Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 12, Number 3 (2016), pp. 2355–2366

http://www.ripublication.com/gjpam.htm

Modeling and Performance Analysis of Hybrid Inverter HVAC System using Colored Hybrid Petri Nets

Jessie Abraham[1]
*Department of Mathematics,
Loyola College, Chennai 600034, India.*

D. Neela
*School of Advanced Sciences,
VIT University, Chennai 600127, India.*

Abstract

In this paper, we model the hybrid inverter based HVAC system using Colored Hybrid Petri Nets (CPN), a non linear hybrid systems modeling tool which combines the strength of Petri Net, a bipartite graph and modular functional programming language. The model gives a representation of the dynamic behavior of the system and enables to find the system performance for different temperature settings. The liveness, performance and executability of this model can be verified using reachability graph.

AMS subject classification:
Keywords: Colored Hybrid Petri Nets, modeling, programming language, performance analysis, reachability graph.

1. Introduction

HVAC (stands for Heating, Ventilation and Air Conditioning) is a control system that regulates a heating and/or air conditioning system [7]. Usually a sensing device is used to compare the actual state (e.g., temperature) with a target state. The control system then draws a conclusion on the action to be taken (e.g., start the blower). The main purpose of a HVAC system is to provide cooling and thermal comfort by maintaining

[1]Corresponding author.

good indoor air quality through adequate ventilation and filtration. The selection of an ideal for heating, cooling and ventilating is a complex design decision that must balance many factors including heating and cooling needs, energy efficiency, humidity control, potential for natural ventilation, adherence to codes and standards, outdoor air quantity and quality, indoor air quality, and cost.

Inverter based HVAC compressor is designed in such a way that it ramps up quickly, providing the energy necessary to achieve the cooling/heating demand of the particular zone. The compressor motor works in tandem with system controls, sensors and a variable frequency drive and varies its speed to maintain the desired comfort level. Hence at partial load conditions, the system performs only at the minimum energy level, thereby minimizing electricity consumption. Modeling this process enables performance analysis of the system and also allows finding the optimum temperature setting for lowest energy consumption. Petri Nets serve this purpose the best [1].

As a powerful event modeling and analyzing tool, Petri Net provides a graphical representation of the dynamic events of the system and enables clear visualization of the system state changes [3]. In particular, Colored Petri Nets [5] are well suited for large and complex systems as it combines the graphical representation with programming language to fire transitions. Different color sets are associated with the places where each token has a value of the predefined type. Programming languages are used to define data types and manipulation of data. They have two types of representations namely, expression and function representation. The expression representation uses guards and arc expressions between multi sets whereas function representations use linear functions. In this system expression representation is used to model the dynamics.

An important tool to guarantee the success functioning of any Petri Net model is by forming a reachability graph to monitor system performance and deadlock situations [6]. In the sections to follow we have modeled the working dynamics of a hybrid inverter HVAC system using colored hybrid Petri Net and shown that the model works efficiently using a reachability graph.

2. Preliminary Definitions

A Petri Net is a directed bipartite graph in which one set of nodes are called places, denoted by a circle and the other set of nodes are called transitions, denoted by rectangular bars. The set of edges connecting the places and transitions are directed arcs. Mathematically it is defined as follows.

Definition 2.1. [6] A Petri Net is a quadruple $PN = (P, T, A, \mu_k)$, where

(i) $P = \{p_1, p_2, \ldots, p_n\}$ denotes a finite set of places. Each place pi has a set of data values called tokens, denoted by small black circles, whose value differs according to the state changes. The number of tokens present in a place can be 0, 1 or more according to the system needs.

(ii) $T = \{t_1, t_2, \cdots, t_m\}$ is a finite set of transitions and $P \cap T = \Phi$.

(iii) $A \subset (P \times T) \cup (T \times P)$ is a set of directed arcs connecting P and T.

(iv) $\mu_k = (M_1, M_2, \ldots, M_r)$ is a finite r dimensional vector of markings. Each M_j denotes the number of tokens in each place p_i after the k^{th} transition.

