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Optimization of Wavelet Neural Networks Model by Setting the Weighted Value of Output Through Fuzzy Rules Takagi-Sugeno-Kang (TSK) Type As a Fixed Parameter

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Abstract

In this paper, we propose a model of fuzzy wavelet neural network (FWNN) which is optimized from wavelet neural network (WNN) model by setting the weighting coefficient of the output using the TSK fuzzy inference type. This coefficient in the proposed FWNN model is seen as exogenous parameters that are not updated in the learning process using the method of gradient descent with momentum. The accuracy and the execution time of the model was illustrated using several univariate time series data cases, and the results were compared with models of WNN that had been previously published. The simulation results for variety of these cases show that the effectiveness and the accuracy of proposed FWNN model is better than the previous model of WNN.

AMS subject classification: 41-04, 41A15, 41A30, 65-04, 65T60, 65Y10, 92B20.
Keywords: Wavelet neural network, fuzzy wavelet, wavelet B-spline, fuzzy inference, TSK fuzzy rules, time series.

1. Introduction

The application of time series analysis is usually used to solve real problems relating to the cases of smoothings, modelings, forecastings and controls ([13]; [7]). Prediction of time series is one field of application and research which is much in demand. Model and result predicted from time series are widely used in various fields such as economy and finance, demographics, geophysics and meteorology, medicine, and various industrial processes.

The development of time series analysis model is in line with the development of science and technology. Initially, method of time series analysis only stands on the statistical methods, and some models that have been produced and commonly used among other methods are Box-Jenkins, autoregressive and moving average (ARMA) methods, methods of autoregressive conditional heteroscedastic (ARCH) and generalized autoregressive conditional heteroscedastic (GARCH) method. However, at the level of application, all of these models have limitations which require the data to be stationary and linear. On the other hand, the data on the observation of a research is generally nonlinear and non stationary (heteroscedastic).

Since the decade of 1990, some researchers have begun to develop a method based on soft computing to solve problems of nonlinear and non stationary time series. Soft computing techniques was originally popularized by L.A. Zadeh in 1994 in his paper entitled “Soft Computing and Fuzzy Logic”. Soft computing technique itself is a collection of some concepts and methods such as fuzzy systems, neural networks (NN), genetic algorithms and hybrid. Some of these methods, either partially or combination, have been widely used and successfully solve the problems of time series data including complex data [11].

Wavelet and NN are two methods based on soft computing which partially have their respective advantages. NN has adaptive capabilities and learns algorithm by itself, generalize ability and resolve complex and cumbersome nonlinear problems, while the wavelet is a basic of functions and has advantages in the process of denoising, data compression and multiresolution. The combination of these two methods, namely NN and wavelet analysis (wavelet transformation), is a relatively new idea in science and technology. Based on these advantages, many research problems can be solved by using WNN, including system modeling, analysis of system reliability, recognition of patterns, and some applications in engineering. Application of WNN in a variety of disciplines includes problem analysis of time series nonlinear [12], application of WNN for image processing [**?**], interpolation wavelet network [5], application of WNN to control disc drives [6], application of WNN in the study of dynamic analysis [14], control design for nonlinear systems [15], and model of WNN based on wavelet B-spline and its application to predict traffic travelers and cement sales volumes [3].

On the other hand, as one of soft computing techniques, fuzzy technique is an effective tool to resolve the phenomena that are unclear, incorrect, or too complex, to be analyzed by conventional mathematics [8]. The combination of fuzzy systems and neural networks as a method of soft computing and wavelet analysis (wavelet transformation) is a relatively new idea in science and technology which has begun since 2005. The combination of these three methods is known as wavelet fuzzy neural network (FWNN). FWNN method is a combination of three methods of mutual support in which the advantages of each method is to be the superiority of FWNN and the weaknesses of the method is partially covered by the advantages of other methods. In line with this, Wang, Hong and Jun (2006) found that the combination of FWNN is able to enhance the ability of these three tools, together as one method in terms of accuracy, generalization ability, speed of convergence, and multiresolution.

In this paper, the optimization is to optimize the numerically determined by the value of approximation error (means square error, MSE) and execution time. The smaller the error value obtained and the faster execution time of an algorithm, then the proposed model more optimal.

