Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 12, Number 3 (2016), pp. 1947–1964

http://www.ripublication.com/gjpam.htm

Portfolio risks of bivariate financial returns using copula-VaR approach: A case study on Malaysia and U.S. stock markets

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Abstract

The recent financial turmoil which causes the financial markets to react in a non-linear way has led to a renewed interest in the modeling of portfolio dependence and risk. Risk can be measured by the traditional VaR measures such as normal VaR and historical simulation. However, it is challenging to estimate the portfolio VaR via parametric methods because of the complexity of modeling the joint multivariate distribution of the assets in the portfolio. Copula model is an alternative method that is able to account for the joint multivariate distribution. The purpose of this study is to evaluate the risks of equally and mixed weighted portfolios of the SP500 and KLCI returns using the VaR based copula (copula-VaR) approach. Comparisons between the copula-VaR estimates with the traditional VaR measures were also conducted. This study reveals that the marginal distribution of the SP500 and KLCI return series can be modeled by the ARMA-GARCH models, while the dependence structure between both indices can be described by the Clayton copula. The backtesting results indicate that the copula-VaR provide better estimates of the portfolio risks compared to the normal VaR and historical simulation. Our study also found that the VaR models produce a more accurate risk estimates when a less volatile asset has a higher investment fraction in the portfolio.

AMS subject classification: 62P05.
Keywords: Copula, Value-at-risk, Backtesting.

1. Introduction

Value-at-risk (VaR) is an industry standard for market risk measure which was first exposed in the 1990s. VaR provides the worst expected loss at a specified confidence level over a specified time horizon. For example, a daily (or weekly monthly/quarterly) VaR of USD 5 million at the 99 percent confidence level indicates that there is 1 percent chance of the occurrence of a loss greater than USD 5 million on the next day (or week/month/quarter). The greatest advantage of VaR is that it can summarize risks in a single number [1]. Portfolio managers can also use VaR to determine the marginal contribution of each asset to the portfolio risks, and thus, providing indication of the expected loss if any asset is removed from the portfolio [2].

VaR can be constructed on the basis of normality assumption and is generally used in Basel I and Basel II [3]. This VaR computation is also called the parametric VaR (or the normal VaR) and requires the normal assumption of the return (or loss) distribution. However, the financial returns (or losses) often exhibit leptokurtic distribution and hence, the normal VaR may become an inappropriate measure of risk. An alternative to the parametric VaR is the nonparametric VaR that is based on historical simulation. The main concept of the nonparametric VaR methodology is that the future losses can be predicted based on past performance [4].

According to Fantazzini [5], the estimation of portfolio VaR may become difficult when the method of parametric VaR is used due to the complexity of modeling the joint multivariate distribution of the assets in the portfolio. The concept of copula has recently been used as an alternative to measure the dependence between two or more variables. The copula method gives more flexibility by modeling the univariate return series separately from their dependence, besides providing the joint multivariate distribution function of the portfolio. Hence, by using the joint multivariate distribution modeled by a copula, we attempt to evaluate the risks of both the equally and mixed weighted portfolios in this study.

Through a Monte Carlo study, Fantazzini [5] reported that the type of copula was not a fundamental aspect in obtaining a good VaR forecast and suggested that the specifications of the generalized autoregressive conditional heteroscedastic (GARCH) for the variance were able to provide a more precise VaR estimates. On the other hand, Aloui et al [6], who performed the extreme-value copula-VaR approach on the equally-weighted portfolios of the bivariate stock returns using GARCH models as the marginal distributions, found that the copula-based VaR model outperformed the parametric and historical simulation methods. The copula-based VaR model also provided improvement in the accuracy of the out-of-sample VaR forecasts for energy portfolios [7]. Another study revealed that the VaR model based on the asymmetric copula outperformed the historical simulation method [8].

Based on the authors' knowledge, there are limited studies that use the copula-VaR methodology for evaluating portfolio VaR. There are also mixed views regarding the ability of the method in providing a better VaR estimates and forecasts of the portfolios. Therefore, the aims of this paper are to apply the copula-VaR approach using GARCH models for evaluating portfolio risks, modeling the dependence of the U.S. and Malaysia stock market indices and observing their tail dependencies. The performance of the copula-VaR over the traditional VaR measures is further investigated. The backtesting of the portfolio VaR is also performed to check the accuracy of the VaR estimates.

The remaining part of this article proceeds as follows: 2) brief theory on copula and its families; 3) methodological procedures; 4) empirical results accompanied with discussion; and 5) conclusion.

2. Copula

A copula is a dependence function that allows the characterization of independence and perfect dependence in a straightforward way [9]. Consider a vector of two variables with a univariate marginal distribution function for each variable. The Sklar's theorem, which was introduced in 1959, states that there exists a copula that links these marginal distributions into a multivariate joint distribution of a vector, which can be mathematically expressed as:

$$H(x, y) = C\left[F(x), G(y)\right] \tag{1}$$

where H denotes the 2-dimensional multivariate distribution function with a range of [0,1], C represents the copula, and F and G are the univariate marginal distribution functions of random variables X and Y, respectively. Equation (1) can be inverted to a copula function in terms of a joint distribution function where the inverses of the two margins are strictly increasing [10]:

$$C(u, v) = H\left[F^{-1}(u), G^{-1}(v)\right] \tag{2}$$

where F^{-1} and G^{-1} are the quasi-inverses of F and G, respectively. The mathematical proofs of equation (2) can be seen in Nelson [10].

