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Degenerate and non degenerate parametric converter in interaction with a single two-level atom

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Abstract

In this paper, we study the interaction between a two-level atom and a quantized three type of the field interaction, two as a degenerate two photon and the third represented the converter form. The solution to the Schrodinger equation is obtained exactly. By using this solution we discuss numerically the atomic inversion, the degree of entanglement through the linear entropy, the variance entropy for chosen values of the detuning and coupling parameters.

AMS subject classification:
Keywords:

1. Introduction

Quantum entanglement plays an basic role in many quantum information and quantum computation tasks. Also many of quantum systems poses quantum correlation that need to be characterized and realized for use in different applications. One of these very important quantum systems is Jaynes-Cumming model (JCM) [1] in quantum optics that describes the interaction between a single-mode quantized field and a single two-level atom. One advantage of JCM is that it is exactly solvable model and we can investigate its entanglement properties analytically. One important application of JCM is in quantum computing for realization of quantum registers, namely cavity and circuit quantum electrodynamics. It is well known that the Quantum entanglement is being seen as a resource for communication, information processing and quantum computing [2], for example in the investigation of quantum teleportation [3, 4], dense coding [5], decoherence in quantum computers and the evaluation of quantum cryptographic schemes [6]. Several authors [7–10] have proposed physically motivated postulates based on the concept of the operational formulation of quantum mechanics [11] to characterize entanglement. However, a first step towards the realization of quantum computation has recently been achieved by creating deterministic entanglement of two ions. It has been shown that trapped and laser-irradiated ions can be modeled by a strongly nonlinear multiquantum JCM [12]. For example, the quantized center of mass motion of the ion in this case is coupled via the laser to the internal (electronic) degree of freedom. This in fact plays a role similar to that played by the cavity light field mode in the conventional JCM. The degree of quantum control that can be achieved in a coupled system of internal and vibrational degrees of freedom was demonstrated in experiments on the generation of nonclassical states of motion of a single trapped ion. However, several ions with well-controlled interaction between them have to be considered in order to realize extended quantum registers. Thus we may conclude that there is a strong link connecting both the JCM and the trapped ion(s) with quantum information. In a previous communication we introduced a modified model of the JCM Hamiltonian. This model represented the interaction between a two-level atom and two electromagnetic fields injected simultaneously within a perfect cavity [13, 14]; the (atomic) field entropy was studied. That treatment concentrated on the isotropic case (where the field frequencies are equal) but ignored the effect of the interaction between the fields themselves. However, in the present work we generalize the Hamiltonian model to include in addition to the JCM the interaction between the two fields in the parametric frequency converter form. Here we should point out that the parametric frequency converter model as it stands is described by a process of exchanging photons between two optical fields of different frequencies. The model can be applied to describe various optical phenomena, e.g. to find analogies between a frequency converter and a beam splitter [15], a two-level atom driven by a single mode of an electromagnetic field [16], Raman scattering [15, 17], etc. The aim of the present work is to investigate the degree of entanglement for the above-mentioned system which will be described by the Hamiltonian given in the next section. The organization of this paper is as follows: in section 2 we introduce our Hamiltonian model and give an exact expression for the wave function $|\psi(t)>$ using the Schrodinger picture. In section 3 we use the analytical results obtained in section 2 to investigate the properties of the atomic inversion. In section 4 we consider the degree of entanglement due to the linear entropy. In section 5 we discuss the variance squeezing. In section 6 we discuss the entropy squeezing. Finally, we summarize the results in section 7.

2. System Hamiltonian and it solution

The time-dependent Hamiltonian which represents such a system is given by

$$\begin{aligned}\frac{\hat{H}}{\hbar} &= \omega_a \hat{a}^\dagger \hat{a} + \omega_b \hat{b}^\dagger \hat{b} + \left(\frac{\omega_0}{2}\right)\hat{\sigma}_z \\ &+ \left(\hat{\sigma}_- - \sigma_+\right)\left[\lambda_1\left(\hat{a}^{\dagger 2} e^{-2i\omega_a t} - \hat{a}^2 e^{2i\omega_a t}\right)\right. \\ &\left.+ \lambda_2\left(\hat{b}^{\dagger 2} e^{-2i\omega_b t} - \hat{b}^2 e^{2i\omega_b t}\right) + \lambda_3\left(\hat{a}^\dagger \hat{b} e^{-i\omega t} - \hat{a}\hat{b}^\dagger e^{i\omega t}\right)\right] \end{aligned} \tag{1}$$

