac{h_2}{k_f}(T_{f,d} - T_2).$$

It can be simplified as

$$\begin{aligned} \frac{d^2 T_{f,d}}{dr^2} + \frac{1}{r}\frac{dT_{f,d}}{dr} - \frac{h_2}{k_f t} T_{f,d} + c = 0, \\ \left.\frac{dT_{f,d}}{dr}\right|_{r=r_{t,d}} = -\frac{h_2}{k_f} T_f + c, \end{aligned} \tag{3.19}$$

where the constant b is defined by $b = \frac{h_2 T_2}{k_f}$. The correction variational functionals which gives the solution for the dry fin region can be expressed by the following iteration

$$T_{f,d,n+1}(r) = T_{f,d,n}(r) - \int_{r_d}^{r} \left(\frac{d^2 T_{f,d,n}}{ds^2} + \frac{1}{s}\frac{dT_{f,d,n}}{ds} - \frac{h_2}{k_f t} T_{f,d,n} + c \right) ds. \quad (3.20)$$

In MVIM we let the initial approximation that is given by

$$T_{f,d,0}(r) \; = T_{f,w}(r_d) - \frac{h_2}{k_f}(T_f - T_2) r = c_1 + c_2 r,$$

where $c_1 = T_{f,w}(r_d)$ and $c_2 = -\frac{h_2}{k_f}(T_f - T_2)$ and we compute $T_{f,d,1}$ by using (3.20)

$$\begin{aligned} T_{f,d,1}(r) &= T_{f,d,0}(r) - \int_{r_d}^{r} \left(\frac{d^2 T_{f,d,0}}{ds^2} + \frac{1}{s}\frac{dT_{f,d,0}}{ds} - \frac{h_2}{k_f t} T_{f,d,0} + c \right) ds \\ &= c_1 + c_2 r - \int_{r_d}^{r} \left(\frac{c_2}{s} - \frac{h_2}{k_f t}(c_1 + c_2 s) + c \right) ds \\ &= c_3 + c_4 r + c_5 r^2 + c_6 lnr, \end{aligned}$$

where the constants c_3, c_4, c_5 and c_6 are given by

$$\begin{aligned} c_3 &= c_1 - c_2 ln(r_d) - \frac{r_d h_2 c_1}{k_f t} - \frac{r_d^2 h_2 c_2}{2k_f t} + r_d c \\ c_4 &= c_2 + \frac{h_2 c_1}{k_f t} - c, \quad c_5 = \frac{h_2 c_2}{2k_f t}, \quad c_6 = -c_2 \end{aligned}$$

At stage $n = 2$, the $T_{f,d,1}(r)$ becomes the initial approximation of the iteration (3.20), so we can compute the approximation function $T_{f,d,2}(r)$ as

$$\begin{aligned} T_{f,d,2}(r) &= c_3 + c_4 r + c_5 r^2 - c_6 lnr \\ &\quad - \int_{r_d}^{r} \left[2c_5 - \frac{c_6}{s^2} + \frac{1}{s}\left(c_4 + 2c_5 s + \frac{c_8}{s} \right) \right. \\ &\quad \left. - \frac{h_2}{k_f t}\left(c_3 + c_4 s + c_5 s^2 + c_6 ln(s) + c \right) \right] ds \\ &= c_7 + c_8 r + c_9 r^2 + c_{10} r^3 + c_{11} lnr + c_{12} rlnr, \end{aligned} \quad (3.21)$$

where the constants $c_7, c_8, c_9, c_{10}, c_{11}, c_{12}$ are given by

$$\begin{aligned} c_7 &= c_3 + c_4 r + c_5 r^2 - c_6 lnr - 4r_d c_5 + r_d c + c_4 lnr_d \\ &\quad - \frac{h_2 r_d}{k_f t} c_3 - \frac{r_d^2 h_2}{2k_f t} c_4 - \frac{r_d^3 h_2}{3k_f t} c_5 - \frac{r_d h_2 lnr_d}{k_f t} + \frac{r_d h_2}{k_f t} c_6, \\ c_8 &= -4c_5 - c + \frac{h_2 c_3}{k_f t} - \frac{h_2 c_6}{k_f t}, \quad c_9 = \frac{h_2 c_4}{2k_f t}, \\ c_{10} &= \frac{h_2 c_5}{3k_f t}, \quad c_{11} = -c_4, \quad c_{12} = \frac{h_2 c_6}{k_f t}. \end{aligned}$$