2. TSK Fuzzy Inference

Method of TSK fuzzy is inference suitable to a model of nonlinear systems with large number of data [9]. TSK fuzzy system is given in IF-THEN, the rules are as follows [10]:

$$R_k : IF\ x_1 \in A_{k1} \wedge x_2 \in A_{k2} \wedge x_3 \in A_{k3} \wedge \cdots \wedge x_m \in A_{km}\ THEN\ z_k = a_{k0} + \sum_{j=0}^{m} a_{2j} x_j \tag{2.1}$$

where R_k represents the k-th fuzzy inference rules. If the fire level is given by

$$\alpha_i = A_{i1}(x_{0i}) \wedge A_{i2}(x_{0i}) \wedge \cdots \wedge A_{im}(x_{0i}), i = 1, 2, \ldots, k$$

then the output of the individual is provided by

$$Z_i^* = a_{i1}(x_{0i}) + a_{i2}(x_{0i}) + \cdots + a_{im}(x_{0i}),\ i = 0, 1, 2, \ldots, k$$

and the overall output of the system is given by

$$Z = \frac{\sum_{i=1}^{k} \alpha_i Z_i^*}{\sum_{i=1}^{k} \alpha_i} \tag{2.2}$$

3. Model of the Wavelet Neural Networks (WNN)

Model of WNN to be discussed here is the model WNN plus which is based on wavelets B-spline proposed by Bahri et al. (2016). This model is a model of WNN coupled with the stages of pre-processing of data in the model architecture as shown in Figure 1. This expansion was developed based on preliminary research of investigators that the implementation of pre-processing stages data on NN models can increase the efficiency (execution time) and effectiveness/accuracy of the model based on MSE indicators and coefficient regression of the model [2].

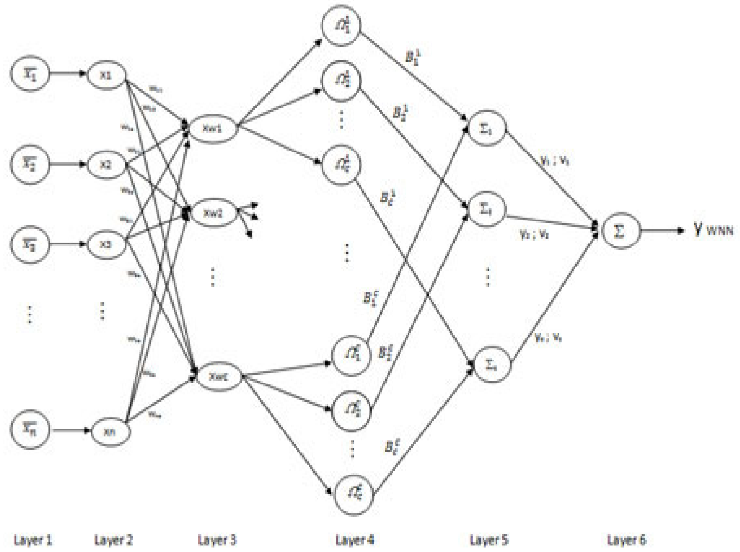

Figure 1: Architecture of WNN Plus based on wavelet B-Spline, Bahri et al. (2016)

Architecture of the WNN Plus in Figure 1 consists of 6 layers with the stages of pre-processing data using a discrete wavelet transform Daubechies type 8-th order 3-rd level with WNN models used are

$$Y_{WNN} = \frac{\sum_{j=1}^{c} v_j y_j}{\sum_{j=1}^{c} v_j} + \beta \tag{3.3}$$

for a constant β and

$$y_j = \alpha \sum_{k}^{c} \psi_k^j (Xw_{jk}), \text{ for some } \alpha \in \mathbb{R} \tag{3.4}$$

$$Xw_k = \sum_{j}^{n} w_{jk} x_j;\ Xw_{jk} = \frac{Xw_j - b_k}{a_k}, \text{ for some } k = 1, 2, \ldots, c, \text{ and } j = 1, 2, \ldots, n \tag{3.5}$$

$$\psi_j^i(z) = \frac{4b^{i+1}}{\sqrt{\left(2(i+1)\,\sigma_w^2\right)}} \cos\left(2f_0(2z-1)\right) \exp\left(\frac{-(2z-1)^2}{2\sigma_w^2(i+1)}\right) \tag{3.6}$$

which $\psi_j^i(z)$ represents j-th variation of the i-th order of wavelets B-spline, $i, j = 1, 2, \ldots, c$ in Equation (3.6) is a numerical model of the wavelets B-spline generated by Unser (1997).