where ω_a and ω_b are the field frequencies, while ω_0 is the atomic frequency. $\lambda_i, i = 1, 2, 3$ are the coupling parameters between the atom and both of degenerate and non-degenerate parametric amplifier, while $\omega = \omega_a - \omega_b$. The operators $\hat{\sigma}_+(\hat{\sigma}_-)$ and $\hat{\sigma}_z$ are obeying the commutation relations

$$[\hat{\sigma}_z, \hat{\sigma}_\pm] = \pm 2\hat{\sigma}_\pm, \qquad [\hat{\sigma}_+, \hat{\sigma}_-] = \hat{\sigma}_z, \tag{2}$$

and the boson operators $\hat{a}$ and $\hat{b}$ satisfy the well known commutation relations

$$\left[\hat{a}, \hat{a}^\dagger\right] = 1, \qquad \left[\hat{b}, \hat{b}^\dagger\right] = 1, \qquad \left[\hat{a}, \hat{b}\right] = 0. \tag{3}$$

In order to the handle the problem we introduce the operators $\hat{A} = \hat{a}\exp(i\omega_a t)$ and $\hat{B} = \hat{b}\exp(i\omega_b t)$ which enable us to rewrite the Hamiltonian in the form

$$\begin{aligned}\frac{\hat{H}}{\hbar} &= \left(\frac{\omega_0}{2}\right)\hat{\sigma}_z + \left(\hat{\sigma}_- - \sigma_+\right)\left[\lambda_1\left(\hat{A}^{\dagger 2} - \hat{A}^2\right)\right. \\ &\left.+ \lambda_2\left(B^{\dagger 2} - \hat{B}^2\right) + \lambda_3\left(\hat{A}^\dagger \hat{B} - \hat{A}\hat{B}^\dagger\right)\right]. \end{aligned} \tag{4}$$

Furthermore, if we define the operators

$$\hat{A} = \hat{u}\cosh\phi + \hat{v}^\dagger \sinh\phi, \qquad \hat{B} = \hat{u}^\dagger \sinh\phi + \hat{v}\cosh\phi, \tag{5}$$

where $\hat{u}$ and $\hat{v}$ have the properties

$$\left[\hat{u}, \hat{u}^\dagger\right] = 1 = \left[\hat{v}, \hat{v}^\dagger\right], \qquad \left[\hat{u}, \hat{v}\right] = 0. \tag{6}$$

Provided we consider ϕ is the rotating angle given by

$$\phi = \frac{1}{2}\tanh^{-1}\left(\frac{\lambda_3}{\lambda_1 + \lambda_2}\right), \tag{7}$$

and define

$$\begin{aligned}\mu_1 &= \lambda_1 \cosh^2\phi + \lambda_2 \sinh^2\phi - \frac{\lambda_3}{2}\sinh 2\phi, \\ \mu_2 &= \lambda_1 \sinh^2\phi + \lambda_2 \cosh^2\phi - \frac{\lambda_3}{2}\sinh 2\phi \end{aligned} \tag{8}$$

then the Hamiltonian (4) takes the form

$$\frac{\hat{H}}{\hbar} = \left(\frac{\omega_0}{2}\right)\hat{\sigma}_z + \mu_1\left(\hat{\sigma}_-\hat{u}^{\dagger 2} + \sigma_+\hat{u}^2\right) + \mu_2\left(\hat{\sigma}_-\hat{v}^{\dagger 2} + \sigma_+\hat{v}^2\right), \tag{9}$$

where we have applied the rotating wave approximation. As one can see it is difficult task to deal with the above equation, however, if we consider the case in which $\lambda_3 = 2\sqrt{\lambda_1\lambda_2}$, then $\mu_1 \to 0$ and consequently $\mu_2 = \lambda_1 + \lambda_2$. In this case the Hamiltonian (9) becomes

$$\frac{\hat{H}}{\hbar} = \left(\frac{\omega_0}{2}\right)\hat{\sigma}_z + \mu_2\left(\hat{\sigma}_-\hat{v}^{\dagger 2} + \sigma_+\hat{v}^2\right). \tag{10}$$