For the Petri Net to be executed, at least one transition should be enabled in the initial state. A transition $t_i \in T$ is said to be enabled iff each of its input p_i is marked with at least one token. The execution of a transition is called firing. Transition firing removes tokens from each input p_i and adds tokens to each output p_i. Firing changes the marking of the net from μ_a to μ_b which is denoted by $\mu_a \xrightarrow{t_i} \mu_b$. A sequence of transition firing is represented by σ and denoted as

$$\sigma = \mu_0 \xrightarrow{t_1} \mu_1 \xrightarrow{t_2} \mu 2 \ldots \xrightarrow{t_m} \mu_m .$$

The marking in each place changes according to the end product of each transition firing. The marking at a place after a transition fires changes from M_j to M_{j+1} and is given by

$$M_{j+1}(p_i) = M_j(p_i) - I\,(p_i) + O\,(p_i)\,, \forall p_i \in P.$$

When there are no enabled transitions, execution of the Net halts and the Net is said to be in a deadlock state. For example, consider the Petri Net in Figure 1. Here,

$$P = \{p_1, p_2, p_3\}\,,$$

$$T = \{t_1, t_2, t_3\}\,,$$

$$\mu_0 = (1, 1, 0)\,.$$

After t_1 fires, $\mu_1 = (0, 0, 1)$ and t_2 gets enabled as seen in Figure 2. t_1 is no longer enabled. After t_2 fires, $\mu_2 = (0, 0, 0)$. The model enters a deadlock state as in Figure 3.

Petri Nets differ from other modeling tools in the fact that they can exhibit all typical characteristics of a dynamic event driven system such as sequential occurrence, concurrency, conflict in decisions and synchronization in the graphical representation.

Colored Petri Nets (CPN) [2, 8, 9] are a type of Petri Nets which are useful when the system to be modeled requires both the states of the system and the events that alter the system states. CPN combines the strength of general Petri Nets and programming languages. Each token is mapped to some expression in the color set by a function. The programming language is used in assigning the function to arcs for enabling the transitions. In case of conflict, the programming language aids in choosing the transition to be enabled. This formalism befits concurrent and conflicting actions in complex systems. CPN assigns simple or complex data values called token color to different tokens and a declaration called color set to each place.

Extending the formal definition of CPN in [4] to hybrid systems leads to the concept of Colored Hybrid Petri Nets. The events which last for a long time and are continuously transformed are taken as continuous events.

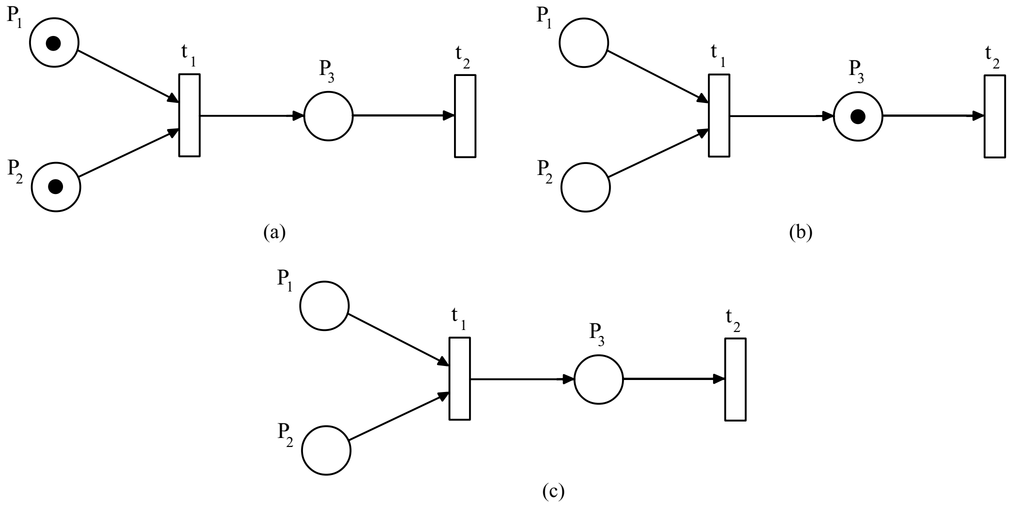

Figure 1: (a) t_1 is enabled (b) t_2 is enabled (c) Model in deadlock state.