4. Model of Proposed Fuzzy Wavelet Neural Network (FWNN)

Model of FWNN proposed in this research is the development of a model of WNN with an architecture that is given in Figure 1.

4.1. Architecture Model of Fuzzy Wavelet Neural Network(FWNN)

The architectural design of FWNN is the development of a WNN model architecture in Figure 1. The development is situated on the determination of the coefficient of the output value y_k for some k-th of the class classification in which the coefficients of WNN models are delivered and updated on a learning process using gradient descent algorithm with momentum. On the contrary, the coefficient of the output y_k is determined by using the fuzzy inference type TSK and not updated through by a learning process. Architecture of the FWNN model developed in this study is shown in Figure 2.

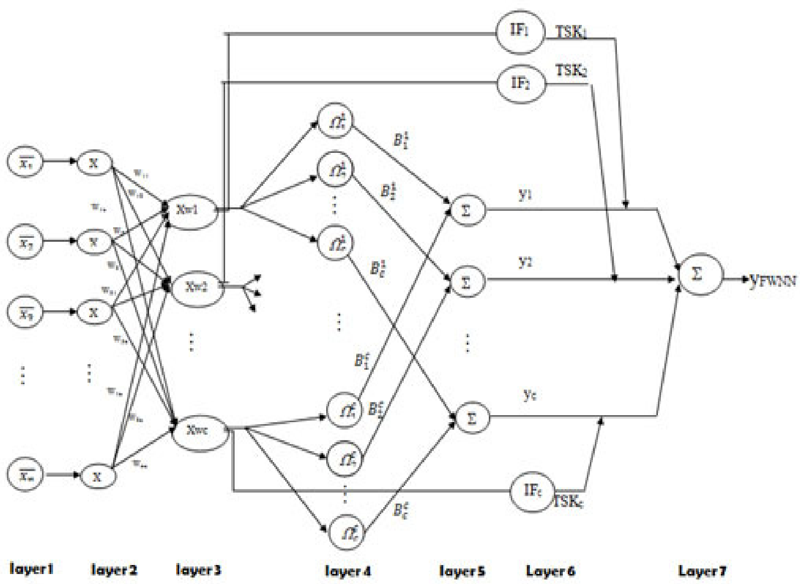

Figure 2: Architecture of proposed FWNN

FWNN feed-forward architecture proposed in Figure 2 consists of seven layers. The first layer is the data input time series and the second layer is the result of the discrete wavelet transform Daubechies Db8 type 3-rd level of the data that has been normalized. In the third layer, any data on the second layer is weighted by matrix W sized $n \times c$ using the following formula

$$Xw_k = \sum_{j}^{n} w_{jk} x_j; \;\; k = 1, 2, \ldots, c \tag{4.7}$$

On layer fourth, any data on the third layer is activated using wavelet discrete transformation based on the B-spline of 1-st order to c-th order and as much as c variations of wavelet B-spline based on variations in the parameters of translation and dilatation, in which $\psi_j^i(\bar{x}_i)$, $i, j = 1, 2, \ldots, c$ states variation j-th of the i-th order of wavelet B-spline.

$$\psi_j^i(z) = \frac{4b^{i+1}}{\sqrt{\left(2(i+1)\,\sigma_w^2\right)}} \cos\left(2f_0(2z-1)\right) \exp\left(\frac{-(2z-1)^2}{2\sigma_w^2(i+1)}\right) \tag{4.8}$$

In the fifth layer, the aggregation of every order of wavelets B-spline uses the following equation.

$$y_j = \alpha_j \sum_k^c \psi_k^j(Xw_{jk}) \tag{4.9}$$

where α_j is a constant, $Xw_{jk}) = \dfrac{Xw_j - b_k}{a_k}$ states k-th variation from Xw_j, $j = 1, 2, \ldots, c$ and parameter a_k and b_k are consecutive declaration of the k-th variation parameter dilations and translations of wavelet B-Spline. The amount of translation and dilatation parameter variation equal to the amount of class data classification is determined by using a Fuzzy c-Means (FCM) clustering algorithm.

