Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 12, Number 3 (2016), pp. 2023–2033

http://www.ripublication.com/gjpam.htm

Modeling and Optimization of Assets Portfolio with Consideration of Profits Reinvestment

Irina Leonidovna Kashirina, Tatiana Vasilievna Azarnova, Yulia Valentinovna Bondarenko, Irina Naumovna Shchepina
Voronezh State University,
Russia, 394006, Voronezh, Universitetskaya sq. 1.

Abstract

The goal of the article: development of mathematical modeling apparatus of portfolio investment processes, applied for the substantiation of investment decisions. The article examines the model of assets portfolio management for a long-term investment period, divided into a certain number of short-term periods, moreover, after each short-term period the portfolio profit is not withdrawn but reinvested. The model is based on the maximization of the portfolio yield growth rate during the whole long-term period. The deviation of the average geometric portfolio yield growth rate for the even growth rate is used as the assessment of such portfolio risk. The article provides the results of the calculation experiment, which demonstrates both distinctive features of the suggested approach and in some cases similarity with the results obtained with the application of the classical Markowitz model. The article provides theoretical explanation of the possible similarity of the modeling results.

AMS subject classification:
Keywords: Portfolio management, portfolio yield, risk assessment, capital growth rate.

1. Introduction

The problems of the formation and management of securities portfolio take an important place both in the theoretical literature and scientific research and in the system of real economy (Davnis et al., 2013), (Berkolaiko, Russman, 2004). Securities portfolio is an integrated financial tool, purposefully developed in accordance with the definite investment strategy and it is the totality of contributions in the selected investment objects (Watsham and Parramore, 1996). The main goal of the securities portfolio management is an intentionto obtain the highest possible yield with the lowest possible (or limited) risk level during a certain period of time. Generaly, this goal is achieved, through the portfolio diversification, i.e. redistribution of investor's funds among different assets and through the optimal choice of parts of the said assets in the portfolio (Barkalov et al., 2014). With the traditional approach in the portfolio, as a rule, they purchase securities of well-known companies, which hold good production and financial indicators. Alongside with this, the portfolios structure is determined not only by the availability or absence of a security in them, but also by its share.

Usually, yield and risk of the portfolio, expected by the end of the investment period, are assessed on the basis of statistical data for previous periods of time of the same length. In addition, the most common approach is the one suggested by H. Markowitz (Markowitz, 1952).

Despite a large number of studies, dedicated to the mentioned topic, the problems of modeling and optimization of financial tools portfolios are still urgent. The development of modern market, globalization, and availability of a broad statistical basis provide the emergence of new methods of financial tools portfolio management, based on new conceptual approaches. The goal of this article is development of assets portfolio formation model, based on the maximization of the portfolio yield growth rate during the period of investment, which takes into account the changes in the portfolio yield during said period and which has principle differences from H. Markowitz's model.

2. Classical approach to portfolio optimization

The problem of the selection of securities portfolio optimal structure was comprehensively studied for the first time by H. Markowitz in 1952 (Markowitz, 1952). The model of portfolio optimization suggested by him became the basis for the studies in the field of modern theory of investment decisions making. According to Markowitz's model the portfolio yield is assessed as weighed by investment shares average yield of each of the assets included in the portfolio, and the risk is measured as average quadratic deviation of the yield. With the set level of risk it is possible to maximize the portfolio yield:

$$d_p = \sum_{i=1}^{n} x_i \cdot m_i \to \max \tag{1}$$

$$\sigma_p = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} v_{ij} \cdot x_i \cdot x_j} \leq R_{\max} \tag{2}$$

$$\sum_{i=1}^{n} x_i = 1 \tag{3}$$

$$x_i \geq 0, \quad i = 1, 2, \ldots, n \tag{4}$$

where – x_i is the share of investment in i-th security; m_i is the expected yield of the i-th security; d_p is the portfolio yield; v_{ij} is covariation between i-th and j-th securities;

σ_p is portfolio risk; R_{max} is a parameter, limiting the maximum risk; n is a number of securities, regarded as the objects of investment.

While formulating the problem of the portfolio optimization Markowitz used theoretical and probabilistic formalization of the concepts "yield" and "risk", assuming, alongside with this, that securities yield conforms to the normal law of distribution.

Modern researchers in the sphere of portfolio optimization single out a number of deficiencies, (Kasimov, 2005) in the Markowitz model.