Consider a portfolio with two financial assets. The joint distribution function of the two financial variables can be decomposed into univariate marginal distributions of the two variables and a copula, and thus, allowing the marginal distributions to be modeled separately from the dependence structure. Earlier studies used the unconditional approach [11],[12] such as the empirical distribution and simple distribution fitting. Our study uses time series models as the marginal distributions of the financial data which are similar to the recent studies implemented in [13],[14],[15]. The dependence structure is described by one of the copula models from the families of Elliptical, Archimedean and extreme-value copulas.

The Gaussian and Student's t copulas, which belong to the Elliptical copula family, share a common property where the distribution of the multivariate data is symmetric. The Gaussian copula has the following function:

$$C_\rho(u,v) = \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi(1-\rho^2)^{1/2}} \exp\left(-\frac{x^2 - 2\rho xy + y^2}{2(1-\rho^2)}\right) dxdy \tag{3}$$

where ρ is the copula parameter and $\Phi^{-1}(.)$ is the inverse of a standard univariate Gaussian distribution function. The relationship between copula parameter ρ and Kendall's τ can be mathematically expressed as $\rho = sin\left(\frac{\pi\tau}{2}\right)$. The Gaussian copula focuses on the central dependence rather than the tails dependence.

On the other hand, the Student's t copula incorporates the tail dependence at both the upper and lower tails. The Student's t copula has another parameter, ν, which is the degrees of freedom that affects the strength of the tail dependence. As ν approaches infinity, the multivariate data distribution converges to the Gaussian copula distribution. The function for the Student's t copula is:

$$C_{\rho,\nu}(u,v) = \int_{-\infty}^{t_\nu^{-1}(u)} \int_{-\infty}^{t_\nu^{-1}(v)} \frac{1}{2\pi(1-\rho^2)^{1/2}} \left(1 + \frac{x^2 - 2\rho xy + y^2}{\nu(1-\rho^2)}\right)^{-\frac{\nu+2}{2}} dxdy \tag{4}$$

In finance literature, an increasing ν implies that the tendency of the presence of extreme comovements decreases. The Student's t copula is often regarded as a dominant copula in modeling the non-linear and non-normal dependences.

The Clayton copula belongs to the Archimedean family that captures only the lower (or left) tail dependence and has an asymmetric dependence structure. The simple closed form of the Clayton copula function with $\alpha \in [-1, 0) \cup (0, \infty)$ is:

$$C_\alpha(u,v) = (u^{-\alpha} + v^{-\alpha} - 1)^{-\frac{1}{\alpha}} \tag{5}$$

where α denotes the copula parameter that controls the dependence. While $\alpha \to 0$ implies independence between the two random variables, perfect dependence is obtained if α approaches infinity [16]. The copula parameter, α , is related to the Kendall's τ through $\alpha = 2\tau/(1-\tau)$. The Clayton copula is a preferable choice for dependence structure especially in cases of stress conditions (i.e. financial turmoil).

Another member of the Archimedean copula is the Gumbel copula. Unlike the Clayton copula, the Gumbel copula exhibits greater dependence only at the right (or upper) tail. The simple closed form function of the Gumbel copula with $1 \leqslant \alpha < \infty$ is:

$$C_\alpha(u,v) = \exp\left[-\left[(-\log u)^\alpha + (-\log v)^\alpha\right]^{\frac{1}{\alpha}}\right] \tag{6}$$

where α denotes the copula parameter that controls the dependence. The relationship between Gumbel's copula parameter, α, and Kendall's τ is $\alpha = 1/(1-\tau)$.

We also consider fitting the Frank, Galambos and Husler-Reiss copulas to the financial data. The Frank copula is an Archimedean copula that exhibit weak dependence between the extreme values. The distribution function of the Frank copula is:

$$C_\alpha(u,v) = -\frac{1}{\alpha}\ln\left(1+\frac{(e^{-\alpha u}-1)(e^{-\alpha v}-1)}{e^{-\alpha}-1}\right) \tag{7}$$

where $\alpha \in (-\infty, 0) \cup (0, \infty)$. The relationship between Frank's copula parameter and Kendall's τ is:

$$\tau = 1 - \frac{4}{\alpha}\left[1 - \frac{1}{\alpha}\int_0^\alpha \frac{t}{e^t-1}dt\right] \tag{8}$$

The Galambos copula, which was introduced by Galambos in 1975, is an extreme-value copula that exhibits greater dependence at the upper (or positive) tail. Its copula function is:

$$C_\alpha(u,v) = uv\exp\left[\left[(-\log u)^{-\alpha} + (-\log v)^{-\alpha}\right]^{-\frac{1}{\alpha}}\right] \tag{9}$$

with $0 \leqslant \alpha < \infty$.

In 1987, Husler and Reiss introduced the Husler-Reiss copula that also captures the upper tail dependence. Its copula function is:

$$C_\alpha(u,v) = \exp\left[-(\ln(u))\phi\left[\frac{1}{\alpha}+\frac{1}{2}\alpha\ln\left(\frac{\ln u}{\ln v}\right)\right] + (\ln(v))\phi\left[\frac{1}{\alpha}+\frac{1}{2}\alpha\ln\left(\frac{\ln v}{\ln u}\right)\right]\right] \tag{10}$$

with $0 \leqslant \alpha < \infty$.

