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Equitable Dominating in an Intunionistic Fuzzy Graphs

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Abstract

Let G be an intuitionistic fuzzy graph. Let u and v be two vertices of G. A subset D of V is called a *fuzzy equitable dominating set* if every $v \in V - D$there exist a vertex $u \in D$ such that $uv \in E(G)$ and $|deg(u) - deg(v)| = 1$ where $deg(u)$ denotes the degree of vertex u and $deg(v)$ denotes the degree of vertex v and $\mu_2(v_i, v_j) = \mu_1(v_i) \wedge \mu_1(v_j)$, $\gamma_2(v_i, v_j) = \gamma_1(v_i) \vee \gamma_1(v_j)$. The minimum cardinality of an intuitionistic fuzzy equitable dominating set is denoted by γ_{eif}. In this paper we introduce the concept of intuitionistic fuzzy equitable dominating set, minimal intuitionistic fuzzy equitable dominating set, and intuitionistic fuzzy equitable independent set further obtain some interesting results for this new parameter in equitable intuitionistic fuzzy graphs.

AMS subject classification:
Keywords: Intuitionistic fuzzy equitable dominating set, minimal intuitionistic fuzzy equitable dominating set, intuitionistic fuzzy equitable independent set.

1. Introduction

The first definition of fuzzy graphs was proposed by Kafmann, from the fuzzy relations introduced by Zadeh. Although Rosenfeld introduced another elaborated definition, including fuzzy vertex and fuzzy edges, and several fuzzy analogs of graph theoretic concepts such as paths, cycles, connectedness and etc. The concept of domination in fuzzy graphs was investigated by A. Somasundaram, S. Somasundaram and A. Somasundaram present the concepts of independent domination, total domination, connected domination of fuzzy graphs. C. Natarajan and S.K. Ayyaswamy introduce the strong (weak) domination in fuzzy graph. The first definition of intuitionistic fuzzy graphs was proposed by Atanassov. The concept of domination in intuitionistic fuzzy graphs was investigated by R.Parvathi and G.Thamizhendhi. In this paper we develop the strong (weak) domination set, connected dominating set, efficient dominating set and independent dominating set of intuitionistic fuzzy graphs. Further introduce a dominating parameter of these sets and investigate the property of this domination parameter in intuitionistic fuzzy graph.

In this paper we introduce the concept of intuitionistic fuzzy equitable dominating set, minimal intuitionistic fuzzy equitable dominating set, and intuitionistic fuzzy equitable independent set and obtain some interesting results for this new parameter in equitable intuitionistic fuzzy graphs.

2. Basic Definitions

An intuitionistic fuzzy graph (IFG) is of the form $G = (V, E)$, where $V = \{v_1, v_2, \cdots, v_n\}$ such that $\mu_1 : V \to [0, 1]$, $\gamma_1 : V \to [0, 1]$ denote the degree of membership and non-membership of the element $v_i \in V$ respectively and $0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1$ for every $v_i \in V$, $(i = 1, 2, \cdots, n)$. $E \subseteq V \times V$ where $\mu_2 : V \times V \to [0, 1]$ and $\gamma_2 : V \times V \to [0, 1]$ are such that

$$\begin{aligned} \mu_2(v_i, v_j) &\leq \mu_1(v_i) \wedge \mu_1(v_j) \\ \gamma_2(v_i, v_j) &\leq \gamma_1(v_i) \vee \gamma_1(v_j) \\ 0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) &\leq 1 \end{aligned}$$

An arc (v_i, v_j) of an IFG G is called an strong arc if

$$\mu_2(v_i, v_j) = \mu_1(v_i) \wedge \mu_1(v_j)\gamma_2(v_i, v_j) = \gamma_1(v_i) \vee \gamma_1(v_j).$$

Let $G = (V, E)$ be an IFG. Then the cardinality of G is defined to be

$$|G| = \left| \sum_{v_i \in V} \left[\frac{1 + \mu_1(v_i) - \gamma_1(v_i)}{2} \right] \right| + \left| \sum_{v_i \in V} \left[\frac{1 + \mu_2(v_i, v_j) - \gamma_1(v_i, v_j)}{2} \right] \right|.$$

Let $G = (V, E)$ be an IFG. The vertex cardinality of G is defined to be

$$|G| = \left| \sum_{v_i \in V} \left[\frac{1 + \mu_1(v_i) - \gamma_1(v_i)}{2} \right] \right|$$

for all $v_i \in V$, $(i = 1, 2, \ldots, n)$.