1. Groundlessness of the assumption of normal distribution of yields. Statistical observations show that distribution of assets yields is characterized by large probabilities of extreme deviations from the average, than it can be typical for normal distribution (i.e. it has so called "thick tails").

2. Covariation between yields of two assets is not a constant value and it changes in time, which, as a result, leads to the wrong assessment of the future risk of investment portfolio (Benati, 2003).

3. In practical calculations according to Markowitz model the future yield of securities is defined as an average arithmetical of a number of their historical yields. The said forecast doesn't include the influence of macroeconomic (the level of GDP, inflation, unemployment, branch indexes, etc.) and microeconomic (liquidity, profitability, company's financial sustainability) factors (Davnis et al., 2012).

4. The asset's risk in the Markowitz model is assessed on the basis of the measure of variability of the relatively of average value of the yield, but the deviation of the yield to a bigger side is not a risk but a super yield of the asset.

5. The Markowitz's model doesn't take into account many important factors and limitations (transaction costs, limitations on the assets shares).

The attempts to develop and improve the Markowitz model have generated a large number of scientific results in the sphere of modeling and optimization of securities portfolio. Particularly, in a number of papers the behavior of securities yield is described by the law of distribution, different from the normal one (Tanaka, Guo 1999), or by non-static methods (Kashirina et al., 2014). Some papers use other than Markowitz's approaches to the assessment of the yield and risk (Fusai, Luciano, 2001), (Kashirina et al., 2008). Some researchers while calculating the risk don't take into account the positive deviations from the average yield (Sortino Price 1994). Many papers take into account the additional limitations of the investor and the external environment (Chang T. J. et al., 2000). In some papers authors use indecipherable logics in order to take into account the indefiniteness of the expected risk and yield (Xia et al., 2000), (Liu, 1999).

Nevertheless, until now there is a number of unsolved problems, which decrease the quality of the portfolio optimization results, in particular:

1. In classical Markowitz model the problem of portfolio management is solved for one fixed period of investment. Such approach doesn't suit very much for long-term investments, as it doesn't allow to take into consideration the changes of the portfolio yield during the said period.
2. Mechanisms of portfolio optimization in many scientific studies are the modifications of the Markowitz's approach and they practically do not contain any scientific novelty.
3. Models, suggested by some authors, often do not contain any theoretical substantiation of the practical advantages in comparison with the classical approach.

That is why the development of the models, which do not contain the above mentioned deficiencies, is urgent.

3. Portfolio optimization model with consideration of profits reinvestment

Let us consider problem of the securities portfolio management for a long-term period, which, in its turn, consists of n short-term periods of investment. Alongside with this, it is assumed that profit, obtained from the investment in i-th short-term period is not withdrawn but reinvested in $(i+1)$-th period.

Let us indicate the portfolio yield in i-th short-term period with d_i, $i = 1, \dots, n$, i.e.

$$d_i = \frac{c_i - c_{i-1}}{c_{i-1}},$$

where c_i – portfolio value in i period, $i = 0, \dots, n$. Then the portfolio yield during n periods equals to

$$d = \frac{c_n - c_0}{c_0} = \frac{c_0 \prod_{i=1}^{n}(1+d_i) - c_0}{c_0} = \prod_{i=1}^{n}(1+d_i) - 1.$$

The value $1 + d_i$ is called capital growth rate in i-th period (ralph Vince, 2007), and value $T = 1 + d = \prod\limits_{i=1}^{n}(1+d_i)$ is an aggregate capital growth rate during all n periods (Yanovsky, Vladykin, 2009). The average arithmetic portfolio growth rate can be calculated by this formula:

$$T_{ca} = \frac{1}{n}\sum_{i=1}^{n}(1+d_i) = 1 + \frac{1}{n}\sum_{i=1}^{n} d_i, \tag{5}$$

and average geometric growth rate equals to

$$T_c = \sqrt[n]{\prod_{i=1}^{n}(1+d_i)}, \tag{6}$$

Let us explain practical meaning of the average geometric capital growth rate. If portfolio yield in each n short-term periods was equal to some constant value d_{cp}, then in order to receive aggregate capital growth rate during n periods, same as the earlier introduced value $T = \prod_{i=1}^{n}(1 + d_i)$, then the equation would have to be met: $(1 + d_{cp})^n = \prod_{i=1}^{n}(1 + d_i)$. Therefore, $T_c = 1 + d_{cp} = \sqrt[n]{\prod_{i=1}^{n}(1 + d_i)}$ (Shvedov, 1999).