The time evolution operator $U(t)$ is given by

$$\begin{aligned} U(t) &= \exp[-i\hat{H}t] \\ &= \begin{bmatrix} \hat{F}_{11} & \hat{F}_{12} \\ \hat{F}_{21} & \hat{F}_{22} \end{bmatrix} \end{aligned} \tag{11}$$

with

$$\begin{aligned} \hat{F}_{11} &= \cos\hat{g}_1 t - \frac{i\omega_0}{2\hat{g}_1}\sin\hat{g}_1 t\ ,\ \hat{F}_{12} = -i\mu_2\frac{\sin\hat{g}_1 t}{g_1}\hat{v}^2 \\ \hat{F}_{21} &= -i\mu_2\hat{v}^{\dagger 2}\frac{\sin\hat{g}_1 t}{\hat{g}_1},\quad \hat{F}_{22} = \cos\hat{g}_2 t + \frac{i\omega_0}{2\hat{g}_2}\sin\hat{g}_2 t \\ \hat{g}_j^2 &= \frac{\omega_0^2}{4} + \nu_j, \quad j = 1, 2 \\ \hat{\nu}_1 &= \mu_2^2\hat{v}^2\hat{v}^{\dagger 2}, \\ \hat{\nu}_2 &= \mu_2^2\hat{v}^{\dagger 2}\hat{v}^2 \end{aligned} \tag{12}$$

Now let us consider the state $|\theta, \phi\rangle$ which acquires both excited state $|e\rangle$ and ground state $|g\rangle$ for the two-level atoms in the following form

$$|\theta, \phi\rangle = \cos\left(\frac{\theta}{2}\right)|e\rangle + \sin\left(\frac{\theta}{2}\right)\exp(-i\phi)|g\rangle, \tag{13}$$

where ϕ is the relative phase of the two atomic levels. To reach the excited state we have to take $\theta \to 0$ while to make the wave function describes the particle in the ground state we have to let $\theta \to \pi$.

Noted that g_j are the generalized Rabi frequencies in this case which depend on

$$\hat{v}^\dagger\hat{v} = B^\dagger B\ \cosh^2\phi - AA^\dagger\sinh^2\phi - (AB + A^\dagger B^\dagger)\sinh\phi\ \cosh\phi \tag{14}$$

Therefor consider the pair operators relation $AB + A^\dagger B^\dagger - (B^\dagger B + AA^\dagger)$ for the two modes, and the difference of the photon number operators $(A^{\dagger}A - B^{\dagger}B\ )$ for the two modes, namely:

$$\begin{aligned} [AB + A^{\dagger}B^{\dagger} - (B^{\dagger}B + AA^{\dagger})]|\xi, q\rangle &= \xi|\xi, q\rangle, \\ (A^{\dagger}A - B^{\dagger}B\ )|\xi, q\rangle &= q|\xi, q\rangle, \end{aligned} \tag{15}$$

where the parameter ξ is a complex variable while the parameter q is an integer. Therefore the entangled states can be expressed in the number state representation of the two modes $|n_a\rangle$ and $|n_b\rangle$as follows

$$|\xi, q\rangle = \sum_{n=0}^{q} C_n|q + n, n\rangle. \tag{16}$$

Now when we introduce in eqn (15) the state given by equation (14) then the recurrence relation among the coefficients C_n is obtained in the form

$$\sqrt{(n+1)(q+n+1)}C_{n+1} + \sqrt{n\ (q+n)}C_{n-1} - (q+2n+1)C_n = \xi C_n, \tag{17}$$

by using the transformation

$$C_{n+k} = \left(\frac{(q+n)!}{(q+n+k)!}\frac{(n+k)!}{n!}\right)^{\frac{1}{2}} \sqrt{\frac{(q+n)!}{n!}} S_{n+k} \tag{18}$$

then the Recurrence relation become to

$$(n+1)S_{n+1} + (q+n)\ S_{n-1} = (q+2n+1+\xi)S_n \tag{19}$$

which is recurrence relation for the Laguerre Polynomials, the explicit form for the eigenstate is

Therefore the entangled state $|\xi, q\rangle$ takes the form

$$|\xi, q\rangle = N_q \sum_{n=0}^{\infty} \sqrt{\frac{n!}{(q+n)!}} L_n^q\,(-\xi)\ |q+n, n\rangle, \tag{20}$$

while N_q is the normalization constant given by

$$N_q = \left[\sum_{n=0}^{\infty} \left|\sqrt{\frac{n!}{(q+n)!}} L_n^q\,(-\xi)\right|^2\right]^{\frac{-1}{2}} \tag{21}$$

So

$$\begin{aligned} \hat{v}^{\dagger}\hat{v}|\xi, q\rangle &= L(n)|q+n, n\rangle \\ L(n) &= N_q \sum_{n=0}^{\infty} \sqrt{\frac{n!}{(q+n)!}} L_n^q\,(-\xi)\,[n\ \exp(-\phi)\cosh\phi \\ &\quad -(q+n+1)\ \exp(\phi)\sinh\phi - \xi] \end{aligned} \tag{22}$$