As discussed previously, each copula family has its own characteristic of tail dependence. The tail dependence can be used as a dependence measure for the lower and upper quadrants of a distribution. The concept of tail dependence is therefore vital for risk managers especially in guarding against risky and unwanted events. Let X and Y be the continuous random variables with F and G distribution functions, respectively. The lower tail coefficient, λ_L, measures the probability of observing small Y values given that X is small:

$$\lambda_L = \lim_{t\to 0} P\left[Y \leqslant G^{-1}(t) | X \leqslant F^{-1}(t)\right] \tag{11}$$

whereas the upper tail coefficient, λ_U, measures the probability of observing large Y values given that X is large:

$$\lambda_U = \lim_{t\to 1} P\left[Y > G^{-1}(t) | X > F^{-1}(t)\right] \tag{12}$$

Table 1 lists the tail dependence coefficients for each copula family that are commonly used in academic research papers.

Table 1: Tail dependence characteristics of copula families.

Copula family	Parameter range	Lower Tail, λ_L	Upper tail, λ_U
Gaussian	$\rho \in (-1, 1)$	0	0
t	$\rho \in (-1, 1)$	$2t_{\nu+1}\left(-\sqrt{\nu+1}\sqrt{\frac{1-\rho}{1+\rho}}\right)$	λ_L
Clayton	$\alpha \in [-1, 0) \cup (0, \infty)$	$2^{-1/\alpha}$	0
Gumbel	$\alpha \in [1, \infty)$	0	$2 - 2^{-1/\alpha}$
Frank	$\alpha \in (-\infty, 0) \cup (0, \infty)$	0	0

3. Methodology

When handling financial data, the foremost statistical analyses conducted are the descriptive statistics such as line plots, measures of central tendency and dispersion, and autocorrelation functions plots. It is important to observe the nature of data before fitting any statistical models. Financial data such as stock prices often have non-constant variance, skewed distribution and fat tails. In addition, the financial data are sometimes not independently and identically distributed (iid). Therefore, the data should be fitted to a time series model if iid problem exists.

The marginal distributions should be identified first before the data can be fitted to the dependence model (copula model). In this study, the marginal distributions are represented by the time series models which are fitted to each return series. The potential time series models can be detected by looking at the autocorrelation plots. The autoregressive (AR), moving average (MA) or autoregressive moving average (ARMA) models are potential models that can be fitted if the data have some serial correlation in the observations and no serial correlation in the squared or absolute observations. However, if the data has high serial correlation in the squared or absolute observations, the autoregressive conditional heteroscedastic (ARCH) or the generalized ARCH (GARCH) models would be more appropriate for capturing the volatility in the data.

The scope of this study covers the standard GARCH and ARMA-GARCH models with normal and non-normal error distributions. The conditional mean and variance equations of the ARMA-GARCH model with Gaussian error distribution are shown in equations below:

$$Y_t = \mu_t + \epsilon_t \tag{13}$$

$$\mu_t = \mu_0 + \sum_{i=1}^{p} \phi_i (Y_{t-i} - \mu_0) + \sum_{i=1}^{q} \theta_i \epsilon_{t-i} \tag{14}$$

$$\epsilon_t = \sigma_t X_t \tag{15}$$

$$X_t \sim N(0, 1) \tag{16}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{P} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{Q} \beta_i \sigma_{t-i}^2 \tag{17}$$

where Y_t denotes the returns of a financial asset at the t-th day, and ϕ_i and θ_i are the autoregressive and moving average parameters, respectively. In order to obtain positive conditional variance and stationarity, it is necessary to impose the following conditions to the GARCH parameters: $\omega, \alpha_i, \beta_i > 0$ and $\sum_{i=1}^{P} \alpha_i + \sum_{i=1}^{Q} \beta_i < 1$. The specifications of ARMA and GARCH models for each return (or loss) series and the distributions assumed for the innovations or errors may differ depending on the financial data.

Besides the normal (or Gaussian) distribution, we also consider the skewed normal, Student's t, skewed Student's t, generalized error (GED) and skewed GED distributions for the innovative distribution of X_t. The Student's t and GED distributions have the shape parameter of ν and κ , respectively, while the skewed version of all distributions has a skewed parameter of ξ.

The best-fit time series model can be selected by conducting several diagnostic tests. First, the QQ plots of residuals are observed to examine if the series follow the assumed innovative distribution. The Ljung-Box test for the serial correlations in residuals and squared residuals is applied if the visual inspection provides unclear decision. The null hypothesis for the Ljung-Box test is there is no autocorrelation in the residual series. Further time series modeling with high-order lags is required if the serial correlation in residuals is present. Next, the Lagrange Multiplier (LM) test is conducted for assessing the presence of heteroscedastic errors. Further GARCH modeling is required should heteroscedastic errors remains present. Finally, information criterion such as the Akaike Information Criterion (AIC), Bayesian Information criterion (BIC), Shibata Information Criterion (SIC) and Hannan-Quinn Information Criterion (HQIC) are used for selecting the best-fit model. The best-fit time series models, which capture the volatility in the SP500 and KLCI and have the smallest information criterion, are then applied to represent the marginal distribution of each SP500 and KLCI.

The dependence modeling can be conducted once the best marginal models have been identified. The standardized residuals from each marginal model, X_i , are first converted to pseudo observations using $u_i = Rank(X_{1,i})/(n+1)$ and $v_i = Rank(X_{2,i})/(n+1)$ for the SP500 and KLCI. These transformations are required because the margins in the copula model are uniform over the interval of 0 to 1 inclusively.