Then, the process of determining the value of the coefficient of the output y_k for some k-th class classification is determined at the sixth layer. This process begins with the fuzzification of input using Gaussian membership functions. Gaussian membership function is given by Equation (4.10).

$$\mu_{ij}(\bar{x}_i) = \exp\left(\frac{(\bar{x}_i - c_{ij})^2}{2\,\sigma_{ij}^2}\right), \text{ for some } i = 1, 2, \ldots, n \text{ and } j = 1, 2, \ldots, m \tag{4.10}$$

with n and m denote the number of input and clusters, $\bar{x}_i$ represents data input, and c_{ij} and σ_{ij} successively declare of the center and width of the Gaussian membership function. Parameters c_{ij} and σ_{ij} are determined using the method of FCM.

The process of fuzzy inference using Takagi-Sugeno-Kang (TSK) type applies the following rules. Define the set as a fuzzy set with membership function as follows:

$$IF \;\; \bar{x}_i \in A_{ij} \;\; THEN \;\; \mu_{ij}(\bar{x}_i) = \exp\left(\frac{(\bar{x}_i - c_{ij})^2}{2\,\sigma_{ij}^2}\right) \tag{4.11}$$

where c_{ij} and σ_{ij} successively declare the center and width of the Gaussian membership function.

$$R^k : IF\, \bar{x}_1 \in A_{k1} \wedge \bar{x}_2 \in A_{k2} \wedge \bar{x}_3 \in A_{k3} \wedge \cdots \wedge \bar{x}_n \in A_{kn} \;\; THEN \;\; TSK_k = \underline{\mu}_k\, L_k(z) \tag{4.12}$$

Vector $\{\bar{x}_1, \bar{x}_2, \cdots, \bar{x}_n$ expresses the input variable results of discrete wavelet transform Daubechies, $L_j(z) = \dfrac{1}{1+\exp z}$ is a logsig function (default of MATLAB), $z_{ij} =$

$\dfrac{Xw_j - b_{ij}}{a_{ij}}$ with a_{ij} and b_{ij} states dilation and translation parameters, and

$$Xw_k = \sum_{k=1}^{m} w_{ik}\, x_{ik},$$

w_{ik} represents the weight of i-th input related to the k-th rule. Consequent of each rule R^k for each $k = 1, 2, \ldots, m$ is given by the following equation

$$TSK_k = \underline{\mu}_k\, L_k(z_{ik}) \tag{4.13}$$

where $\underline{\mu}_k = \min\{\mu_{ik} \,:\, i = 1, 2, \ldots, n \;\text{ and}\, j = 1, 2, \ldots, m\}$.

Finally, the seventh layer is a weighted aggregation of the results of the fifth layer with weight given by Equation (4.13) is

$$y_{FWNN} = \frac{\sum_{j=1}^{c} v_j\, y_j}{\sum_{j=1}^{c} v_j} + \beta \tag{4.14}$$

for a constant β and $v_j = TSK_j$ for each $j = 1, 2, \ldots, c$.

4.2. Learning Parameters of FWNN Model

Proposed FWNN model in the training uses a type of supervised training. Training FWNN is done to minimize the cost function

$$E = \frac{1}{2}\left(y_i - y_i^t\right)^2 \tag{4.15}$$

where y_i is the output of the FWNN model and y_i^t states output targets. By using the gradient descent algorithm with momentum, the weight of the dilation and translation parameters will be updated using Equation (4.16)–(4.18)

$$w_{jk}(t+1) = w_{ij}(t) + \eta_w \frac{\partial E}{\partial w_{jk}} \tag{4.16}$$

$$a_j(t+1) = {}_j(t) + \eta_a \frac{\partial E}{\partial_j} \tag{4.17}$$

$$b_j(t+1) = b_j(t) + \eta_b \frac{\partial E}{\partial a_j} \tag{4.18}$$

where η_w, η_a, and η_b is successive states of learning rate for weighting parameter w_{ij}, dilation and translation parameters of wavelets B-spline are determined based of the Banakar Theorem [4].