The wave function for any dynamic system at any time $t > 0$ can be evaluate through the form $|\psi(t) >= \hat{U}(t)|\psi(0)>$, we can write the wave function in the form

$$\begin{aligned} |\psi(t)\rangle &= \sum_{n=0}^{\infty}\left[\hat{D}(t)|q+n,n,e\rangle + \hat{T}(t)|q+n,n,g\rangle\right], \\ &= |\psi_e>|e> + |\psi_g>|g> \end{aligned} \tag{23}$$

where

$$\begin{aligned} \hat{D}(t) &= N_q\sqrt{\frac{n!}{(q+n)!}}L_n^q(-\xi)\left[\hat{F}_{11}\cos\frac{\theta}{2} + \hat{F}_{12}\sin\frac{\theta}{2}\exp(-i\phi)\right] \\ \hat{T}(t) &= N_q\sqrt{\frac{n!}{(q+n)!}}L_n^q(-\xi)\left[\hat{F}_{21}\cos\frac{\theta}{2} + \hat{F}_{22}\sin\frac{\theta}{2}\exp(-i\phi)\right] \end{aligned} \tag{24}$$

Now after obtained the explicit form of the final state of the system $|\psi(t)\rangle$, we are therefore in a position to discuss the statistical properties of the system. Now let us turn our attention to the time evolution of the operators $\hat{S}_x(t)$, $\hat{S}_y(t)$ and $\hat{S}_z(t)$.

$$\begin{aligned} S_z(t) &= <\psi_e|\psi_e> - <\psi_g|\psi_g> \\ S_x(t) &= <\psi_e|\psi_g> + <\psi_g|\psi_e> \\ S_y(t) &= -i<\psi_e|\psi_g> - <\psi_g|\psi_e> \end{aligned} \tag{25}$$

$$\begin{aligned} |\psi_e> &= N_q\sum_{n=0}^{\infty}\left\{\begin{array}{c} \sqrt{\frac{n!}{(q+n)!}}L_n^q(-\xi)\cos\frac{\theta}{2}\left(\cos(g_1t) - \frac{i\,\omega_0}{2}\frac{\sin(g_1t)}{g_1}\right) \\ -i\sqrt{\frac{(n+2)!}{(q+n+2)!}}L_{n+2}^q(-\xi)\,\mu_2\sin\frac{\theta}{2}e^{-i\phi}\frac{\sin(g_1t)}{g_1}\sqrt{(L(n)+2)(L(n)+1)} \end{array}\right\}|q+n,n> \\ |\psi_g> &= N_q\sum_{n=0}^{\infty}\left\{\begin{array}{c} -i\sqrt{\frac{(n-2)!}{(q+n-2)!}}L_{n-2}^q(-\xi)\,\mu_2\frac{\sin(g_1t)}{g_1}\sqrt{L(n)(L(n)-1)} \\ +\sqrt{\frac{(n)!}{(q+n)!}}L_n^q(-\xi)\,e^{-i\phi}\sin\frac{\theta}{2}(\cos(g_2t) + \frac{i\,\omega_0}{2}\frac{\sin(g_2t)}{g_2}) \end{array}\right\}|q+n,n> \end{aligned} \tag{26}$$

Using the above results we are able to discuss some statistical properties of the present system. This is seen in the forthcoming sections where we start with the atomic inversion.

3. Atomic inversion

In this section we discuss the revival–collapse phenomenon in the evolution of the atomic inversion for the system Hamiltonian (1). This phenomenon is a purely quantum mechanical effect and has its origin in the granular structure of the photon number distribution of the initial field [18]. The phenomenon has been realized experimentally in the sense that the state of the atomic beam leaving the cavity is monitored by ionization detectors [19]. It is defined as the difference between the probabilities of finding the atom in the exited state and in the ground state. When the atom starts in its excited state, the atomic inversion is given by,

$$W(t)=\frac{N_q^2}{2}\sum_{n=0}^{\infty}\frac{n!|L_n^q(-\xi)|^2}{(q+n)!}\left\{\left|\left(\cos g_1 t-\frac{i\omega_0}{2g_1}\sin g_1 t\right)\right|^2 -\mu_2^2(L(n)+2)(L(n)+1)\frac{\sin^2 g_1 t}{g_1^2}\right\} \tag{27}$$

Now we plot the atomic inversion against the scaled time $\mu_2 t$ when the atom initially starts in its excited state and the field is prepared in state (20). In Fig. (1a) we see the flipping from upper to lower state occurring partially and rise slow oscillations and revivals of the atomic inversion, the revival at $\mu_2 t = 2n\pi$ $(n = 1, 2..)$ which is in compliance with the expected calculation where $\lambda_3 = 0$, $\xi = 0.3$ and $q = 0$ (identical two photon case) as shown in Fig. 1a. To see the influence of the converter coupling in the atomic inversion we set $\lambda_3 = 0.9$ and the other parameters same as the previous case. We show that the converter terms leads to a regular fluctuations of the function between 0.5 and -0.2 as should be expected and the periodic phenomena of the revivals as the previous case with more fluctuations as shown in Fig. 1b.