we also compute the function $T_{f,d,3}$ at stage $n = 3$ as

$$\begin{aligned} T_{f,d,3} &= c_7 + c_8 r + c_9 r^2 + c_{10} r^3 + c_{11} lnr + c_{12} rlnr \\ &\int_{r_d}^{r} \left[2c_9 + 6c_{10}s - \frac{c_{11}}{s^2} + \frac{c_{12}}{s} + \frac{1}{s}\left(c_8 + 2c_9 s + 3c_{10}s^2 + \frac{c_{11}}{c_{12}} \right.\right. \\ &\left.\left. + c_{12} lns \right) - \frac{h_2}{k_f t}(c_7 + c_8 s + c_9 s^2 + c_{10}s^3 + c_{11} lns + c_{12} slns) + c \right] ds \\ &= c_{13} + c_{14} r + c_{15} r^2 + c_{16} r^3 + c_{17} r^4 + c_{18} lnr + c_{19} lnr^2 + c_{20} rlnr + c_{21} r^2 lnr \end{aligned} \tag{3.22}$$

where the constants $c_{13}, c_{14}, \ldots, c_{20}$ and c_{21} are given by

$$\begin{aligned} c_{13} &= r_d c - \frac{r_d h_2 ln(r_d)}{k_f t} c_{11} - \frac{r_d^2 h_2 ln(r_d)}{2k_f t} c_{12} + 2r_d c_9 + \frac{r_d h_2}{k_f t} c_{11} + \frac{r_d^3}{2} c_{10} \\ &- \frac{r_d^2 h_2}{2k_f t} c_8 - \frac{r_d^3 h_2}{3k_f t} c_9 - \frac{r_d^4 h_2}{4k_f t} c_{10} + c_7 + 2ln(r_d) c_{12} + ln(r_d) c_8 \\ &+ \frac{ln(r_d)^2}{2} c_{12} - \frac{r_d h_2}{k_f t} c_7 + \frac{r_d^2 h_2}{4k_f t} c_{12} \\ c_{14} &= c_8 - \frac{h_2 c_{11}}{k_f t} + \frac{h_2 c_7}{k_f t} - c - 4c_9, \quad c_{15} = \frac{h_2 c_8}{2k_f t} - \frac{h_2 c_{12}}{4k_f t} + c_9 - \frac{9c_{10}}{2} \\ c_{16} &= c_{10} + \frac{h_2 c_9}{3k_f t}, \quad c_{17} = \frac{h_2 c_{10}}{4k_f t}, \quad c_{18} = c_{11} - c_8 - 2c_{12} \\ c_{19} &= -\frac{c_{12}}{2}, \quad c_{20} = c_{12} + \frac{h_2 c_{11}}{k_f t}, \quad c_{21} = \frac{h_2 c_{12}}{2k_f t} \end{aligned}$$

The rest of the components of the iteration formula (3.20) were obtained by computing the symbolic Maple program in the same manner.

4. Analytical Solutions of MVIM and Discussion

In this chapter, we present the analytical solutions for temperature distribution of the annular fin under partially-wet surface condition. Symbolic and analytical solutions of the results in various situations are solved by Multistage Variation Iteration Method that is developed by the full source codes in Maple program. We use some collected information, as shown in Table 1.

4.1. The solution for tube wall region

Applying the MVIM iterative formula and the temperature at the tube wall region $T = 2$ celsius, we give the initial approximation in the form $T_{w,0} = 2 + a_2 lnr$. It obtains the

Table 1: Set of Parameters used to construct the symbolic computations

Parameters	Values
Half fin thickness (t)	0.00125
Half fin pitch (p)	0.005
Wall thermal conductivity (k_w)	237
Fin thermal conductivity (k_f)	237
Heat transfer coefficient at T_1, (h_1)	47400
Heat transfer coefficient at T_2, (h_2)	4740
Inner radius of the wall (r_i)	0.005
Outer radius of the wall (r_o)	0.0075
Fin tip radius (r_t)	0.015

following results:

$$\begin{aligned} T_{w,1}(r) &= 2 + a_2 lnr, \\ T_{w,2}(r) &= 2 + a_2 lnr, \\ &\vdots \end{aligned}$$

Further calculation will confirm that $T_{w,1} = T_{w,2} = \cdots = 2 + a_2 lnr$, so the results are the exact solution. The coefficient a_2 can be evaluated from the specific boundary condition

$$\left.\frac{dT_w}{dr}\right|_{r=r_i} = \frac{a_2}{r_i} = -\frac{h_1(T_1 - T_w)}{k_f}$$

Hence $a_2 = 0.025$, so the MVIM solution of the temperature distribution at tube wall region is in the form

$$T_w(r) = 2 + 0.025 lnr, \quad 0.005 \le r < 0.0075 \tag{4.23}$$

and the graph of MIVM solution is shown as the Figure 2 (Left).