Let G = (V, E) be an IFG. An edge cardinality of G is defined to be be

$$|G| = \left| \sum_{v_i \in V} \left[\frac{1 + \mu_2(v_i, v_j) - \gamma_1(v_i, v_j)}{2} \right] \right|$$

for all $(v_i, v_j) \in V \times V$.

Let $G = (V, E)$ be an IFG. A set $D \subseteq V$ is said to be a dominating set of G if every $v \in V - D$ there exist $u \in D$ such that u dominates v.

An intuionistic fuzzy dominating D of an IFG, G is called minimal dominating set of G if every node $u \in D$, $D - \{u\}$, is not a dominating set in G.

An intunionistic fuzzy domination number $\gamma_{if}(G)$ of an IFG, G is the minimum vertex cardinality over all minimal dominating sets in G.

3. Intunionistic Fuzzy Equitable Dominating

Definition 3.1. Let G be an in intuitionistic fuzzy graph. Let u and v be two vertices of G. A subset D of V is called an *intunionistic fuzzy equitable dominating set* if for every $v \in V - D$ there exist a vertex $u \in D$ such that $uv \in E(G)$ and $|deg(u) - deg(v)| = 1$ and

$$\mu_2(v_i, v_j) = \mu_1(v_i) \wedge \mu_1(v_j),$$
$$\gamma_2(v_i, v_j) = \gamma_1(v_i) \vee \gamma_1(v_j).$$

The minimum cardinality of an intunionistic fuzzy equitable dominating set is denoted by γ_{eif}.

Definition 3.2. A vertex $u \in V$ is said to be *degree equitable* in an intunionistic fuzzy graph with a vertex $v \in V$ if $|deg(u) - deg(v)| \leq 1$ and $\mu_2(v_i, v_j) = \mu_1(v_i) \vee \gamma_1(v_j)$.

Definition 3.3. An intunionistic fuzzy equitable dominating set D is said to be a *minimal fuzzy equitable dominating set* if no proper subset of D is an intunionistic fuzzy equitable dominating set.

Defition 3.4. Let $u \in V$. An intunionistic fuzzy equitable neighbourhood of u denoted by $N_{eif}(u)$ is defined as

$$N_{eif}(u) = \{v \in V | v \in N(u)\}, |deg(u) - deg(v)| \leq 1,$$
$$\mu_2(v_i, v_j) = \mu_1(v_i) \wedge \mu_1(v_j), \ \gamma_2(v_i, v_j) = \gamma_1(v_i) \vee \gamma_1(v_j)$$

and $u \in I_e \iff N_{eif}(u) = \emptyset$.

The cardinality of $N_{eif}(u)$ is called an intunionistic fuzzy equitable degree of u and it is denoted by $d_{eif}(u)$.

Definition 3.5. The maximum and minimum intunionistic fuzzy equitable degree of a vertex in G are denoted respectively by $\Delta_{eif}(G)$ and $\delta_{eif}(G)$. That is

$$\Delta_{eif}(G) = \max_{u \in V(G)} \left| N_{eif}(u) \right|$$

and

$$\delta_{eif}(G) = \min_{u \in V(G)} \left| N_{eif}(u) \right|.$$

Definition 3.6. Let G be an intunionistic fuzzy graph. Then $D \subseteq V$ is said to be a *strong (weak) intunionistic fuzzy equitable dominating set* of G if every vertex $v \in V - D$ is strongly (weakly) dominated by some vertex u in D. We denote a strong (weak) intunionistic fuzzy equitable dominating set by seifd-set (weifd-set).