The copula parameters can be computed by inverting the Kendall's τ. The latter is a copula-based measure described as:

$$\tau = -1 + 4 \int_{[0,1]^2} C(u, v) dC(u, v) \tag{18}$$

The formulas relating the Kendall's τ and copula parameter have been provided in the previous section.

The best copula model is selected using the goodness-of-fit test of Cramer Von Mises (S_n) statistics which measures the closeness of the empirical copula C_n compared to the fitted copula C_{θ_n}. The null hypothesis for this test is that the data is distributed according to the assumed copula family. The formula of (S_n) statistic is:

$$S_n = \sum_{i=1}^{n} \left[C_n(u_i, v_i) - C_{\theta_n}(u_i, v_i) \right) \tag{19}$$

Besides the goodness-of-fit measure, the tail dependence coefficients are also computed to assess the strength of the dependence of the extreme negative and/or positive returns of the SP500 and KLCI. The formula for the tail dependence coefficients can be seen in Table 1.

The value-at-risk (VaR) of a portfolio based on the ARMA-GARCH-copula model (hereafter referred to as the copula-VaR) is then computed. The following steps are conducted to obtain the simulated returns series which are used in the estimation of the portfolio risk:

1. Simulate uniform variates from the best-fit copula model.

2. Transform the variates into standardized residuals.

3. Compute the returns of each SP500 $(Y_{1,t})$ and KLCI $(Y_{2,t})$ using the standardized residuals and the conditional mean and variance terms observed in the original series.

Consider a one-period global portfolio with two assets, $R = wY_1 + (1 - w)Y_2$, where Y_1 and Y_2 are the two assets and w is the fraction invested in asset 1. The VaR of the simulated portfolio at q confidence level is equal to the $(n \times q)$th percentile of the distribution of R , mathematically expressed as,

$$VaR_q = F_{-R}^{-1}(q) \tag{20}$$

The accuracy of the copula-VaR estimates of the portfolio is then ensured by applying the Kupiec's proportion of failures (POF) test where the null hypothesis is that the number of exceptions observed in a sample size T follows a binomial distribution with parameter T and α. The Kupiec's Likelihood Ratio test statistics complies with the Chi-square distribution with 1 degree of freedom. The copula-VaR model provides good estimates if the null hypothesis of the Kupiec's test is not rejected. It should be expected that the number of exceedances from the copula-VaR is closer to its expected number of exceedances.

The violation ratios (VR) are also computed to check the performance of the VaR model by dividing the expected number of exceedances in the forecasted period with the total number of actual exceedances. The VaR model is invalid if the VR is lower than 0.5 or exceed 1.5 [4].

4. Empirical Results and Discussion

4.1. Data

The dataset is obtained from Bloomberg and consists of the daily stock price indices of FTSE Bursa Malaysia Kuala Lumpur Composite Index (KLCI) and Standard and Poor's 500 Index (SP500) for the period of January 2000 to December 2012. These indices are often used in academic researches as proxies for the financial stock market in their respective countries. The original data is transformed using log difference, $Y_t = \log P_t - \log P_{t-1}$, where P_t represents the price at the t-th trading day. Figures 1 and 2 show the time series plots of SP500's and KLCI's prices respectively, together with their returns, squared returns series, autocorrelation function (ACF) of returns and ACF squared returns.

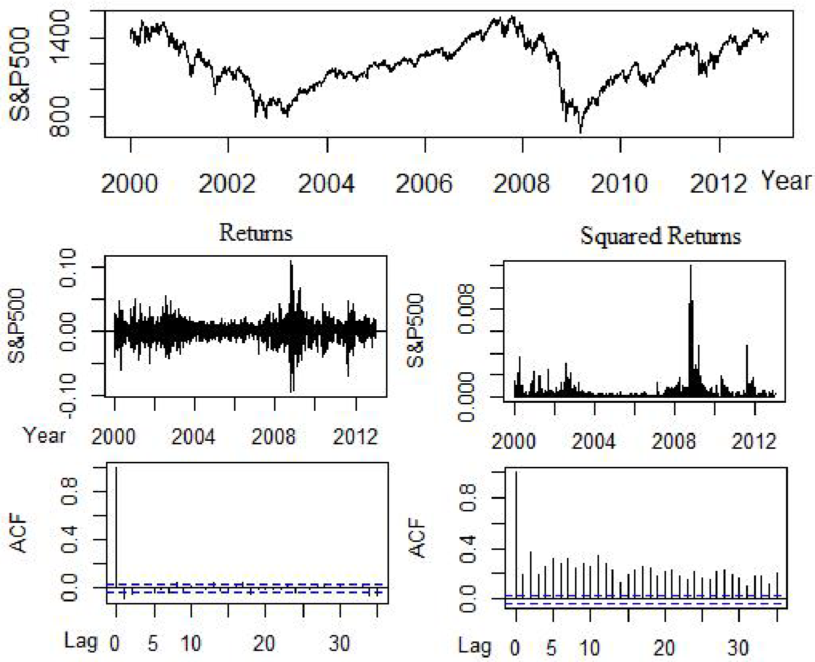

Figure 1: Time Series Plots of SP500's price, returns and squared returns series and autocorrelation function (ACF) plots of returns and squared returns.