Thus, maximum gain of the capital during n periods will ensure the portfolio, for which the value of the average geometric capital growth rate is maximal:

$$T_c = \sqrt[n]{\prod_{i=1}^{n}(1 + d_i)} \to \max, \tag{7}$$

Sustainability of such portfolio is evidently connected not with dispersion of yields d_i around their average value, but with the deviation of the average capital growth $(1 + d_i)$ from some constant growth rate $(1 + d_{cp})$ (Yanovsky, Vladykin, 2010). As for the portfolio with fixed growth rate there is the equation $T_{ca} = T_c$ (average arithmetic is equal to average geometric), then the risk connected with the instability of such portfolio can be assumed equal to the difference between mentioned values:

$$R = T_{ca} - T_c = 1 + \frac{1}{n}\sum_{i=1}^{n} d_i - \sqrt[n]{\prod_{i=1}^{n}(1 + d_i)}. \tag{8}$$

Due to the known mathematics ratio between average arithmetic and average geometric values, $R \geq 0$ as an alternative variant of the indicator of portfolio risk it is possible to use the value equal to the difference between 1 and ratio between average geometric and average arithmetic values:

$$R = 1 - \frac{T_c}{T_{ca}} = 1 - \frac{\sqrt[n]{\prod_{i=1}^{n}(1 + d_i)}}{1 + \frac{1}{n}\sum_{i=1}^{n} d_i}. \tag{9}$$

In this case $0 \leq R < 1$.

Now, let us now consider the objective of constructing the portfolio with optimal average geometric capital growth rate.

Supposing for the inclusion in the portfolio's regard there is k different financial instruments. Let's indicate the share of i asset in the investor's portfolio with x_i, $i = 1, 2, \ldots, k$, and the yield of i-asset in j-short-term period, $i = 1, 2, \ldots, k$, $j = 1, \ldots, n$. with d_{ij}. Then the average arithmetic capital growth rate can be calculated according to the formula:

$$T_{ca} = \frac{1}{n}\sum_{j=1}^{n}\left(\sum_{i=1}^{k}(1+d_{ij})x_i\right) = \frac{1}{n}\sum_{j=1}^{n}\left(\sum_{i=1}^{k}x_i + \sum_{i=1}^{k}d_{ij}x_i\right)$$
$$= \frac{1}{n}\sum_{j=1}^{n}\left(1+\sum_{i=1}^{k}d_{ij}x_i\right) = 1 + \frac{1}{n}\sum_{j=1}^{n}\sum_{i=1}^{k}d_{ij}x_i, \tag{10}$$

alongside with this, normalization of shares is taken into account: $\sum_{i=1}^{k} x_i = 1$. The formula for the calculation of the average geometric capital growth rate will acquire the view:

$$T_c = \sqrt[n]{\prod_{j=1}^{n}(\sum_{i=1}^{k}(1+d_{ij})x_i)} = \sqrt[n]{\prod_{j=1}^{n}\left(\sum_{i=1}^{k}x_i + \sum_{i=1}^{k}d_{ij}x_i\right)} = \sqrt[n]{\prod_{j=1}^{n}\left(1+\sum_{i=1}^{k}d_{ij}x_i\right)} \tag{11}$$

Thus, the objective of the portfolio formation with maximum geometric capital growth rate during n short-term periods can be put down like this:

$$T_c = \sqrt[n]{\prod_{j=1}^{n}\left(1+\sum_{i=1}^{k}d_{ij}x_i\right)} \to \max, \tag{12}$$

$$R = 1 - \frac{\sqrt[n]{\prod_{j=1}^{n}\left(1+\sum_{i=1}^{k}d_{ij}x_i\right)}}{1+\frac{1}{n}\sum_{j=1}^{n}\sum_{i=1}^{k}d_{ij}x_i} \le R_{\max}, \tag{13}$$

$$\sum_{i=1}^{k}x_i = 1, \tag{14}$$

$$x_i \ge 0, i = 1, 2, \ldots, k. \tag{15}$$

Here $R_{\max}$ is a parameter, which limits the portfolio maximum risk level. Instead of the limitation (13) it is also possible to use the alternative limitation (13), based on the application of the formula (8):

$$R = 1 + \frac{1}{n}\sum_{j=1}^{n}\sum_{i=1}^{k}d_{ij}x_i - \sqrt[n]{\prod_{j=1}^{n}\left(1+\sum_{i=1}^{k}d_{ij}x_i\right)} \le R_{\max} \tag{13a}$$

But for practical calculation the limitation appeared to be more convenient (13) as the risk in this case is an unlimited value, taking values from the segment [0.1].