Definition 2.2. [5, 10] A Colored Hybrid Petri Net is a decuple

$$CHPN = \{HP, HT, A, \Sigma, C, N, E, S, I, G\}$$

where,

(a) $HP = \{P_D, P_C\}$, where P_D denotes a finite set of discrete places and P_C denotes a finite set of continuous places. Tokens in P_D represent commands and signals whereas tokens in P_C represent time variant states and values.

(b) $HT = \{T_D, T_C\}$, where T_D denotes a finite set of discrete transitions and T_C indicates a finite set of continuous transitions. A continuous transition must have at least one continuous output place with which it forms a loop. The loop can be represents by bidirectional arcs. Continuous transitions keep on firing till a halting condition is imposed and hence can be associated with time.

(c) A is a finite set of connecting arcs such that $HP \cap A = A \cap HT = \Phi$.

(d) Σ denotes the finite set of all color set declarations.

(e) $C : HP \to \Sigma$ defines color functions which maps places in HP to colors in Σ. V is a finite set of color variables which assigns the colors defined by C to the places in HP.

(f) $U = P \cup T$ denotes the set of nodes.
$N : A \to (HP \times HT) \cup (HT \times HP)$ assigns node functions corresponding to source node and the destination node to the directed arcs.
$F : U \to A_s$ maps each node u into the set of all its surrounding arcs, that is, arcs with u as a source node or destination node and is defined as $F(u) = \{a \in A \mid \exists\, u' \in X : [N(a) = (u, u') \vee N(a) = (u', u)]\}$.

(g) E denotes the set of expressions which are mapped to arcs, tokens and transitions and $\forall(u, u') \in HP \times HT \cup HT \times HP : [E(u, u') = \sum_{(a \in (u,u')} E(a)]$.

(h) $S : A \to E$ is a function which maps each arc to its corresponding arc expression. For input to a decision making place the arc expression is of the form *act (event, state)*. The function enabling expressing the event and state variables in one expression is called *binding function*.

(i) $I : P \to E$ is an initialization function which maps each place into an initialization expression and evaluates multiset of tokens with a color corresponding to the color of the place $C(p_i)$.

(j) G is a guard function also known as monitoring function which is an arc expression used prominently in conflict situations to restrict event transitions on the basis of conditions for firing. General functionality of a guard is given by

$$\begin{aligned} G(t_i) : &If\ check(state, event) \\ &Then\ act(result(event, state), newstate) \\ &Else\ Empty. \end{aligned}$$

For instance, a switch is off and it is to be switched on. Here off and on are the state denoted by s and switching is the event denoted by e. A guard expression for this event occurrence is given by
If $(s = off, e = switch)$
Then $(switched, on)$
Else Empty
$G : T \to E$ maps the transitions to corresponding guard expression. When evaluated, the monitor expression gives either of the two Boolean values $(True,\ False)$. A transition t_i in conflict is enabled only if in addition to the firing constraint, the guard function evaluates to true.

The marking of places involved with monitoring function is transformed as follows. When $G(t_i)$ = True,

$$M'(p_i) = M(p_i) - 1, \forall p_i \in Input(t_i).$$

$$M'(p_k) = M(p_k) + 1, \forall p_k \in Output(t_i).$$

The various symbols used in CHPN are listed in Figure 2.

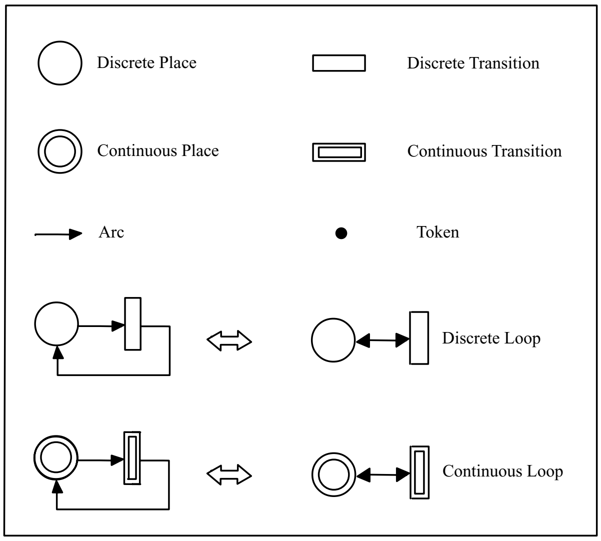

Figure 2: Symbols used in the model.