Several volatility clustering are observed in Figure 1. The volatility cluster that occurred in 2000-2002 was the Dotcom bubble burst, while in 2007-2009, the housing bubble and credit crisis in the U.S. and U.K. had caused severe impact to the SP500 index. The incident of the 'August 2011 stock market fall' shook the SP500 index, but the shock was less severe than the previous financial disaster. In terms of statistical perspective, the average return rate is constant over time while the volatility is varying over time. Based on the ACF plot, the returns of SP500 has less serial autocorrelation. However,

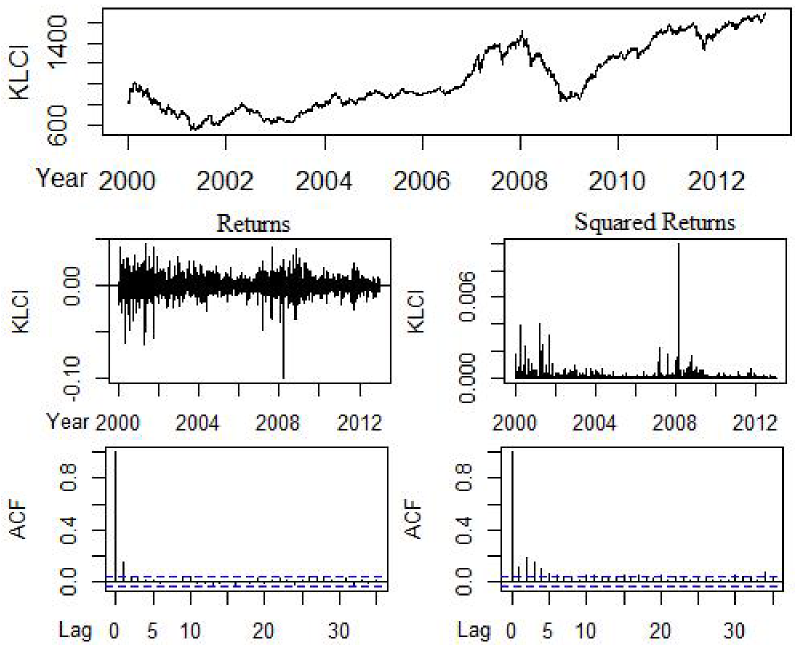

Figure 2: Time Series Plots of KLCI's price, returns and squared returns series and autocorrelation function (ACF) plots of returns and squared returns.

high serial correlation were detected for the squared returns, and thus, confirming the existence of volatility clusters.

The Dotcom bubble burst also affected the performance of KLCI in 2000-2002 as can be observed in Figure 2. The volatility clustering can also be seen in the line plot of squared returns in Figure 2. The global financial crisis in 2008 which rose from the credit crisis in the U.S. caused the highest difference between the daily price index. However, the volatility of returns lasted quickly afterwards. The 'August 2011 stock market fall' did not significantly affect the KLCI performance in 2011 as compared to the SP500. Referring to the ACF plot for squared residuals, the moderate serial correlation implies that the KLCI returns have less volatility clusters.

Table 2 provides the statistics of daily return series including the mean, median, standard deviation (SD), skewness and kurtosis. The statistical values from the Jarque-Bera (JB) test for normality and the Augmented Dickey Fuller (ADF) test for unit root are also shown. The correlation between KLCI and SP500 is measured using a non-parametric correlation measure.

The SP500 has a negative average return rate with a larger standard deviation, while the KLCI has a positive mean returns but a smaller standard deviation. In terms of skewness and kurtosis, both the KLCI and SP500 have left skewed return distributions and excess kurtosis. These statistical findings imply that both stock indices have more positive returns than extreme negative returns. The JB and ADF statistics for both series are significant at 1 percent, indicating that the rejection of hypotheses that returns are normally distributed and non-stationary. The correlation value which is positive but close

Table 2: Summary statistics of daily returns series.

Property	KLCI	SP500
Mean	2.08189e-04	−5.94411e-06
Median	5.699e-05	1.632e-04
SD	0.0087060	0.0132625
Skewness	−0.8873386	−0.1606828
Kurtosis	10.348514	7.707455
JB statistic	15571.57	8405.525
ADF statistic	56.8516	63.0855
Correlation	0.0413	

to zero indicates that the relationship between the SP500 and KLCI is weak. In general, the empirical properties of the KLCI and SP500 returns conform to the general stylized facts described by McNeil, Frey and Embrechts [17] and Pfaff [18].

4.2. Marginal models and dependence estimation

Since the returns are not independently and identically distributed, the time series models are utilized for modeling the marginal distribution of each return series. The returns series are fitted to the univariate time series models before applying the copula model. The results of ARMA-ARCH models are not shown here but they are available upon request. In brief, the ARMA-ARCH models are not suitable to capture the volatility clustering in the return series. The ARCH errors are still present in the residuals, and hence, the ARMA-GARCH models are applied to the return series. Tables 3 and 4 present the ARMA-GARCH parameter estimates, diagnostic tests and information criterion for the KLCI and SP500 returns series, respectively.

The results in Table 3 show that all parameters are significant at 1 percent level, and the ARMA(1,0)-GARCH(1,1) model with generalized error distribution (GED) has the lowest information criterion. However, this model does not provide a good fit for the KLCI because of the highly significant Ljung-Box Q-statistic of the residuals at lags 10 and 20. The ARMA(1,0)-GARCH(1,1) with skewed GED distribution is also rejected due to similar reasons. The ARMA(1,0)-GARCH(1,1) model with normal and skewed normal distributions are rejected because the residuals deviates from the assumed distribution in their QQ-plots. Following Ruppert [19] who stated that small autocorrelations are often statistically significant for large sample data, the ARMA(1,0)-GARCH(1,1) with Student's t distribution is selected as the marginal model for KLCI.