The minimum scalar cardinality of a seifd-set (weifd-set) is called the strong (weak) intunionistic fuzzy equitable domination number of G and it is denoted by $\gamma_{seif}(G)(\gamma_{weif}(G))$.

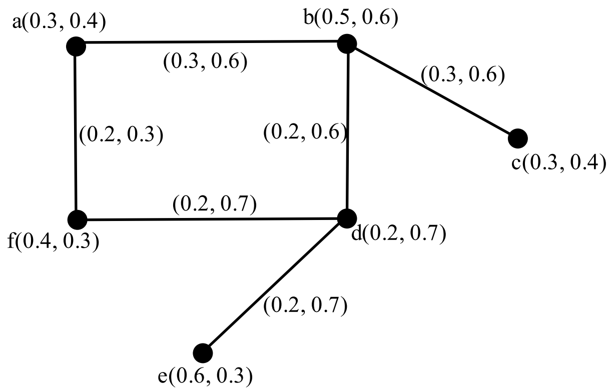

Figure 1: Example for

In Fig 1, $deg(a) = 0.45$, $deg(b) = 0.9$, $deg(c) = 0.45$, $deg(d) = 1.2$, $deg(e) = 0.25$, $deg(f) = 0.25$. An intunionistic fuzzy equitable fuzzy dominating set $D = \{b, d\}$, $\gamma_{eif}(G) = 0.7$.

Theorem 3.7. A dominating set D of G is a minimal intunionistic fuzzy equitable dominating set iff for each $d \in D$ one of the following two conditions holds:

(i.) $N_{eif}(d) \cap D = \varnothing$

(ii.) There is a vertex $c \in V - D$ such that $N_{eif}(c) \cap D = \{d\}$.

Proof. Let D be a minimal intunionistic fuzzy equitable dominating set and $d \in D$. Then $D_d = D \setminus d$ is not an intunionistic fuzzy equitable dominating set and hence there exists $x \in V \setminus D_d$ such that x is not dominated by any element of D_d. If $x = d$ we get (i) and if $x \neq d$ we get (ii). The converse is obvious. ■

Theorem 3.8. Let G be an IFG of order P, then

(i.) $\gamma_{eif}(G) \leq \gamma_{seif}(G) \leq p - \Delta_{eif}(G)$

(ii.) $\gamma_{eif}(G) \leq \gamma_{weif}(G) \leq p - \delta_{eif}(G)$.

Proof. Every strong equitable intuitionistic fuzzy dominating set is an equitable Intuitionistic fuzzy dominating set of G.

$$\gamma_{eif}(G) \leq \gamma_{seif}(G).$$

Similarly every weak intuitionistic fuzzy equitable dominating set in an equitable intuitionistic fuzzy dominating set of G.

$$\gamma_{eif}(G) \leq \gamma_{weif}(G).$$

Let $u, v \in V$. If $d_{eif}(u) = \Delta_{eif}(G)$ and $d_{eif}(v) = \delta_{eif}(G)$. Clearly $V - N_{eif}(u)$ is a strong intuitionistic fuzzy equitable dominating set and $V - N_{eif}(v)$ and is a weak intuitionistic fuzzy equitable dominating set. Therefore

$$\gamma_{seif}(G) \leq \left|V - N_{eif}(u)\right|_{eif}$$

and

$$\gamma_{weif}(G) \leq \left|V - N_{eif}(v)\right|_{eif}.$$

i.e.

$$\gamma_{seif}(G) \leq p - \Delta_{eif}(G)$$

and

$$\gamma_{weif}(G) \leq p - \delta_{eif}(G).$$

■

Theorem 3.9. Let G be an IFG without isolated vertices. Let D be a minimal equitable dominating set of G. The $V \setminus D$ is an intuitionistic fuzzy equitable dominating set of G.