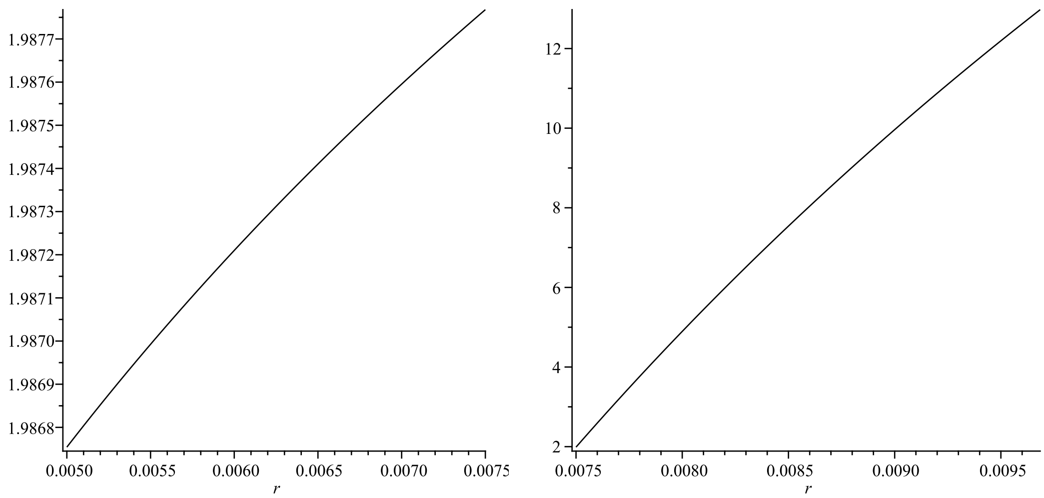

Figure 2: Plot of the temperature solution in tube wall region (Left) and plot of the temperature solution in the wet fin region (Right)

4.2. The solution for wet fin region

We begin with giving the initial approximation $T_{f,w,0} = 1.9955 - 1.0342r$ of MVIM iterative formula, it obtains directly the other components as

$$
\begin{aligned}
T_{f,w,1} &= 7.0835 - 4.8747r - 0.0827r^2 + 1.0340lnr, \\
T_{f,w,2} &= 30.9625 - 7.7360r - 0.4727r^2 + 5.9087lnr \\
&\quad +0.1654rlnr - 0.0044r^3, \\
T_{f,w,3} &= 69.1965 - 5.9966r - 1.0783r^2 + 13.3138lnr + 1.1108rlnr \\
&\quad -0.0296r^3 - 0.0827lnr^2 + 0.0132r^2ln(r) - 0.0002r^4, \\
T_{f,w,4} &= 123.0336 + 3.0715r - 1.4692r^2 + 17.0887lnr + 3.2146rlnr \\
&\quad -0.0864r^3 - 0.6381lnr^2 \\
&\quad +0.1021r^2lnr - 0.0014r^4 - 0.00001r^5 - \frac{0.1654}{r} \\
&\quad -0.0132rlnr^2 + 0.0007r^3lnr.
\end{aligned}
$$

The graph of the MVIM solution $T_{f,w,4}$ for the temperature distribution in the wet fin region is shown in Figure 2 (Right).

4.3. The solution for dry fin region

Lastly we let the initial approximation for MVIM iterative formula as $T_{f,d,0} = 12.1470 - 0.2090r$. The results of MVIM with Maple program obtain the other components as

$$
\begin{aligned}
T_{f,d,1} &= 13.3466 - 23.7932r - 0.1659r^2 + 0.209lnr, \\
T_{f,d,2} &= 123.9031 - 45.1408r - 19.0559r^2 + 24.0022lnr + 0.3319rlnr - 0.0878r^3, \\
T_{f,d,3} &= 333.5752 + 146.8384r - 54.6308r^2 + 68.4793lnr + 38.4436rlnr \\
&- 10.1738r^3 - 0.1659lnr^2 + 0.2635r^2lnr - 0.0349r^4, \\
T_{f,d,4} &= -257.1388 + 742.8949r + 92.469r^2 - 155.2464lnr \\
&\quad +146.6514rlnr - 38.9495r^3 \\
&\quad -19.3878lnr^2 + 30.7848r^2ln(r) - 4.0735r^4 - \frac{0.3319}{r} - 0.0111r^5 \\
&\quad -0.2635rlnr^2 + 0.1395r^3lnr.
\end{aligned}
$$

The graph of the MVIM solution for the temperature distribution in dry fin region $(0.009685 < r < 0.015)$ is shown in Figure 3 (Left).