Table 1: Portfolio, obtained on the basis of Markowitz model.

VSMPO	Diksi	Video	GUM	Magnit	Apple	MTS	NMLK	Rosneft	Sberbank
x1	x2	x3	x4	x5	x6	x7	x8	x9	x10
0.215	0	0	0	0.605	0.18	0	0	0	0

Table 2: Portfolio, obtained on the basis of model (12)-(15).

VSMPO	Diksi	Video	GUM	Magnit	Apple	MTS	NMLK	Rosneft	Sberbank
x1	x2	x3	x4	x5	x6	x7	x8	x9	x10
0.341	0	0	0	0.559	0.1	0	0	0	0

4. Results

Despite the principle differences in the approaches to the portfolio formation, the calculation experiment showed that the portfolios formed on the basis of model (12)-(15), in which a long-term period of investment is divided into n short-term periods, are close in their composition to the portfolios, obtained on the basis of the Markowitz model. Thus, Table 1 provides the structure of the portfolio, obtained on the basis of the Markowitz model with the target function, which expresses the average monthly portfolio yield. Alongside with this, the value of the portfolio risk, expressed by the average quadratic deviation of its yield, was limited by the value 0.1.

Table 2 provides the structure of the portfolio, obtained on the basis of the model (12)-(15) by 12 short-term periods, each of which has the duration of one month. Herewith, we used the value equal to 0.01in order to limit the portfolio risk, expressed by the ratio between average arithmetic and average geometric growth rate by formula (13).

It was experimentally noted that the portfolios, formed on the basis of the Markowitz model and models (12)-(15), are close in their composition, if the ratio of the maximum risks set for them is in the limits $12 \div 15$.

In order to more vividly understand the specificity of the model (12)-(15), let's examine the example of the formation of the portfolio with two assets, for which we know the growth rates in two short-term periods.

Table 3 demonstrates, that despite the fact that asset II growth rates differ (they are

Table 3: Example 1: Initial data for the portfolio of two assets.

Month	Asset I value	Assets II value	Asset I growth rates	Asset II growth rates
1	10	5	-	-
2	20	15	2	3
3	40	30	2	2
Average arithmetic asset growth rate			2	2.5
Average geometric asset growth rate			2	2.45

Table 4: Portfolio structure depending on the set value of maximum risk.

Model (8)-(11)			The Markowitz model		
Rmax	x1	x2	Rmax	x1	x2
>0.03	0	1	>0.5	0	1
0.02	0.006	0.994	0.45	0.1	0.9
0.01	0.343	0.657	0.3	0,4	0.6
0.001	0.812	0.188	0.1	0,8	0.2
0.0001	0.942	0.058	0.01	0.98	0.02

equal to correspondingly 3 and 2) in the first and second short-term periods, the average geometric growth rate of the said asset differs slightly from its average arithmetic growth rate, which means low risk valuefor model (12)-(15), connected with investment in this asset. But asset I has permanent growth rate, i.e. its risk assessment in model (12)-(15) is zero. Alongside with this, asset II seems to be more attractive for investment by model (12)-(15), as its average geometric growth rate is considerably higher than that of the first asset (2.45 against 2.0). Experimental calculation according to model (12)-(15), presented in Table 4, shows that asset 1 will be included in portfolio only with rigid enough limitation of the portfolio risk, and it will be impossible to form the portfolio only with the first asset with zero limitation of the risk. For comparison the same table also presents the calculations for portfolio formation on the basis of the Markowitz model (on the basis of the same initial data). It is possible to note that in Markowitz model the risk of the first asset is also zero (as its yield is constant), but it is also impossible to form the portfolio of only the first asset with zero limitation of risk.