3. System modeling

3.1. Model description

The rotational speed of a compressor motor is given by $s_c = \frac{120f}{n}$, where f is the frequency supplied to the motor. We observe that the speed is directly proportional to the supplied frequency. In addition, it is also proportional to the load conditions. Hence changing the supplied frequency according to temperature modulation and load changes alters the speed of rotation of the compressor motor. This function is performed by a variable frequency drive which acts as an inverter drive with three components: AC to DC converter, DC link and DC to Quasi sinusoidal AC inverter. The drive along with digital programmable controllers and sensors vary the frequency supplied to the motor. This variable frequency supply to motor decreases the electricity consumption by a considerable amount.

3.2. System functioning

The room temperature sensor sends activation signal to the HVAC Micro Controller (MC) when temperature is set. The Micro Controller in turn sends maximum frequency reference signal to the signal control unit. The signal is regulated and compared with a carrier waveform generated using an oscilloscope, in a block called Pulse Width Modulation (PWM) generator which sends a pulse width modulated switching signal to Insulated Gate Bipolar Transistors in a Variable frequency drive. High frequency switching of these transistors generates a quasi sinusoidal voltage waveform of full frequency. The voltage is filtered to give a sinusoidal waveform which is supplied to the motor rotor. This continues till the set temperature (t) is reached. When the set temperature is at-

Table 1: Model entities

Input place	Transition	Output place
Pre-condition	Event	Post-condition
Input signal	Signal processor	Output signal

Table 2: Places and transitions in the model

Places	Transitions
p_1: Temperature is set	t_1: Temp sensor senses room temp
p_2: Sensor produces signal corresponding	t_2: Sensor sends activation signal to MC
to temperature	t_3: MC sends maximum frequency reference signal
p_3: MC analyzes signal	t_4: MC sends minimum frequency reference signal
p_4: Freq. reference signal is fed to regulator	t_5: Modulating signal enters generator
p_5: V/Hz ratio is maintained	t_6: Comparator compares modulating signal
p_6: Modulating signal is incident	and carrier waveform
on comparator	t_7: Generator sends high freq. switch signal
p_7: Carrier waveform is fed to comparator	t_8: Generator sends low freq. switch signal
p_8: Switch signal initiated.	t_9: Particular frequency switching stops
p_9: Transistors switch on/off	t_{10}: Sensor senses modulation in temp
p_{10}: Quasi sine pulsating	t_{11}: Modulation signal is sent to MC
waveform is generated	t_{12}: MC decreases freq. reference signal
p_{11}: Filtered sine voltage is fed to motor	t_{13}: MC increases freq. reference signal
p_{12}: Motor shaft rotates	t_{14}: Sensor senses temperature cut-off
p_{13}: Room temp alters	t_{15}: Cut off signal is sent to MC
p_{14}: Temperature is cut-off	t_{16}: frequency reference cuts off
p_{15}: cutoff signal is produced	t_{17}: Switching signal stops

tained, a modulating signal is sent to the micro controller and it lowers the frequency reference signal. By the same process as the initial stage, the frequency and motor to the motor are decreased and remain in the low state till an alteration in temperature from set temperature i.e.Δt occurs. Proceeding in the same way, frequency to motor is increased or decreased depending on whether Δt $>$ set temp or Δt $<$ set temp till temperature is cut off. When temperature is cut off, the whole process comes to a halt.

3.3. CHPN model

In this model, Each place is associated with one or more color sets which represents either a state or an event. The color sets are assigned to different arcs by a variable corresponding to the set. The color sets p_1 to p_{15} and transitions t_1 to t_{17} represent either of the two situations in Table 1.