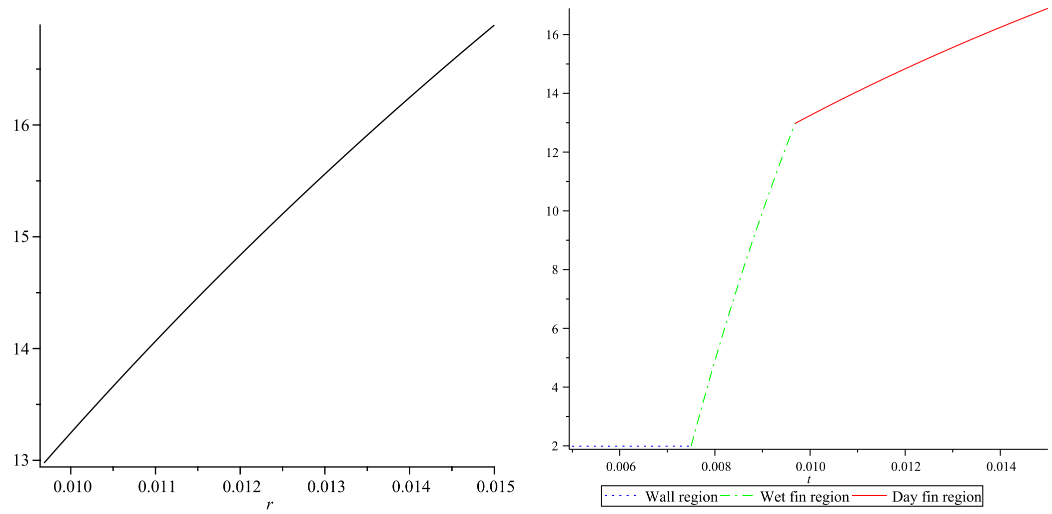

Figure 3: Plot of the solution $(0.009685 < r < 0.015)$ in dry fin region (Left) and Temperature distribution along the fin with $T = 2$ (Right).

From Figure 3 (Right) and Figure 4, the temperature distribution is separated into three different regions. The innermost region is the tube wall region. As a result of pure heat conduction occurring in the region, the temperature almost linearly increases as radius increases. The temperature at the inner tube wall region is about $T = 2$ and $T = 12.5$ celsius. The middle region shown in Figure 3 (Right) and Figure 4 is the wet fin region. The wet fin region is the region where the fin temperature is below the dew point temperature of the air. The results show that the temperature increases with

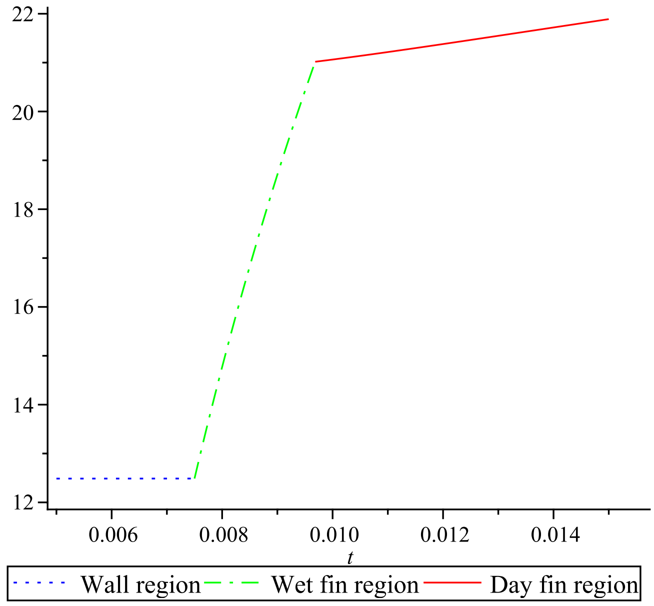

Figure 4: Temperature distribution along the fin with $T = 12.5$

a higher rate than other regions because the latent heat transfer rate has a considerable effect on the total heat transfer rate. The dry fin interface is about 0.009685 m. The fin base temperature is 13.50007 celsius and the dew point temperature is 21.426 celsius. The wet-dry interface is the position where the fin temperature is equal to the dew point temperature of the air. Moistures or water droplets form at this point. The outermost region is the dry region. At this region, the fin temperature is above the dew point temperature of the air and the fin surface is completely dry.