As soon as we take the difference between the photon number parameter $q = 6$, the function increased oscillations during the revival period see Fig. 1c. Also the slow oscillations in the pervious cases washed by more rapidly oscillations as observed in Fig. 1c. More increasing in the coupling parameter value leads to an increase in the oscillations during revival periods, with a spreading out of the revival periods around their centers. This can be seen for the case in which $\lambda_3 = 0.9$ see Fig. 4d. It should be noted that the shape of the function for previous cases are the same; however, the difference between them is just occurring in the period of the collapses and revivals.

4. Linear entropy

Here we aim to discuss the degree of entanglement between the subsystem by using the linear entropy, which is a convenient way to study the coherence properties of the density operator as a function of time. The linear entropy is defined by [20–22] to quantify the amount of entanglement of the subsystem, which is defined by

$$\zeta_F=\frac{1-\left(\langle\sigma_z\rangle^2+\langle\sigma_x\rangle^2+\langle\sigma_y\rangle^2\right)}{2} \tag{28}$$

It should be noted that, when the state-vector description of each individual system of the ensemble is possible, a necessary and sufficient condition for the ensemble to be

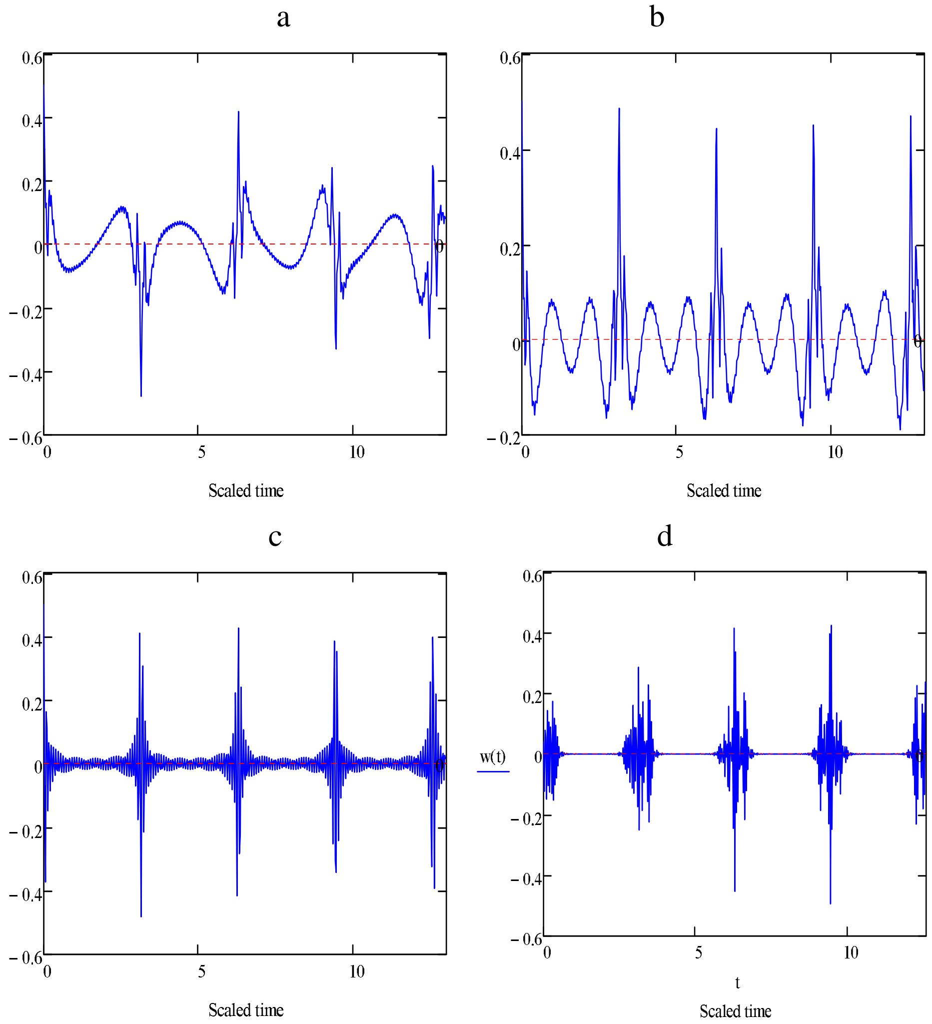

Figure 1: The atomic inversion for the two-level atom initially in its excited state and the field is inintially in state (20) for fixed value of $\xi = 0.3$, $\Delta = 0$. Where a)$q = 0, \lambda_3 = 0$, b)$q = 0, \lambda_3 = 0.9$, c) $q = 6, \lambda_3 = 0$, d) $q = 6, \lambda_3 = 0.9$.