Despite the general similarity of the results, received with the help of models (12)-(15) and Markowitz's model it is easy to give the example in which model (12)-(15) provides a more rational result. According to the initial data, presented in table 5, asset I looks more attractive for investment according to the Markowitz model as its average yield is higher than that of asset II. For the model (12)-(15), on the contrary,asset IIis more attractive, as its average geometric growth rate is higher, than that of asset I. Alongside with this, the results of the model (12)-(15) seem more substantiated, as by the results of the two periods the value of asset I dropped by 15%, while the value of the asset II, on the contrary, increased by 15%.

In fairness, it should be noted that the dispersion of the asset I yield in this example is higher than that of asset II and that is why with the corresponding limitation for maximum portfolio risk the Markowitz model would also give the preference to asset II.

5. Results discussion

Some theoretical explanation of the similarity of the results of model (12)-(15) and that of Markowitz can be seen on the example of the portfolio, built for two short-term periods, for which average geometric capital growth rate,calculated by formula (2), equals $T_{c\text{Ð}^{3}} = \sqrt{(1+d_1)(1+d_2)}$. Let us indicate average arithmetic value of yields

Table 5: Example 2: Comparison of initial data for the portfolio of two assets.

Month	Asset I value	Asset II value	Asset I growth rates	Assets II growth rates	Asset I yield	Asset II yield II
1	20	20	-	-	-	-
2	10	21	0.5	1.05	-0.5	0.05
3	17	23	1.7	1.095238	0.7	0.095238
Average yield			-	-	0.1	0.072619
Average geometric growth rate			0.921954	1.072381	-	-

during two periods with d, i.e. $d = \dfrac{d_1 + d_2}{2}$. Then the average quadratic deviation of the yield during mentioned periods will be the value:

$$\sigma = \sqrt{\frac{1}{2}\left(d_1 - \frac{d_1 + d_2}{2}\right)^2 + \frac{1}{2}\left(d_2 - \frac{d_1 + d_2}{2}\right)^2} = \sqrt{\left(\frac{d_1 - d_2}{2}\right)^2} = \left|\frac{d_1 - d_2}{2}\right| \tag{16}$$

Assuming for certainty that $d_1 \geq d_2$, then the yields can be presented as: $d_1 = d + \sigma$, $d_2 = d - \sigma$.

Thus, we can write down, that

$$T_c = \sqrt{(1 + d_1)(1 + d_2)} = \sqrt{(1 + d + \sigma)(1 + d - \sigma)} = \sqrt{(1 + d)^2 - \sigma^2} \tag{17}$$

From formula (17) it is evident, that average geometric capital growth rate will increase with the growth of the average portfolio yield and it will decrease with the growth of its dispersion, which corresponds to the main principles of the Markowitz model.

Now, let's calculate portfolio risk magnitude by formula (9):

$$R = 1 - \frac{\sqrt{(1 + d)^2 - \sigma^2}}{1 + d} \leq R_{\max} \tag{18}$$

Hence:

$$1 - R_{\max} \leq \sqrt{\frac{(1 + d)^2 - \sigma^2}{(1 + d)^2}} \Rightarrow \frac{\sigma^2}{(1 + d)^2} \leq R_{\max}^2 + 2R_{\max}.$$

With regard of non-negativity of the values $(1 + d)$ and σ we finally receive:

$$\sigma \leq (1 + d)\sqrt{R_{\max}^2 + 2R_{\max}} \tag{19}$$

Thus, limitation (18) connects average quadratic deviation of the portfolio σ (risk according to Markowitz) with average arithmetic capital growth rate $(1 + d)$, meaning, that risk revealed according to the Markowitz model, should not grow faster, than the average capital growth rate, multiplied by the constant, calculated depending on the set value of the maximum risk $R_{\max}$.

Although it should be noted once again that this vivid illustration is given for the case with two short-term periods.

6. Conclusion

The article examines the assets portfolio management model, based on the maximization of the average geometric capital growth rate during the investment period. The suggested model of portfolio optimization is alternative to the traditional approach of H. Markowitz (Markowitz, 1990). The model differs by taking into account the changes in portfolio yield and risk during the period of investment and by a different approach to the assessment of the yield and risk of securities. By the results of the calculation experiment we made the conclusion that model (12)-(15), in general, gives results similar to the results of the Markowitz model but there are examples of the initial data, for which results of model (12)-(15) seem to be more preferable. The direction of further research can be the consideration of certain cases of portfolio optimization with set limitations (consideration of limitations by the share of securities in the portfolio, consideration of transaction costs etc.).