Although the results are not shown here, the ARMA(1,2)-GARCH(1,1) model is fitted to the SP500 returns series. The model does not provide acceptable statistical results and failed the diagnostic tests. Based on the results in Table 4, the AR(1) coefficient ϕ_1 is significant at 1 percent level for all models specified suggesting that the current return rate is affected by the previous day's return rate. The MA(1) coefficient θ_1 is significant at 1 percent level, indicating that the current return rate is also affected by the

Table 3: ARMA(1,0)-GARCH(1,1) models estimates for KLCI.

	Normal	S.Normal	t	S.t	GED	S.GED
Parameter estimates and standard errors						
μ	0.000399*	0.000345*	0.000374*	0.000356*	0.000283*	0.000305*
	(0.00011)	(0.00011)	(9.73E-05)	(0.00011)	(8.85E-05)	(0.00011)
ϕ_1	0.15800*	0.15423*	0.11629*	0.11601*	0.08002*	0.079962*
	(0.01849)	(0.01841)	(0.01720)	(0.01721)	(0.01477)	(0.01381)
ω	6.80E-07*	6.52E-07*	9.64E-07*	9.61E-07*	7.83E-07*	7.87E-07*
	(1.76E-07)	(1.70E-07)	(2.92E-07)	(2.92E-07)	(2.57E-07)	(2.58E-07)
α_1	0.09146*	0.089117*	0.10883*	0.10858*	0.096119*	0.096374*
	(0.01046)	(0.01027)	(0.01647)	(0.01644)	(0.01497)	(0.01501)
β_1	0.90426*	0.90641*	0.88755*	0.88771*	0.89806*	0.89786*
	(0.01054)	(0.01036)	(0.01564)	(0.01562)	(0.01504)	(0.01506)
Skew, ξ		0.94629*		0.99188*		1.00540*
		(0.01786)		(0.02187)		(0.01541)
Shape			4.55390*	4.56190*	1.12390*	1.12220*
			(0.38080)	(0.38260)	(0.03854)	(0.03842)
Diagnostic tests						
$Q(10)$	0.1118	0.0904	0.0023	0.0022	0.0000	0.0000
$Q(20)$	0.4811	0.4395	0.0550	0.0530	0.0003	0.0003
$Q^2(10)$	0.1717	0.1527	0.3737	0.3732	0.2605	0.2613
$Q^2(20)$	0.3252	0.3069	0.5622	0.5624	0.5624	0.4903
LM test	0.1804	0.1666	0.3556	0.3556	0.2896	0.2904
Information criterion						
AIC	-6.966664	-6.968618	-7.056898	-7.056348	-7.060983	-7.060429
BIC	-6.957625	-6.957771	-7.046051	-7.043693	-7.050136	-7.047774
SIC	-6.966668	-6.968624	-7.056904	-7.056357	-7.060989	-7.060437
HQIC	-6.963433	-6.96474	-7.053020	-7.051824	-7.057106	-7.055905

Note: Shape parameters are ν for t and skewed t distributions and κ for GED and skewed GED distributions. The parameter estimates are significant at 1 percent level*. Standard errors are provided in parentheses. The p-values are provided for Ljung-Box Q-statistics at lag 10 for residuals ($Q(10)$) and squared residuals ($Q^2(10)$) and Lagrange Multiplier (LM) test.

previous day's residual. In addition, α_2 and β_1 are found to be significant at 1 percent level, implying that the current volatility of return is influenced by the previous 2-day's residual and the previous day's volatility. Based on the diagnostic tests, all models have insignificant Q-statistics for the residuals and squared residuals. The Lagrange multiplier (LM) test is also insignificant which concurs with the null hypothesis that no ARCH errors are present in the series. The best model for the marginal of SP500, which

Table 4: ARMA(1,2)-GARCH(2,1) models estimates for SP500.