Values of the partial derivatives in Equation (4.16)–(4.18) is given by the following equations

$$\frac{\partial E}{\partial w_{jk}} = \frac{\partial E}{\partial y_{FWNN}} \frac{\partial y_{FWNN}}{\partial w_{jk}} = \frac{\partial E}{\partial y_{FWNN}} \frac{\partial}{\partial w_{jk}} \left(\frac{\sum_{k=1}^{c} v_k\, y_k}{\sum_{k=1}^{c} v_k} + \beta \right) \tag{4.19}$$

$$\frac{\partial E}{\partial a_k} = \frac{\partial E}{\partial y_{FWNN}} \frac{\partial y_{FWNN}}{\partial a_k} = \frac{\partial E}{\partial y_{FWNN}} \frac{\partial}{\partial a_k} \left(\frac{\sum_{k=1}^{c} v_k y_k}{\sum_{k=1}^{c} v_k} + \beta \right) \tag{4.20}$$

$$\frac{\partial E}{\partial b_k} = \frac{\partial E}{\partial y_{FWNN}} \frac{\partial y_{FWNN}}{\partial b_k} = \frac{\partial E}{\partial y_{FWNN}} \frac{\partial}{\partial b_k} \left(\frac{\sum_{k=1}^{c} v_k y_k}{\sum_{k=1}^{c} v_k} + \beta \right) \tag{4.21}$$

5. Empirical Results

Empirical studies related to the applications of developed FWNN models will be simulated using time series data. Time series data used includes the following four cases: (1) the data volume of tourist visits to the NTB, (2) the volume of cement sales of PT. Semen Gresik Tbk., (3) dynamic system data quoted Banakar et al. (2006), and (4) the Box Jenkins time series data constructed of delay differential equations Mackey-Glass [4].

For case 1, the data of tourist visits to NTB in the of period January 2007 to December 2013 can be seen in Figure 5.1a. Data $x(t)$ is predicted by using a data sequence $x(t-18)$, $x(t-16)$, $x(t-14)$, $x(t-12)$, $x(t-9)$, $x(t-6)$, $x(t-4)$, $x(t-3)$, $x(t-2)$, and $x(t-1)$. From this structure, there are 66 pairs of data that is organized into 50 data for the training data and 16 data for testing data. For case 2, Figure 5.1b shows the data on cement sales of the PT. Semen Gresik Tbk. period of January 2006 to March 2015. The data $x(t)$ predicted by using a data sequence $x(t-13)$, $x(t-12)$, $x(t-9)$, $x(t-6)$, $x(t-4)$, $x(t-3)$, $x(t-2)$, and $x(t-1)$. From this structure there are 98 pairs of data that is organized into 75 data for training data and 23 data for testing data.

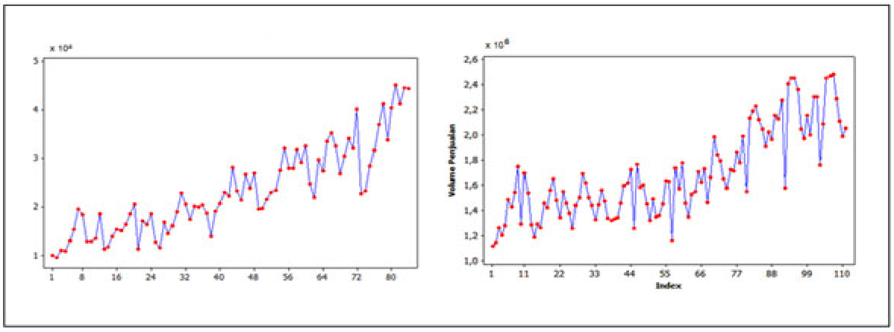

Figure 3: (a) Data of tourist visits to NTB (left) and (b) data of cement sales of PT. Semen Gresik Tbk. (right)

Table 1 shows a comparison between various parameters that play a role in computer simulations for cases 1 and 2. Figure 4 shows a comparison of actual data with the output generated by the WNN model (left) and FWNN model (right) for each, case 1 at the upper and case 2 at the bottom.

For case 3, the data used is data dynamical system constructed/generated by the

Table 1: Comparison of parameters between WNN model and FWNN Model associated of the case 1 and Case 2.