When temperature is set, p_1 has a token and t_1 is enabled. By firing t_1, the action to be selected is determined to p_2. The decision making input is in the form of a *(state, event)* pair which creates the token to be put in p_3. Since the process is in initial phase, t_2 is enabled and fires. In the same way t_3 is enabled and fires, depositing a token in p_4. t_4 is enabled followed by t_5 whose firing gives token to p_5 and p_6 respectively. Once p_6 gets a token, t_6 which already has a token incident on it from p_7 gets enabled. t_6 fires and

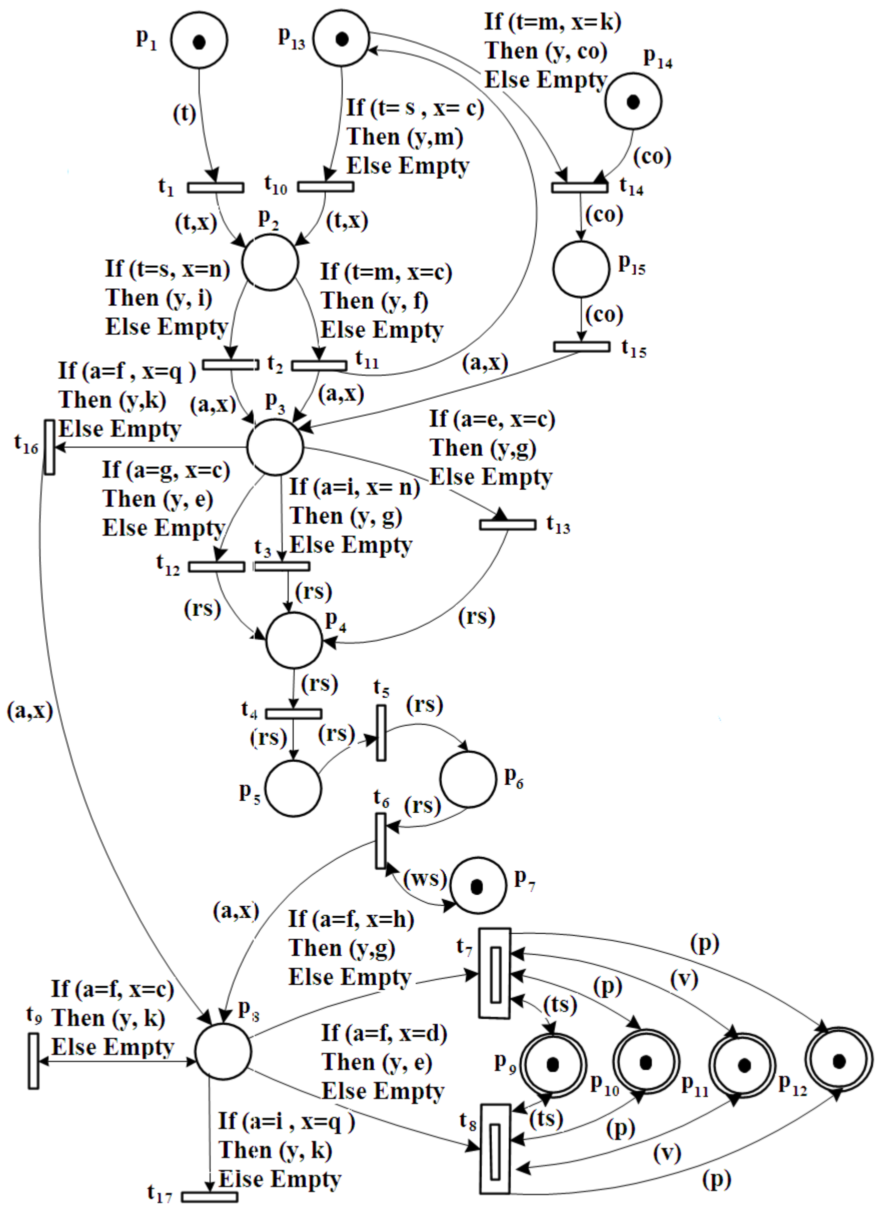

Figure 3: CHPN model.