5. Conclusion

According to the symbolic computations, the temperature distribution is separated into three different regions. As a result of pure heat conduction occurring in the region, the temperature almost linearly increases as the radius increases. The wet fin region is the region where the fin temperature is below the dew point temperature of the air. The MVIM computations show that the temperature increases with a higher rate than that of the other two regions. This is due to the condensation of water droplets occurring in the region because the latent heat transfer rate has a considerable effect on the total heat transfer rate. The wet-dry interface is the position where the fin temperature is equal to the dew point temperature of the air. Moisture begins to condense at this position. At this region, the fin temperature is above the dew point temperature of the air and the fin surface is completely dry. The symbolic computations also show that the width of the wet fin region increases as the relative humidity increases.

Acknowledgement

This research was supported by the Science and Technology Research Institute of King Mongkut's University of Technology North Bangkok for financial fund.

References

[1] J.E.R. Coney, H. Kazeminejad and C.G.W. Sheppard. Dehumidification of air on a vertical rectangular fin: a numerical study. Proc. Instn. Mech. Engrs, Part C 203, (1989): 165–175.

[2] Wu. G. and Bong, T.-Y., Overall efficiency of a straight fin with combined heat and mass transfer. ASHRAE Transactions, Part I, (1994): 367–374.

[3] H. Kazeminejad, Analysis of one dimensional fin assembly heat transfer with dehumidification, Int. J. Heat Mass Transfer 38, (1995): 455–462.

[4] M.M. Salah El-Din, Performance analysis of partially-wet fin assembly, Appl. Therm. Eng. 18, (1998): 337–349.

[5] Paisarn Naphon, Study on the heat transfer characteristics of the annular fin under dry-surface,pertially wet-surface, and fully wet-surface conditions, Int. J. Communications in Heat Mass Transfer, 33, (2006): 112–121.

[6] J. H. He, Variational iteration method kind of non-linear analytical technique: Some examples, Int. J. Non-Linear Mechanic, 34(1999) 699–708.

[7] J. H. He, Variational iteration method for delay differential equations, Commun. Nonlinear Sci. Numer. Simul, 2 (1997), 235–236.

[8] J.H. He, Variational iteration method for autonomous ordinary differential systems, Appl. Math. Comput, 114 (2000) 115–123.

[9] J.H. He, Y.Q. Wan and Q. Guo, An iteration formulation for normalized diode characteristics, Int. J. Circuit Theory Appl. 32(2004) 629–632.

[10] J.H. He, Variational iteration method-Some recent results and new interpretations, J. Comput. Appl. Math, (in press).

[11] M. Inokuti, H. Sekine and T. Mura, General use of the Lagrange multiplier in nonlinear mathematical physics, in: S. Nemat-Nassed (Ed.), Variational Method in the Mechanics of Solids, (Pergamon Press, 1978, pp. 156–162).

[12] A. Wazwaz, A study on linear and nonlinear Schrodinger equations by the variational iteration method, Chaos, Solitons and Fractals, 37 (2008) 1136–1142.

[13] M. Dehghan and F. Shakeri, Application of He's variational iteration method for solving the Cauchy reaction-diffusion problem, Comput. Appl. Math, 214 (2008) 435–446.

[14] A. S. V. Ravi Kanth and K. Arana, He's variational iteration method for treating nonlinear singular boundary value problems, Comput. Appl. Math, 60 (2010) 821–829.

[15] G. Rezazadeh, H. Madinei and R. Shabani, Study of parametric oscillation of an eletrostatically actuated microbeam using variational iteration method, Appl. Math. Model, 36 (2012) 430–443.

[16] G. Yang and R. Chen, Choice of an optimal initial solution for a wave equation in the variational iteration method, Comput. Appl. Math, 61 (2011) 2053–2057.

[17] M. Torvattanabun and S. Duangpithak, Numerical Simulations of Fokker-Plank equation by variational iteration method, Int. J. Math. Analysis, 5 (2011) 2193–2201.

[18] M. Torvattanabun and S. Koonprasert, Convergence of the Variational Iteration Method for Solving a First-Order Linear System of PDEs with Constant Coefficients, Thai Journal of Mathematics, Special Issue Annual Meeting in Mathematics, (2009) 1–13.

[19] M. Torvattanabun and S. Koonprasert, Variational Iteration Method for Solving Eighth-Order Boundary Value Problems, Thai Journal of Mathematics, Special Issue Annual Meeting in Mathematics, (2010) 121–129.

[20] S. Duangpithak and M. Torvattanabun, Variational Iteration Method for Solving Nonlinear Reaction-Diffusion-Convection Problems, Appl. Math. Sci., 6 (2012), 843–849.

[21] B. Batlha, M.S.M. Noorani, I. Hashim, E.S. Ismail, The multistage variational iteration method for a class of nonlinear system of ODEs, Phys. Scripta 76 (2007) 1–5.