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Additive Properties for Measurable Set on Cauntable Union Measurable Set

Sukoriyanto, Sisworo, and Imam Supeno
Mathematics Department, FMIPA,
Universitas Negeri Malang,
Indonesia Jalan Semarang 5,
Malang, 65114, Indonesia.

Abstract

In this paper it will look for condition countable collection of measurable sets $\{Ai\}$ that applies $m\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} m(E_i)$. Looking for the condition of countable collection measurable set $\{Ai\}$ and proving theorems done through study of properties measurable set.

AMS subject classification:
Keywords: Measurable, aditif properties, Union set.

1. Introduction

Measure theory is one part of the real analysis are widely used in other sciences, such as knowledge of trade that can be used to compare the methods of science trade [2], physics used to look at the dynamic range the quantum phase [13], the linear program to determine the linear models [11], the health sector in the determination of the effectiveness of the treatment [8], the field of psychology in the establishment obscurity information criteria [15], the field of risk management in the determination of the complicated decision-making [19], field techniques for measuring turbine rotation by using similarity cosinus [12].

The development of modern measure theory is characterized by the introduction of concept outer measure. At that time the outer measure is defined as the infimum of the total length of the interval that covers the set [9]. While [4] tried to decompose the Lebesgue measure to determine the singularity of measure, according to [3] measurable functions can be rearranged and can determined of it convergent on the measurement space. According to [10] measure can be developed through the boundary and symmetric divergence. While [18] to continue the development of [9] on subaditif finite outer measure. With the development of outer measure, has many problems that can be solved them if the set is interval then the outer interval equal to the length of the interval, but theoretically outer measure has a weakness, because it does not meet the outer measure additive properties that $m*(A \cup B) \neq m*(A) + m*(B)$. That's why the researchers tried to cover up the weakness of the outer measure. Among the researchers are [14], [16], which defines the measure by using the concept outer measure.

By using the concept of measure, important problems in the real analysis can be developed such as in [1] concerning the properties of the open set, that a union of any collection of open set in R is open and on another litertur namely [6] If G_1 and G_2 is set to open in R, then $G_1 \cap G_2$ is open set. By using the concept of measure the two theorems can be developed, one by [7] which is a union of sequence measurable set is measurable set. Even problems on Real Analysis are not applicable, by using the concept of measure the problem can be proven to be valid. As an example that if A and B are open set in R then A - B is not necessarily open set in R, using the concept of measure it can be shown that if A and B measurable set then A - B is measurable set [17].

On the outer measure concept it does not apply the additive properties of that $m^*(\cup A_i) \neq \sum m^*(A_i)$, and according to [9] for the outer measure the properties of sub-additive effect the following example if $\{E_i\}$ is countable collection set, then $m^*(\cup A_i) \leqslant \sum m^*(A_i)$, it is interesting to conduct research on the properties of the additive to the concept of measure. Also on the operation of two measurable sets, [17] can prove that if A and B are two measurable sets then A-B is measurable set.

According to [5] if A and B are measurable set, then $m(A \cup B) = m(A) + m(B) - m(A \cap B)$. This means that $m(A \cup B) \leqslant m(A) + m(B)$ for an A and B measurable set, so far the discussion $m(\cup A_i) = \sum m(A_i)$ recently conducted by [9], but the results of the discussion is not yet at the identification condition of the set $\{A_i\}$, therefore it is interesting to develop research what condition must be met on the measurableset $\{A_i\}$ so that $m(\cup A_i) = \sum m(A_i)$.

2. Materials and Methods

This research uses literature review. This research requires the data or information derived from books, journal or newsletters related to problems studied. Step research began reviewing the definition of difference of measurable set by using source reduction [17], [9], [7], and [1]. If we have a C and D are set, chances of that happening are as follows;