described in terms of a pure state is that $\zeta_F = 0$. However, for the case in which $\zeta_F > 1$, the field is in a statistical mixture state. For a two-level system one can see the maximally mixed ensemble corresponds to $\zeta_F = \frac{1}{2}$.

In Fig. 2 we have plotted the function of purity ζ_F against the scaled time $\mu_2 t$ for different values of the parameters q and λ_3 and fixed that the atom initially in the excited state and the field in the state (20) with parameter to $\xi = 0.3$. The starting time we see that $\zeta_F = 0$, but after a short interval ζ_F increases and the coherence of the field loses. The linear entropy of the field ζ_F shows local maxima and minima, corresponding to the entanglement and disentanglement between the subsystem. The first case in absence

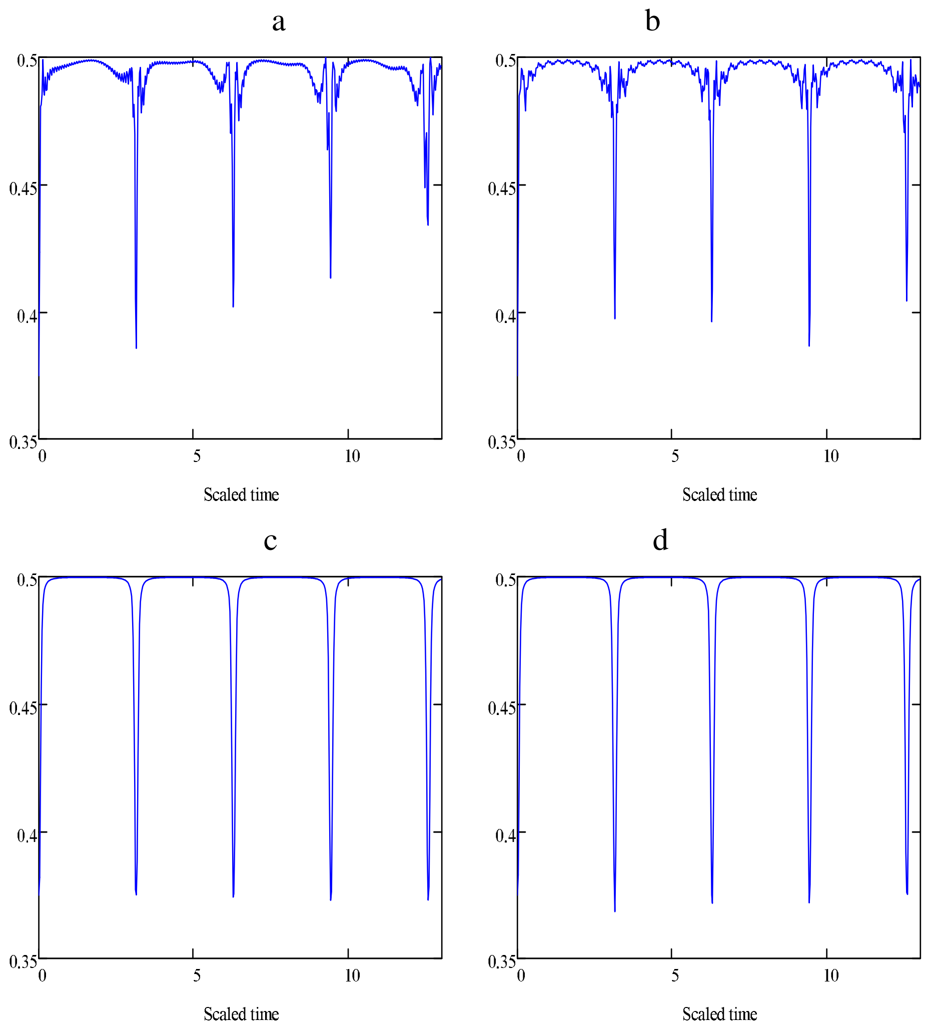

Figure 2: The linear entropy and same conditions same as Figure (1)