puts a *(state, event)* bound token in p_8. As a result of decision making, the continuous transition t_7 is enabled and fired. During the firing process, the token value in p_9, p_{10}, p_{11} and p_{12} are continuously updated. Since the presence of tokens in continuous places do not affect the liveness of the process, continuous transitions continue till a halting command is given. When set temp is reached, token in p_{13} is activated and t_{10} is enabled. t_{10} fires, depositing a token in p_2 and the firing process continues same as during the initial phase till transition t_6. When p_8 gets a token, t_9 fires, bringing t_7 to a halt. As t_7

stops, t_8 starts firing immediately and continues till t_9 fires again. As long as the system is on, t_{10} fires according to temperature modification and the remaining transition firings take place according to the state of modification. When temperature is cut off, token in p_{14} gets activated and t_{14} fires, followed by t_{15}. This gives a token to p_3 and t_{16} is enabled and fired. p_8 receives a token which enables t_{17}, the firing of which gives rise to a deadlock state, signaling the end of the process. The CHPN model of this system is given in Figure 3.

Table 3: Color set declarations

Color set	Represents
Color T = real	Temperature with range $13°C$ to $30°C$. s denotes the
with s/m	set temp. m indicates the modulated temp (Δt).
Color A = enum	Multi modulating states. i represents no frequency.
with i/f/g/e/k	f denotes change in frequency. g indicates high
	frequency. e denotes low frequency. k denotes halting.
Color X = int	Controlling signal. n denotes activation signal. q
with n/q/c/h/d	denotes deactivation signal. c denotes changing signal
	h denotes increased signal. d denotes decreased signal.
Color Y = int	Signal processing.
Color CO = int	Cut-off signal.
Color RS = int	Reference signal.
Color WS = real	Waveform signal with range [0,1]
Color TS = int	Switching of transistors. It can have only 2 states
	(on/off) and hence indicated by an integer.
Color P = int	Pulse form signal.It can only have high/low state
	and hence denoted by an integer.
Color V = real	Sinusoidal voltage waveform with range [-1, 1].

Signals are denoted as integers because they are composed of bits which are of integer type. Waveforms and temperature are indicated as real as they are continuously transformed to different real values in the given range over a period of time.

3.4. Model Performance Analysis

Performance analysis of the model can be done graphically. In Petri Net terms this analysis is named as liveness analysis wherein the uninterrupted firing of each transition is verified. If the system is live, then the functioning is smooth. Liveness analysis of the model can be performed by means of a reachability graph. The reachability graph is drawn with markings obtained as a result of each transition. If each marking in the graph is reachable from some other marking after, then the model is live and the process execution is smooth. If a marking is not reachable, then there is some deadlock situation which indicates problem in the model. Hence liveness analysis is mandatory for the effective functioning of the model.

Table 4: Variables for color sets

variables	color sets	places
t	T	p_1, p_2, p_{13}
a	A	p_3, p_8
x	X	p_2, p_3, p_8, p_{13}
y	Y	p_2, p_3, p_8, p_{13}
co	CO	p_{13}, p_{14}, p_{15}
rs	RS	p_4, p_5, p_6
ws	WS	p_7
ts	TS	p_9
v	V	p_{11}
p	P	p_{10}, p_{12}

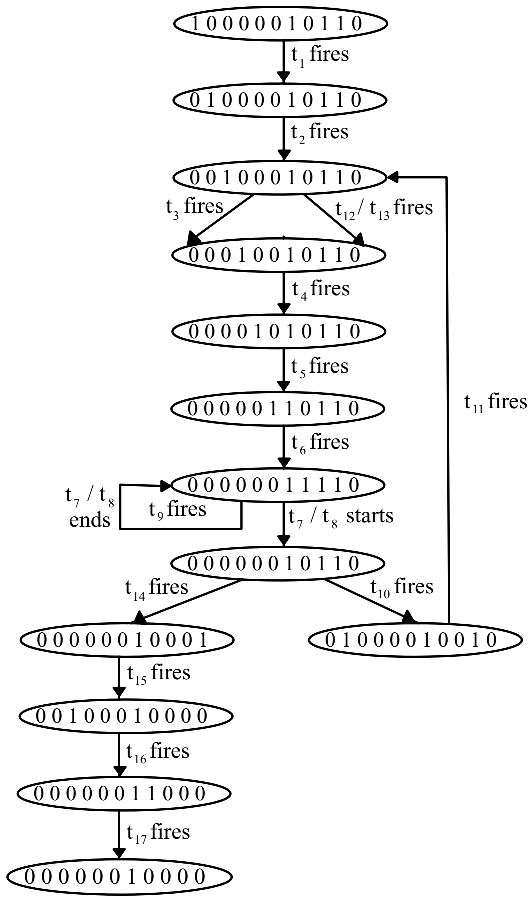

Figure 4: Reachability graph

Let

$$D = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_{13}, p_{14}, p_{15}\}$$