1. $C \cap D \neq \varnothing, C \not\subset D$, and $C \not\supset D$

2. $C \subset D$
3. $D \subset C$
4. $C \cap D = \varnothing$

If viewed from four possibilities were obtained following illustration,

1. For $C \cap D \neq \varnothing$, $C \not\subset D$, and $C \not\supset D$ this is imposible $m(C \cup D) = m(C) + m(D)$, examples let $C = (2,6)$, and $D = (4,8)$, then $m(C) = 4$ and $m(D) = 4$, whereas $m(C \cup D) = 6$ and $m(C) + m(D) = 8$, so, it is not equal.
2. For $C \subset D$ this is imposible $m(C \cup D) = m(C) + m(D)$, examples, let $C = (3,6)$ and $D = (2,8)$, then $m(C) = 3$ and $m(D) = 6$, whereas $m(C \cup D) = 6$ and $m(C) + m(D) = 3 + 6 = -9$, so, it is not equal.
3. For $D \subset C$ this is imposible $m(C \cup D) = m(C) + m(D)$, examples, let $D = (4,6)$ and $C = (3,8)$ then $m(C) = 2$ and $m(D) = 5$, then $m(C \cup D) = 5$, whereas $m(C) + m(D) = 2 + 5 = 7$, so, it is not equal.
4. For $C \cap D = \varnothing$ this is possible $m(C \cup D) = m(C) + m(D)$ examples, let $C = (1,4)$ and $D = (6,8)$, then $m(C) = 3$, and $m(D) = 2$, then $m(C \cup D) = 5$ and $m(C) + m(D) = 5$, whereas $m(C \cup D) = m(C) + m(D)$. But, in mathematics, many examples of these statements apply not guarantee the correct ring, and so there needs to be a proof.

From the examples shows that the relationship between two sets should be disjoint to each other and collection of measurableset should be countable. Therefore, it can be formulated a theorem to show the problem, namely "let $\{E_i\}$ countable collection disjoint measurable set then $m\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} m(E_i)$"

3. Results and Discussions

3.1. Outer Measure

Let F is collection of countable open interval. Every $J \in F$, sum $\sum_{I \in J} l(I)$ real positif number. Let E is set, and C is subset F with C is colection J from open interval $\{I_i\}$ such that $E \subset \bigcup_i I_i$. if we write $C = \{J : J \subset F \text{ and } J \text{ cover } E\}$. Outer measure $m*(E)$ from E is $m*(E) = inf\{\sum_i l(I_i) : \{I_i\} \text{ open interval and } E \subset \bigcup_i I_i.\}$.

Theorem 3.1. If A and B are two sets with $A \subset B$, then $m^*(A) \leqslant m^*(B)$.

Proof. Let $\{I_n\}$ countable collection disjoint open interval such

$$B \subset \bigcup_n I_n$$

Because A ⊂ B, then

$$A \subset \bigcup_n I_n$$

Implies $m^*(A) \leqslant \sum_{n=1}^{\infty} l(I_n)$.

So $m^*(A) \leqslant m^*(B)$. ■

Theorem 3.2. Let $\{E_n\}$ are collection countable set, then $m^*\left(\bigcup_n E_n\right) \leqslant \sum_n m^*(E_n)$.

Proof. Take $m^*(E_n) = \infty$ for same $n \in N$ then the inequality is trivial.

Note the right-hand side of the above inequality. Because $\sum_n m^*(E_n) = \infty$, for $m^*(E_n) = \infty$, then, regardless of the value $m^*\left(\bigcup_n E_n\right)$ will satisfy inequality $m^*\left(\bigcup_n E_n\right) \leqslant \sum_n m^*(E_n)$.

For $m^*(E_n) < \infty$ for every $n \in N$ given $\varepsilon > 0$, $\exists$ countable collection open interval $\{I_{n,i}\}$ $i = 1,2,3,\cdots \ni E_n \subset \bigcup I_{n,i}$ then

$$\sum_i l(I_{n,i}) < m^*\left(E_n + 2^{-n}\varepsilon\right) \ \ni \bigcup_n E_n \subset \bigcup_n \bigcup_i I_{n,i}\,.$$

(base infimum theorem).

Because $\{I_{n,i}\}_{n,i}$ countable collection open interval and cover $\bigcup_n E_n$, then

$$\begin{aligned} m^*\left(\bigcup_n E_n\right) &\leqslant \sum_{n=1}^{\infty}\sum_{i=1}^{\infty} l(I_{n,i}) < \sum_{n=1}^{\infty}\left(m^*(E_n) + 2^{-n}\varepsilon\right) \\ &= \sum_{n=1}^{\infty} m^*(E_n) + \varepsilon \end{aligned}$$

Because

$$\sum_{n=1}^{\infty} 2^{-n} = \sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n = \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1$$

Geometry series with rasio $\frac{1}{2}$.

Because $\varepsilon > 0$ then $m^*\left(\bigcup_n E_n\right) \leqslant \sum_n m^*(E_n)$.