Parameters	Case 1		Case 2	
	WNN Model	FWNN	WNN Model	FWNN
Number of Class Clasification	7	7	7	7
Fixed Parameters				
value of	6	5	6	5
center and stdev of Gauss-MF	-	140	-	112
total of fixed parameter	**6**	**145**	**6**	**121**
Parameters are updated				
the weight matrix W	70	70	56	56
the weight matrix V	7	-	7	-
parameter of dilatation, a	7	7	7	7
parameter of translation, b	7	7	7	7
total of parameter in the update	**91**	**84**	**77**	**70**
Number of Epoch	1200	1200	500	500
Execution Time (second)	866.359	843.099	1000.547	654.400
Performance of Model	**0.024481**	**0.015098**	**0.027106**	**0.019873**

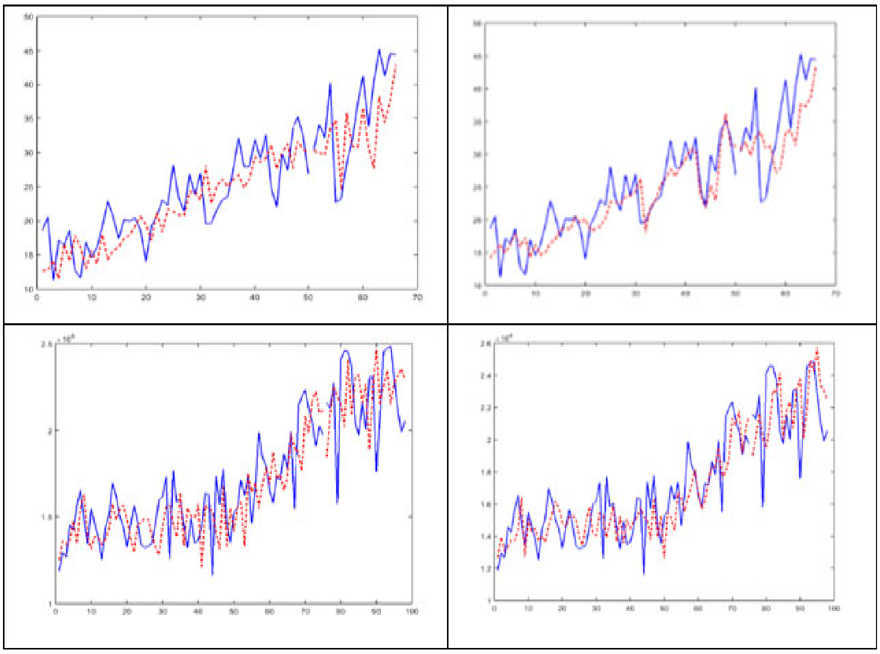

Figure 4: Comparison between the actual data (solid blue line) with the output generated by the model (red dotted line), the left column for WNN model and the right column for FWNN Model

equation

$$x(n+1) = \frac{5\,n(n)}{1+x(n)^2} - 0.5x(n) - 0.5x(n-1) + 0.5x(n-2) \tag{5.22}$$

with the initial value,$x(0) = 0.2$,$x(1) = 0.3$ and $x(2) = 1$ [4]. Time series data collected 120 data which are organized into 90 data used as training data and 30 data as the testing data.

The results of computer simulations of 750 epoch show a comparison between the actual data with the model output for each WNN model (left) and FWNN model (right) as given in Figure 5.

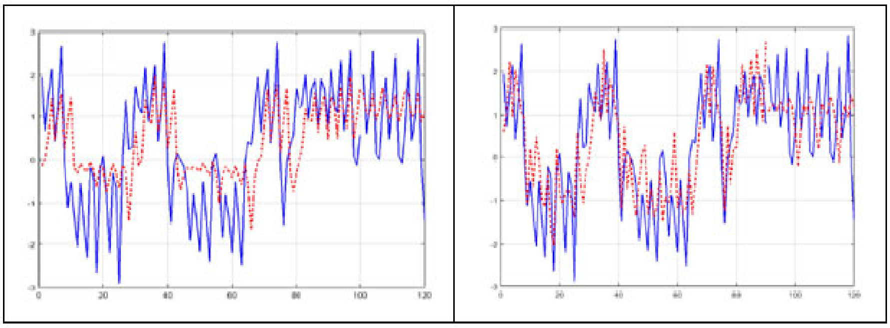

Figure 5: Comparison between the actual data (solid blue line) with the output generated by the model (red dotted line), the left column to WNN model and the right column for Model FWNN.

For case 4, the data used is data constructed/generated by delay differential equations Mackey-Glass [4] given by the following equation.

$$\dot{x}(n+1) = \frac{0.2\,x(t-\tau)}{1+x^1 0(t-\tau)} - 0.1x(t) \tag{5.23}$$

with the initial value $x(0) = 1.2$, $\tau = 17$ and $x(t) = 0$, for $t < 0$ and this data is also available in MATLAB with the file name: mgdata.dat. In this case, the data $x(t-18)$, $x(t-12)$, $x(t-6)$, and $x(t)$ are used as input and the data $x(t+6)$ as output. The amount of data used for the computer simulation is 400 data that is organized into 150 data is used as training data and 250 data for testing data.