Proof. Let d be any vertex in D. There exist a vertex $x \in N(d)$ such that $c \in V \setminus D$ from Theorem 3.7. Thus every vertex of D is dominated by some vertex of $V \setminus D$. ■

Defition 3.10. The maximum and minimum an intunionistic fuzzy equitable degree of a vertex in G are denoted respectively by $\Delta_{eif}(G)$ and $\delta_{eif}(G)$. That is

$$\Delta_{eif}(G) = \max_{u \in V} \left| N_{eif}(u) \right|$$

and

$$\delta_{eif}(G) = \min_{u \in V} \left| N_{eif}(u) \right| .$$

4. Equitable Independent Domination in Fuzzy Graphs

Definition 4.1. A set S of vertices of a fuzzy graph is said to be an intunionistic fuzzy equitable independent set, if for any $u \in S$, $v \notin N_{eif}(u)$, for all $v \in S - \{u\}$ and

$$\begin{aligned} \mu_2(v_i, v_j) &= \mu_1(v_i) \wedge \mu_1(v_j), \\ \gamma_2(v_i, v_j) &= \gamma_1(v_i) \vee \gamma_1(v_j), \forall\, u, v \in S. \end{aligned}$$

Theorem 4.2. If D is an intunionistic fuzzy equitable independent dominating set of G then D is a both minimal intunionistic fuzzy equitable dominating set and a maximal intunionistic fuzzy equitable independent set. Conversely any maximal intunionistic fuzzy equitable independent set D in G is a fuzzy equitable independent dominating set of G.

Proof. If D is an intunionistic fuzzy equitable independent dominating set of G. $D = D \setminus \{d\}$ is not an intunionistic fuzzy equitable dominating set for every $d \in D$ and $D \cup \{x\}$ is not an intunionistic fuzzy equitable independent for every $v \notin D$ so that D is a minimal intunionistic fuzzy equitable dominating set and a maximal intunionistic fuzzy equitable independent set.

Conversely, let D be a maximal intunionistic fuzzy equitable independent set in G. Then for every $x \in V \setminus D$, $D \cup \{x\}$ is not an intunionistic fuzzy equitable independent and hence x is dominated by some element of D. Thus D is an intunionistic fuzzy equitable independent dominating set of G. ■

Theorem 4.3. Let G be a fuzzy graph. Then $\gamma_{eif}(G) = \beta_0^{eif}(G)$.

Proof. Let S be an intunionistic fuzzy equitable independent set of vertices in G such that $|S| = \beta_0^{eif}(G)$. Then G contains no large intunionistic fuzzy equitable independent set. This means that every vertex v in $V - S$ is adjacent to atleast one vertex of S. Therefore S is an intunionistic fuzzy equitable dominating set. Thus $\gamma_{eif}(G) = |S|$. Therefore $\gamma_{eif}(G) = \beta_0^{eif}(G)$. ■

Theorem 4.4. An intunionistic fuzzy equitable independent set S is maximal intunionistic fuzzy equitable independent set iff it is an intunionistic fuzzy equitable independent set and intunionistic fuzzy equitable dominating set.

Proof. Suppose an intunionistic fuzzy equitable independent set S is maximal intunionistic fuzzy equitable independent set. Then for every vertex u in $V - S$, the set $S \cup \{u\}$ is not an intunionistic fuzzy equitable independent set, that is for every vertex u in $V - S$ there is a vertex v in S such that u is adjacent to v. Thus S is an intunionistic fuzzy equitable dominating set. Hence S is both intunionistic fuzzy equitable independent and intunionistic fuzzy equitable dominating set.

Conversely suppose that a set S is both intunionistic fuzzy equitable independent and intunionistic fuzzy equitable dominating set. We show that it is maximal intunionistic fuzzy equitable independent set. Suppose S is not maximal intunionistic fuzzy equitable independent set. Then there exist a vertex u in $V - S$ such that $S \cup \{u\}$ is an intunionistic fuzzy equitable independent set. But if $S \cup \{u\}$ is an intunionistic fuzzy equitable independent set, then no vertex in S is adjacent to u. Hence S is not an intunionistic fuzzy equitable dominating set, which is a contradiction. Therefore S is maximal intunionistic fuzzy equitable independent set. ∎

5. Conclusion

This paper we have introduced the concepts of an intunionistic fuzzy equitable domination and intunionistic fuzzy equitable independent. The various dominations in an intunionistic fuzzy graph will be reported in forth coming paper.

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