Measurable Set

Definition 3.3. E is measurable set, if $\forall A \subset R$, then $m*(A) = m*(A\cap E) + m*(A\cap E^c)$, and if E is measurable set, then $m*(E) = m(E)$.

Theorem 3.4. If D and E are measurable set, then $D \cup E$ is measurable.

Proof. Because D is measurable set,
Base definition, $\forall A \subset R$, then

$$m*(A) = m*(A\cap D) + m*(A\cap D^c).$$
$$= m*(A\cap D) + m*((A\cap D^c)\cap E) + m*((A\cap D^c)\cap E^c)$$

because E measurable

$$= m*(A\cap D) + m*((A\cap E)\cap D^c) + m*((A\cap D^c \cap E^c)$$
$$\geqslant m*(A\cap (D\cup E)) + m*(A\cap (D\cup E)^c)$$

because $A\cap(D\cup E) = (A\cap D)\cup(A\cap E)\cap D^c)$
then $m(A) \geqslant m*(A\cap(D\cup E)) + m*(A\cap(D\cup E)^c)\ldots.1)$
because $A = (A\cap(D\cup E)\cup(A\cap(D\cup E)^c))$, then
$m(A) \leqslant m*(A\cap(D\cup E)) + m*(A\cap(D\cup E)^c)\ldots.2)$

from 1) and 2) then $m(A) = m*(A\cap(D\cup E)) + m*(A\cap(D\cup E)^c)$. So $D\cup E$ measurable. ■

Aditif Measurable Set Properties

Thorem 3.5. Let $\{E_i\}$ countable collection disjoint measurable set then

$$m\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} m(E_i).$$

Proof. To prove $m\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} m(E_i)$. Then, must be shown

$$m\left(\bigcup_{i=1}^{\infty} E_i\right) \leqslant \sum_{i=1}^{\infty} m(E_i)$$

and

$$\sum_{i=1}^{\infty} m(E_i) \leqslant m\left(\bigcup_{i=1}^{\infty} E_i\right).$$

Base from theorem, for every $n \in N$

$$m\left(\bigcup_{i=1}^{n} E_i\right) = \sum_{i=1}^{n} m(E_i).$$

Will prove

$$\sum_{i=1}^{\infty} m(E_i) \leqslant m\left(\bigcup_{i=1}^{\infty} E_i\right).$$

We know that

$$\bigcup_{i=1}^{n} E_i \subset \bigcup_{i=1}^{\infty} E_i$$

then

$$m^*\left(\bigcup_{i=1}^{n} E_i\right) \leqslant m^*\left(\bigcup_{i=1}^{\infty} E_i\right).$$

Because $\bigcup_{i=1}^{n} E_i$ and $\bigcup_{i=1}^{\infty} E_i$ measurable set, then

$$m\left(\bigcup_{i=1}^{n} E_i\right) \leqslant m\left(\bigcup_{i=1}^{\infty} E_i\right)$$
$$\Leftrightarrow \sum_{i=1}^{n} m(E_i) \leqslant m\left(\bigcup_{i=1}^{\infty} E_i\right),$$

thus, if $n \to \infty$, inequality becomes

$$\sum_{i=1}^{\infty} m(E_i) \leqslant m\left(\bigcup_{i=1}^{\infty} E_i\right)$$

Will prove

$$m\left(\bigcup_{i=1}^{\infty} E_i\right) \leqslant \sum_{i=1}^{\infty} m(E_i)$$

Base theorem

$$m\left(\bigcup_{i=1}^{n} E_i\right) \leqslant \sum_{i=1}^{n} m(E_i)$$

because

$$\sum_{i=1}^{n} m(E_i) \leqslant \sum_{i=1}^{\infty} m(E_i),$$
$$\text{then } m\left(\bigcup_{i=1}^{n} E_i\right) \leqslant \sum_{i=1}^{\infty} m(E_i),$$

thus, if $n \to \infty$, inequality becomes

$$m\left(\bigcup_{i=1}^{\infty} E_i\right) \leqslant \sum_{i=1}^{\infty} m(E_i) \tag{1}$$

From (1) and (2), we have

$$m\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} m(E_i).$$

■

4. Conclusion

Measurable of the open set and close set can be discussed in depth with the discovery of proof theorem Let $\{E_i\}$ countable collection disjoint measurable set then $m\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} m(E_i)$.

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