The results of computer simulations of 200 epoch show a comparison between the actual data with the model output for each WNN model (left) and FWNN model (right). It is shown in Figure 6.

Table 2 shows disclosing comparison of various parameters that plays a role in computer simulations related to special cases 3 and 4, and a summary comparison of the accuracy of the model (MSE) and execution time for each case between WNN model and FWNN model are given in Table 3.

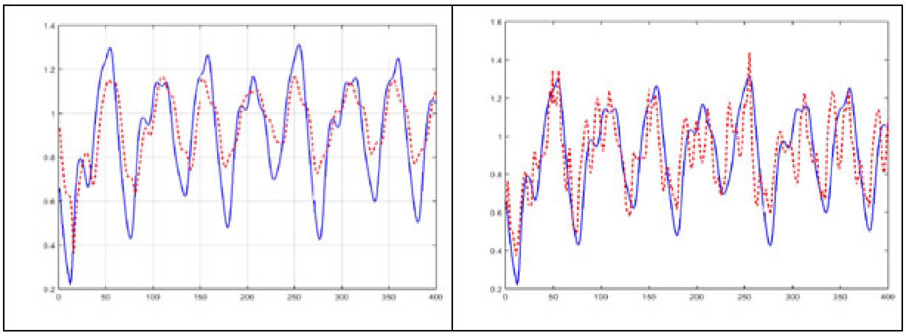

Figure 6: Comparison between the actual data (solid blue line) with the output generated by the model (red dotted line), the left column to WNN model and the right column for Model FWNN.

Table 2: Comparison between the model parameters of WNN and FWNN associated with case 3 and case 4

Parameters	Case 3		Case 4	
	Model WNN	Model	Model WNN	Model
Number of Class Clasification	7	7	12	12
Fixed Parameters				
value of	6	5	6	5
center and stdev of Gauss-MF	-	112	-	96
total of fixed parameter	**6**	**112**	**6**	**101**
Parameters are updated				
the weight matrix W	56	56	48	48
the weight matrix V	7	-	12	-
parameter of dilatation, a	7	7	12	12
parameter of translation, b	7	7	12	12
total of parameter in the update	**77**	**70**	**84**	**72**
Number of Epoch	750	750	200	200
Execution Time (second)	2911.527	2368.844	1900.205	1705.076
Performance of Model	**0.039099**	**0.033201**	**0.024474**	**0.016423**

Based on Figure 4, 5 and 6, it is shown visually that the effectiveness of FWNN model is better than WNN model. A number of parameters that are updated by the model in the learning process are shown in Table 1 and 2. The WNN model has more updated parameters than the FWNN model. Summary of accuracy or performance of the model and execution time are given in Table 3, where for each simulated case, the accuracy and the execution time of FWNN model is seen better than the WNN model. It is indicated by the value of MSE FWNN model which is smaller than the value of MSE WNN model, and the execution time of FWNN model which is faster than model of WNN.

Table 3: Accuracy model (MSE) and execution time (second) for each case simulated

	WNN Model		FWNN Model	
	MSE	Execution Time	MSE	Execution Time
case 1	0.021904	866.359	0.015098	843.099
case 2	0.027106	1000.547	0.019873	654.400
case 3	0.039099	2911.527	0.033201	2368.844
case 4	0.024474	1900.205	0.016423	1705.076

6. Conclusion

FWNN model developed in this study is a modified by WNN plus model [3], especially on the determination of the weighted value coefficient of output using TSK fuzzy inference type and the coefficient is not updated in the learning process of FWNN. Computer simulation results for each case simulated visually show that the effectiveness of the FWNN model is better than WNN model. These results were confirmed by a number of parameters that will be updated by the model in the learning process where the number of parameters FWNN model was less than the WNN model, which shown by the execution time of FWNN model is faster than WNN model. Based on the accuracy or performance model (MSE) each simulated case also shows that the FWNN model is better than WNN model. Therefore, in general it can be concluded that the FWNN model is a form of optimization of the WNN model.

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