of the converter terms and identical photons i.e $\lambda_3 = 0$ and $q = 0$, the field purity still exhibits local maxima and minima, but minimum of disentanglement is not taking place at $\zeta_F = 0$ and the field never reach a pure state. It is evident the function displays regular fluctuations with interference between the patterns. Also, we can observe a decrease in its minimum as the time develops as observed in Fig. 2a. For the converter parameter taken into account adjust $\lambda_3 = 0.9$ the entanglement between the field and the atom more increasing and the minimum values are periodic and reach to 0.35. In general the atom is steady when it is in the ground state $|g>$. However, when the atom is in the excited state $|e>$, the factors, such as the spontaneous emission, the interaction between the field and the atom, the converter terms will lead to more increasing of the entanglement see Fig. 2b. When the $q = 6$ and $\lambda_3 = 0$, the function displays similar behavior to that of the previous case. However, it decreases its minimum (compared with the previous case), showing strong entanglement, but it approaches its maximum as the time increases; see Fig. 2c. More an increase in the periods of strong entanglement, where the function nearly approaches the maximum value of the mixed state. After add the converter terms, we see that the minimum values reduced to 0.37 and the linear entropy never attains a 0 value (i.e., disentanglement) when the system is either in its upper or lower state (i.e., a pure state), while more regions of strong entanglement are occurring between the revivals periods as observed in Fig. 2d.

5. The variance squeezing

It is well known that the variance squeezing is built up on the concept of the uncertainty principle relations in order to discuss the quantum fluctuations. The argument was to use the entropic uncertainty relations for two-level system rather than the Heisenberg uncertainty relations. This has been discussed by the authors of refs. [23–33]. In fact, for the quantum mechanical system with two physical observables represented by the Hermitian operators $\hat{A}$ and $\hat{B}$ satisfying the commutation relation $[\hat{A}, \hat{B}] = i\hat{C}$, one can write the Heisenberg uncertainty relation in the form

$$\langle(\Delta\hat{A})^2\rangle\langle(\Delta\hat{B})^2\rangle \geq \frac{1}{4}|\langle\hat{C}\rangle|^2, \tag{29}$$

where $\langle(\Delta\hat{A})^2\rangle = (\langle\hat{A}^2\rangle - \langle\hat{A}\rangle^2)$.Consequently, the uncertainty relation for a two-level atom characterized by the Pauli operators $\hat{\sigma}_x$, $\hat{\sigma}_y$ and $\hat{\sigma}_z$, satisfying the commutation relation $[\hat{\sigma}_x, \hat{\sigma}_y] = i\hat{\sigma}_z$ can also be written as $\Delta\hat{\sigma}_x\Delta\hat{\sigma}_y \geq \frac{1}{2}|\langle\hat{\sigma}_z\rangle|$.

Fluctuations in the component $\Delta\hat{\sigma}_\alpha$ of the atomic dipole is said to be squeezed if $\hat{\sigma}_\alpha$ satisfies the condition

$$V(\hat{\sigma}_\alpha) = \left(\Delta\hat{\sigma}_\alpha - \sqrt{\left|\frac{\langle\hat{\sigma}_z\rangle}{2}\right|}\right) < 0, \qquad \alpha = x \quad or \quad y. \tag{30}$$

Now to discuss the variance squeezing we have plotted several figures for the function $V(\hat{\sigma}_y)$ against the scaled time $\mu_2 t$. In our computational program we have taken into consideration the field to be initially in the state (20) and the atom in its excited state $(\theta = 0)$. In figure (3) we plot the variance squeezing for various values for the amplifier parameter λ_3 and the difference of the photon number q. For the first case in absence of the converter terms i.e $\lambda_3 = 0$ and $q = 0$ the squeezing can be seen four times during the considered period of time. The first observation occurred at $\mu_2 t = n\pi$ $(n = 1, 2, 3, \ldots)$. In this case it has been found that maximum squeezing is equal to -0.05. It is also noted that the amount of squeezing in the third period of time is too more compared with the second and fourth periods of time see Fig. 3a. When we consider a case in which $\lambda_3 = 0.9$, a small amount of squeezing can be seen after the onset of the interaction as well as at the end of the considered period of time. In this case the squeezing is pronounced, and the quadrature variance shows another three periods of decreasing