denote the set of discrete places and

$$C = \{p_9, p_{10}, p_{11}, p_{12}\}$$

be the set of continuous places. Only marking of places in D are considered for reachability graph construction. This is because tokens in P_C only represent state changes and gets deposited in the same place throughout the process and so liveness of the model is not affected. The initial marking of the model is (1 0 0 0 0 0 1 0 1 1 0). After the sequence of transitions from t_1 to t_{17}, the final marking is (0 0 0 0 0 0 1 0 0 0 0). The system gets into a deadlock state when t_{17} fires, determining the halting of the process. This model is live and the performance is uninterrupted. The performance analysis can be seen in Figure 4.

4. Conclusion

For any dynamic equipment, modeling and performance analysis is very important as it enhances the functioning under different working conditions. In this paper, we have modeled the different signal processing and state changes in a hybrid inverter HVAC system using Colored Hybrid Petri Net. The continuous and discrete characteristics of the model are integrated together and executed. This model can be used to test the performance for different temperature and frequency settings in order to decide the most efficient setting for lowest electricity consumption. The combination of graphical and computational tools in the model makes it easier to visualize and understand the dynamic complexities. Though performance analysis can be done by incidence matrix and state equations, reachability graph is more simple and effective in determining deadlock situations. During conflicts, the guard function helps in decision making and uninterrupted simulation.

Colored Hybrid Petri Nets being an enhancement of Colored Petri Nets give a more detailed representation of the model and thus can be used in modeling more complex dynamic systems. In future, a computer aided simulation tool can be created based on CHPN with time duration for each transition firing, so that complicated processes can be modeled in an efficient way.

References

[1] Almeida, A.G.S., Villani, E., Junqueira, F., and Miyagi, P.E., 2010, "Hybrid Approach to Air Conditioning Systems Modeling Using Petri Net, 2010," ABCM Symp. Mechatron., 4, pp. 345–354.

[2] David, R., and Alla, H., 2010, Discrete, Continuous and Hybrid Petri Nets, Springer Berlin Heidelberg, Germany.

[3] Herajy, M., Schwarick, M., and Heiner, M., 2013, "A Hybrid Petri Net Model of the Eukaryotic Cell Cycle," Lecture Notes in Comput. Sci., 8100, pp 123–141.

[4] Jensen, K., 1997, “A Brief Introduction to Coloured Petri Nets,” Lecture Notes in Comput. Sci., 1217, pp. 203–208.

[5] Jensen, K., 2005, “Colored Petri Nets: A High Level Language for System Design and Analysis,” Lecture Notes in Comput. Sci., 483, pp. 342–416.

[6] Murata, T., 1989, “Petri Nets: Properties, Analysis and Applications,” Proc. IEEE, 77(4), pp. 541–580.

[7] Villani, E., and Miyagi, P. E., 2004, “A Hybrid Petri Net Modeling Approach for HVAC Systems in Intelligent Buildings,” Revista Controle and Automação, 15, pp. 135–148.

[8] Wells, L., 2002, “Performance Analysis Using Colored Petri Nets,” Proc. 10th IEEE/ACM International Symposium on Modeling, Analysis and Simulation of Computer and Telecommunication Systems (MASCOTS’02), Fort Worth, Texas. pp. 217–221.

[9] Xu, Y., Xie, X., 2011, “Modeling and Analysis of Security Protocols Using Colored Petri Nets,” J. Comput., 6, pp. 19–27.

[10] Zhou, J., 2013, “Petri Net Modeling for the Emergency Response to Chemical Accidents,” J. Loss. Prevent. Proc., 26, pp. 766–770.