	Normal	S.Normal	t	S.t	GED	S.GED
Parameter estimates and standard errors						
μ	5.9E-05***	5.9E-05***	5.9E-05***	5.9E-05***	5.9E-05***	5.9E-05***
	(0.00003)	(0.00002)	(0.00004)	(0.00003)	(0.00005)	(0.00003)
ϕ_1	0.82556*	0.75446*	0.87779*	0.74325*	0.88230*	0.71687*
	(0.10240)	(0.08365)	(0.08589)	(0.09982)	(0.11050)	(0.10820)
θ_1	-0.88859*	-0.83516*	-0.93889*	-0.81660*	-0.93540*	-0.78765*
	(0.10390)	(0.08587)	(0.08763)	(0.10170)	(0.11540)	(0.11210)
θ_2	0.025249	0.011162	0.033088	0.010201	0.030411	0.008879
	(0.02465)	(0.02502)	(0.02388)	(0.02537)	(0.02804)	(0.02578)
ω	1.83E-06*	1.66E-06*	1.48E-06*	1.44E-06*	1.62E-06*	1.55E-06*
	(3.65E-07)	(3.44E-07)	(4.30E-07)	(4.12E-07)	(4.48E-07)	(4.27E-07)
α_1	0.0012761	1.00E-08	1.00E-08	1.00E-08	1.00E-08	1.00E-08
	(0.01230)	(0.01189)	(0.01493)	(0.01450)	(0.01478)	(0.01436)
α_2	0.09882*	0.098577*	0.11130*	0.10833*	0.10881*	0.10544*
	(0.01607)	(0.01578)	(0.02054)	(0.01980)	(0.02029)	(0.01954)
β_1	0.88820*	0.88991*	0.88435*	0.88589*	0.88410*	0.88619*
	(0.01070)	(0.01045)	(0.01324)	(0.01279)	(0.01365)	(0.01312)
Skew		0.86273*		0.89015*		0.90889*
		(0.01895)		(0.02106)		(0.01875)
Shape			6.5625*	7.0644*	1.2695*	1.3085*
			(0.79350)	(0.91300)	(0.04559)	(0.04772)
Diagnostic tests						
$Q(10)$	0.8636	0.1609	0.8834	0.5265	0.8821	0.7097
$Q(20)$	0.3650	0.0577	0.3882	0.2251	0.4139	0.3327
$Q^2(10)$	0.4671	0.5144	0.5686	0.5965	0.5543	0.5826
$Q^2(20)$	0.9137	0.9228	0.9178	0.9333	0.9239	0.9387
LM test	0.5577	0.5959	0.6391	0.6772	0.6322	0.6716
Information criterion						
AIC	-6.275094	-6.288362	-6.309703	-6.316396	-6.322281	-6.327949
BIC	-6.260632	-6.272092	-6.293432	-6.298317	-6.30601	-6.309871
SIC	-6.275105	-6.288376	-6.309717	-6.316413	-6.322295	-6.327966
HQIC	-6.269924	-6.282546	-6.303886	-6.309933	-6.316465	-6.321487

Note: Shape parameters are ν for t and skewed t distributions and κ for GED and skewed GED distributions. The parameter estimates are significant at 1*, 5** and 10*** percent levels. Standard errors are provided in parentheses. The p-values are provided for Ljung-Box Q-statistics at lag 10 for residuals ($Q(10)$) and squared residuals ($Q^2(10)$) and Lagrange Multiplier (LM) test.

is selected based on the smallest information criterion, is the ARMA(1,2)-GARCH(2,1) model with skewed GED distribution.

The standardized residuals obtained from each marginal distribution model are then transformed into pseudo observations. The estimated Kendall's τ is 0.06554923 which is significant at 1 percent level. This copula-based value suggest that there is some dependency between the KLCI and SP500, but the relationship is weak as reflected by the Kendall's τ value. Using pseudo observations, the copula parameters are then estimated. The copula parameter estimates, goodness-of-fit test and tail dependence index are shown in Table 5.

Table 5: Copula parameter estimates, goodness of fit (GOF) test and tail dependence index.

Copula	Parameter	Std. Error	GOF	Lower tail	Upper tail
Gaussian	0.10278	0.01853	0.050632*	0	0
$t(\nu)$	0.09415	0.01821	0.060525*	0.006612648	0.006612648
	(11.69245)	(2.66261)			
Clayton	0.14029	0.02716	0.031081	0.007149931	0
Gumbel	1.07015	0.01358	0.086903*	0	0.08883707
Frank	0.592006	0.003745	0.045172*	0	0
Galambos	0.2824	0.0206	0.091824*	0	0.08591285
Husler-Reiss	0.58104	0.02837	0.093105*	0	0.08524234

Note: For t copula, the parameter value and standard error for degrees of freedom ν are provided in parentheses. For goodness-of-fit (GOF) test, the statistics provided are significant* at 1 percent and 5 percent except for Clayton copula.

All copula models, with the exception of the Clayton copula, are rejected due to the high significance of the goodness-of-fit (GOF) test, indicating that the models are unsuitable to capture the dependence structure. The Clayton copula with copula parameter 0.14029 is chosen as the best model to describe the dependence structure between KLCI and SP500. The estimated Clayton copula model implies that the joint dependence of the two indices do not have symmetric distribution where the dependence at the lower tail is slightly greater than the upper tail. In other words, the KLCI and SP500 returns series are correlated in times of crisis rather than in times of blooming. To further support our finding on the dependence structure of KLCI and SP500, the comparison between the empirical plot of pseudo observations (left) and the simulated samples of Clayton copula (right) are shown in Figure 3. A slight concentration of points can be observed at the bottom-left side of each plot.

4.3. Risk evaluations and backtesting

For the purpose of evaluating the portfolio risk of SP500 and KLCI, the simulated returns of SP500 ($Y_{1,t}$) and KLCI ($Y_{2,t}$) are generated using:

$$\begin{cases} Y_{1,t} = 0.000059 + (0.71687)Y_{2,t-1} + (0.008879)\sigma_{1,t-2}X_{1,t-2} \\ -(0.78765)\sigma_{1,t-1}X_{1,t-1} + \sigma_{1,t}X_{1,t} \\ \sigma^2_{1,t} = 1.55 \times 10^{-6} + (1 \times 10^{-8})\sigma^2_{1,t-1}X^2_{1,t-1} + (0.10544)\sigma^2_{1,t-2}X^2_{1,t-2} \\ +(0.88619)\sigma^2_{1,t-1} \\ X_{1,t} \sim SkewGED(\xi = 0.90889, \kappa = 1.3085) \end{cases} \tag{21}$$