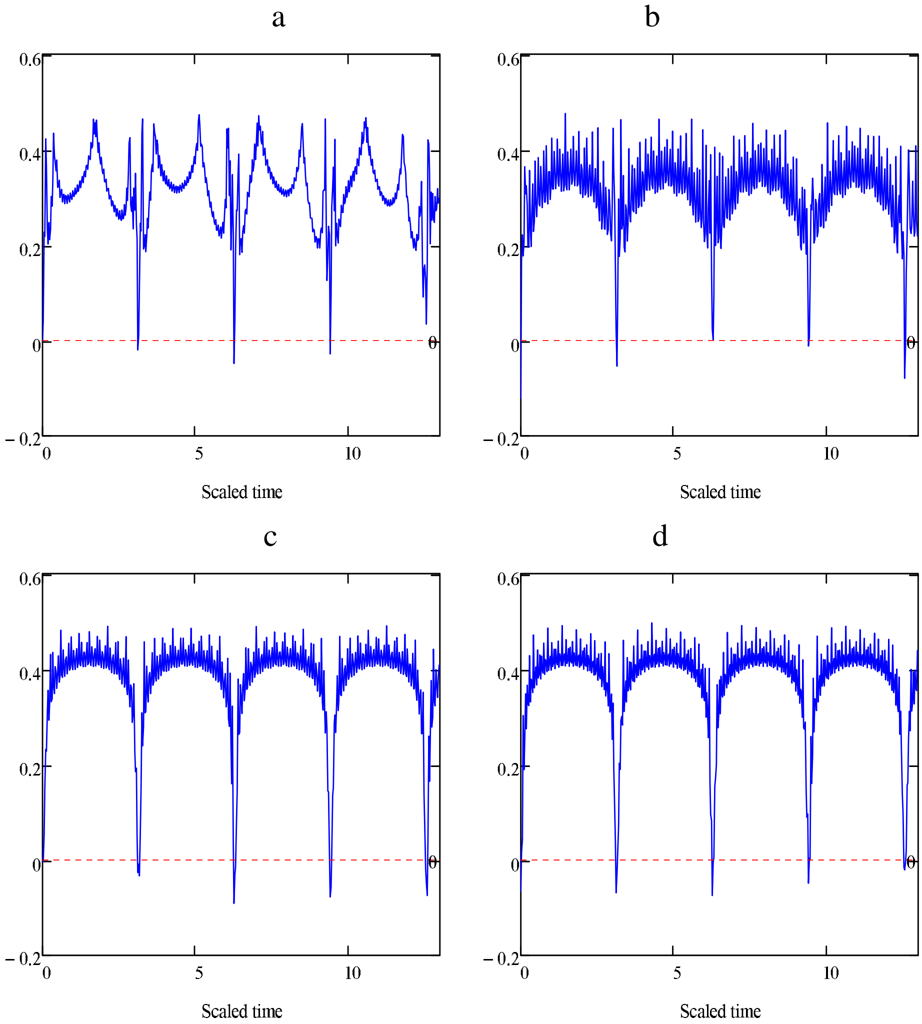

Figure 3: The variance squeezing and same conditions same as Figure (1)

where the maximum value of the squeezing occurred at the value -0.112 followed by increasing of the squeezing phenomenon which observed in Fig. 3b. When we consider a case in which $\lambda_3 = 0$ and $q = 6$, we see that similar behavior can be seen and function $V(\hat{\sigma}_y)$ still shows squeezing. In this case squeezing is also pronounced, but its value has been slightly increased where the minimum value of the squeezing is observed at -0.1. In the meantime it is noted that more fluctuations built up, with a slight shifting in function after the onset of the interaction see Fig. 3c. When we take $\lambda_3 = 0.9$ and $q = 6$, we see that more increasing in the value of the coupling parameter leads to a decrease in the amount of squeezing, also more fluctuations besides decreasing in the squeezing value can be reported (maximum squeezing has the value -0.04) see Fig. 3d.

6. Conclusion

In the present communication we have introduced the problem of the interaction between a two-level atom and a quantized three type of the field interaction, two as a degenerate two photon and the third represented the converter form. Under a certain condition of the Heisenberg chain the Hamiltonian model can be obtained. The canonical transformations is used to remove two coupling parameters and hence we have managed to deal with the Hamiltonian model. The time-dependent evolution operator is used to derive the wave function which enabled us to calculate the expectation value for some dynamical operators. For instance we discussed the phenomenon of collapses and revivals through the atomic inversion where we realized that the coupling λ_3 more oscillation appears. Furthermore we noted that in presence of q and absence of λ_3 the amplitude of the function after onset of interaction is more wider than the opposite case. The degree of entanglement is examined and it is found that the system is too sensitive to the variation of q and λ_3 parameters. Finally we considered the phenomenon of squeezing where we use the variance squeezing. It has been shown that both of q or λ_3 parameters are effective on this phenomenon of squeezing. Also we noted that the squeezing increases in presence of the q parameter more than the amount of λ_3.

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