References

[1] Barkalov, S.A., Ereshko, F.I., Kanaeva N.A. (2014). Diversification models analysis. *Sistemy Upravlenia i Informatsionnye Tekhnologii.*, V. 55, #1.1. P. 112–117.

[2] Benati, S. (2003), The optimal portfolio problem with coherent risk measure constraints. EuropeanJournalofOperationalResearch. #150. P. 572–584.

[3] Berkolaiko, Z., Russman, I.B. (2004), On some methods of assets portfolio formation and management. Part 1. *Ekonomicheskaya nauka sovremennoi Rossii*. #1. P. 18–32.

[4] Berkolaiko, Z., Russman, I.B. (2004), On some methods of assets portfolio formation and management. Part 2. *Ekonomicheskaya nauka sovremennoi Rossii.* #1. P. 25–36.

[5] Chang T.J., Meade N., Beasley J.E., Sharaiha Y.M. (2000), Heuristics for cardinality constrained portfolio optimization, Computers & Operational Research. #27. P. 1271–1302.

[6] Davnis, V. V., Satkin, S., Fetisov Vł., (2013), Optimal portfolio of securities in the conditions of globalization: approaches and models. *Sovremennaya ekonomika: problem i resheniya*. # 8 (44). P. 134–146.

[7] Davnis V. V., Satkin, S., Razinsky Yu. (2012), Problems of real investment on the basis of portfolio decisions. *Sovremennaya ekonomika: problem i resheniya.* # 11 (35). P. 149–155.

[8] Fusai G., Luciano E. (2001), Dynamic value at risk under optimal and suboptimal portfolio policies. European Journal of Operational Research. #135. P. 249–269.

[9] Simov Yu. F. (2005), Introduction in the theory of optimal portfolio of securities. 224 p.

[10] Kashirina, I. L., Berkolaiko, M. Z., Ivanova K. G. (2008), Application of Russman's D grades for assets portfolio management. *Vestnik VGU, Series: Sistemny Analizi Informatsionnye Tekhnologii*, #1P. 102–110.

[11] Kashirina, I. L. (2014), Securities portfolio management on the basis of the prognostic methods of reaching boundary conditions in dual calculation environment. *Ekonomika I Menedzhment System Upravlenia.* V.1. #1. P. 032–039.

[12] Liu L. (1999), Approximate portfolio analysis. *European Journal of Operational Research.* #119. P. 35–49.

[13] Markowitz H. (1952), Portfolio Selection. *Journal of Finance*. #7. P. 77–91.

[14] Markowitz H. (1990), Mean Variance Analysis in Portfolio Choice and Capital Markets. Basil. Blackwell.

[15] Ralph Vince (2007), The handbook of portfolio mathematics: formulas for optimal allocation & leverage. Hoboken, New Jersey: *Wiley & Sons, Inc.* 422 p.

[16] Shvedov, A. (1999), Theory of efficient portfolios of securities. M., Published by GU VShE. 144 p.

[17] Sortino F., Price L. (1994), Performance Measurement in a Downside Risk Framework, *Journal of Investing*. Vol. 3, # 3. P. 59–65.

[18] Tanaka H., Guo P. (1999), Portfolio selection based on upper and lower exponential possibility distributions. *European Journal of Operational Research. #114. P. 115–126.*

[19] Terry J. Watsham, Keith Parramore (1996), *Quantitative Methods in Finance*, Cengage Learning EMEA. 395 p.

[20] Xia Y., Liu B., Wang S., Lai K.K. (2000), A model for portfolio selection with order of expected returns. *Computers & Operational Research*. #27. P. 409–422.

[21] Yanovsky, L. P., Vladykin S. N. (2009), SARM theory and portfolio analysis taking into account the investment horizon and optimal rates of capital growth. *Vestnik of the Russian State University of Trade and Economics (RGTEU)*, #7. P. 98–103.

[22] Yanovsky, L. P., Vladykin S. N. (2009), Portfolio selection taking into account investment horizon. *Finansy i Kredit.* 2009. #29 (365). P. 12–18.

[23] Yanovsky, L. P., Vladykin S. N. (2010), On the connection between the selection of portfolio refinancing strategy and the investor's risk avoidance level. *Sovremennaya ekonomika: problem i resheniya.* # 1. P. 139–148.