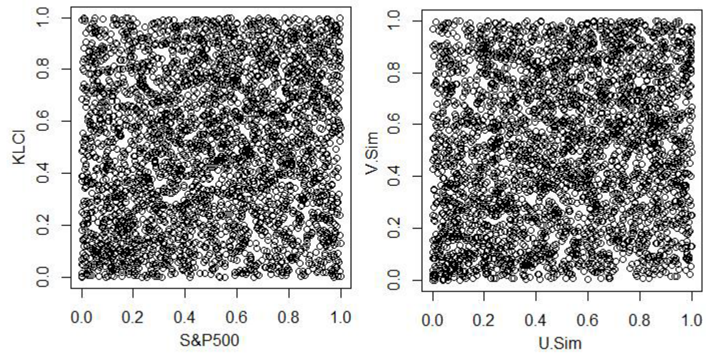

Figure 3: Plots of pseudo observations (U, V) and simulated Clayton copula with parameter of 0.14029.

$$\begin{cases} Y_{2,t} = 0.000374 + (0.11629)Y_{2,t-1} + \sigma_{2,t}X_{2,t} \\ \sigma_{2,t}^2 = 9.64 \times 10^{-7} + (0.10883)\sigma_{2,t-1}^2 X_{2,t-1}^2 + (0.88755)\sigma_{2,t-1}^2 \\ X_{2,t} \sim t(\nu = 4.5539) \end{cases} \tag{22}$$

where $X_1 = (X_{1,1}, X_{1,2}, \ldots, X_{1,t-1}, X_{1,t})$ and $X_2 = (X_{2,1}, X_{2,2}, \ldots, X_{2,t-1}, X_{2,t})$ are the standardized residuals which were transformed from the simulated samples of Clayton copula model.

The one-step ahead value-at-risk (VaR) of the portfolio is evaluated for every 250 days and then forecasted for the next 3139 days. Therefore, the first VaR estimation sample is given by the first 250 days. The predicted VaR estimates are then compared with the observed (actual) portfolio returns and the number of exceedances is recorded for backtesting purposes. Table 6 presents the backtesting results based on the normal VaR (norm), historical simulation (HS) and copula-VaR (Cop) at several significant levels and several mixtures of portfolio weights.

The results in Table 6 show that the copula-VaR models perform better than the normal VaR and historical simulation in terms of violation ratio (VR). From the results of Kupiec's test, the copula-VaR also proved to be the best model since it produces the largest p-value.

It can also be observed that the normal VaR overestimates the risk of the KLCI-SP500 portfolio. These findings agree with the past studies that the normal VaR produces inaccurate risk estimates of a portfolio [6],[7]. A good VaR model that provides accurate risk estimates is suggested to have a VR ranging from 0.8 to 1.2 [4].

Interestingly, the copula-VaR models for the 3:7 weighted portfolio have a more accurate risk estimates compared to the equally weighted and 7:3 weighted portfolios. These results may suggest that a higher investment fraction of a less volatile asset in a portfolio produces a more accurate VaR estimates.

Table 6: Number of exceedances, violation ratio (VR) and backtesting.

Property	VaR 1 percent			VaR 5 percent			VaR 10 percent		
	Norm	Hs	Cop	Norm	Hs	Cop	Norm	Hs	Cop
Equally weighted portfolio									
No. of exceedances	67	45	35	67	45	41	67	45	37
VR	2.16	1.45	1.13	2.16	1.45	1.13	2.16	1.45	1.13
Kupiec's test	0.000	0.022	0.526	0.000	0.022	0.100	0.000	0.022	0.328
3:7 weighted portfolio									
No. of exceedances	63	42	33	63	42	35	63	42	30
VR	2.03	1.35	1.06	2.03	1.35	1.13	2.03	1.35	0.97
Kupiec's test	0.000	0.071	0.071	0.000	0.071	0.526	0.000	0.071	0.800
7:3 weighted portfolio									
No. of exceedances	72	56	39	72	56	42	72	56	37
VR	2.16	1.45	1.13	2.16	1.45	1.13	2.16	1.45	1.13
Kupiec's test	0.000	0.000	0.189	0.000	0.000	0.071	0.000	0.000	0.328

Note: The p-values for Kupiec's test is provided in the table.

5. Conclusion

This paper has evaluated the risk of equally and mixed weighted portfolios using the copula-VaR approach and compare the VaR estimates with the traditional VaR measures. The copula model was used to model the dependence between the assets in the portfolio.

We found that the joint distribution of SP500 and KLCI have asymmetric characteristic where the lower tail dependence was significant, implying that the extreme negative returns of the SP500 and KLCI have some dependence. These results imply that SP500 and KLCI returns are correlated in times of crisis rather than in times of market blooming. However, these dependencies were weak as reflected by the estimated copula parameter and the tail dependence index. This finding conforms to the early expectation that the SP500 and KLCI were found to be weakly associated based on the non-parametric correlation analysis.

The results from our study imply that a good diversification opportunity exist for the global portfolio comprising of the Malaysian and the U.S stocks. Our study also reveals that the value-at-risk (VaR) based on copula-GARCH provides a better one-step ahead estimates compared to the traditional VaR methods (normal VaR and historical simulation). The results of violation ratios (VR) for the copula-VaR at different significant levels and several mixtures of portfolio weights were within the range of a good VaR model. Our finding also raises the possibility that when a less volatile asset has a higher investment fraction in a portfolio, the VaR models may produce a more accurate risk estimates. However, further work is required to validate this claim.

Acknowledgment

The authors gratefully acknowledge the financial support from the Ministry of Higher Education Malaysia (FP055-2014A, FRGS/1/2015/SG04/UKM/02/2 and GUP-2015-002).

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