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Total Vertex and Total Pathos Vertex Semientire Block Graph

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Abstract

In this paper, we introduce the concept of (i) the total vertex semientire block graph and (ii) the total pathos vertex semientire block graph of a graph. We obtain some properties of these graphs. We study the characterization of graphs whose total vertex semientire block graph and the total pathos vertex semientire block graphs are planar, outer planar, Eulerian and Hamiltonian.

Keywords: Inner vertex number, Line graph, Outer planar, Vertex Semientire graph.

Mathematics subject classification: 05C

1. Introduction

All graphs considered here are finite, undirected without loops or multiple edges. Any undefined term or notation in this paper may be found in Harary [2].

For a graph G(p, q) if B = { u_1, u_2, u_3, · · ·,u_r; r ≥2} is a block of G, then we say that point u_1 and block B are incident with each other, as are u_2 and B and so on. If two distinct blocks B_1 and B_2 are incident with a common cut vertex then they are called adjacent blocks.

By a plane graph G we mean embedded in the plane as opposed to a planar graph. In a plane graph G let e_1=uv be an edge. We say e_1 is adjacent to the vertices u and v, which are also adjacent to each other. Also an edge e_1 is adjacent to the edge e_2 =uw. A region of G is adjacent to the vertices and edges which are on its boundary, and two regions of G are adjacent if their boundaries share a common vertex.

The crossing number c(G) of G is the least number of intersection of pairs of edges in any embedding of G in the plane. Obviously G is planar if and only if c(G)=0.

The *edgedegree* of an edge e = {a, b} is the sum of degrees of the end vertices a and b. Degree of a block is the number of vertices lies on a block. *Blockdegee* B_v of a vertex v is the number of blocks in which v lies. *Blockpath* is a path in which each edge in a path becomes a block. If two paths p_1 and $p_2$contain a common cutvertex then they are adjacent paths and the *pathdegree* P_v of a vertex v is the number of paths in which v lies. Degree of a region is the number of vertices lies on a region. The *regiondegree* R_v of a vertex v is the number of regions in which the vertex v lies. Pendant pathos is a path p_i of pathos having unit length.

The inner vertex number i(G) of a planar graph G is the minimum number of vertices not belonging to the boundary of the exterior region in any embedding of G in the plane. A graph G is said to be minimally non-outerplanar if i(G) = 1.

A new concept of a graph valued functions called the vertex semientire block graph $e_{vb}(G)$ of a plane graph G was introduced by Venkanagouda in[8] and is defined as the graph whose vertex set is the union of the set of vertices, blocks and regions of a graph G in which two vertices are adjacent if and only if they are adjacent vertices, vertices lie on the blocks and vertices lie on the regions.

The pathos vertex semientire block graph of a tree T denoted by $P_{vb}(T)$ is the graph whose vertex set is the union of the vertices, regions, blocks and path of pathos of T in which two vertices are adjacent if and only if they are adjacent vertices of T or vertices lie on the blocks of T or vertices lie on the regions of Torthe adjacent blocks of T and the vertices lies on the path of pathos of T. Clearly the number of regions in a tree is one. This concept was introduced by Venkanagouda in [7].

The block graph B(G) of a graph G is the graph whose vertex set is the set of blocks of G in which two vertices are adjacent if the corresponding blocks are adjacent. This graph was studied in [2].

The path graph P(T) of a tree is the graph whose vertex set is the set of path of pathos of T in which two vertices of P(T) are adjacent if the corresponding path of pathos have a common vertex.

Let v_1,v_2,v_3 be the pendant vertices of $K_{1,3}$. The graph $K_{1,3}(P_n)$ is obtained from $K_{1,3}$ by attaching 1 time to any one pendant vertex of $K_{1,3}$ as shown in figure 1.1.

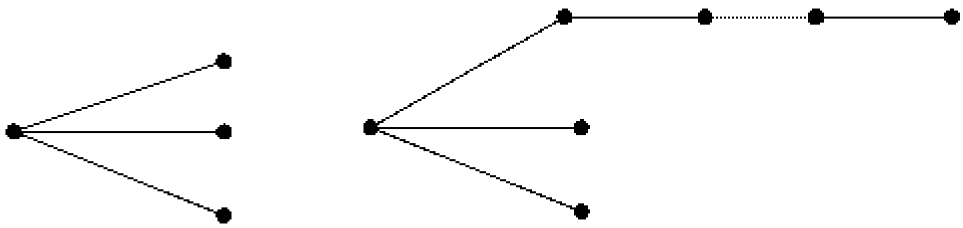


Figure 1.1

The following will be useful in the proof of our results.

Theorem 1 [4]. If G be a connected plane graph then the vertex semientire graph $e_v(G)$is planar if and only if G is a tree.

Theorem 2 [3]. Every maximal outerplanar graph G with p vertices has 2p – 3 edges.
Theorem 3 [8]. For any (p, q) graph G with b blocks andr regions, vertex semientire block graph $e_{vb}(G)$ has (p + b +r)vertices and $q+\sum_{i=1}^{k} d(b_i)+\sum_{j}^{l} d(r_j)$ edges, where $d(b_i)$ is the block degree of a blockb_i and $d(r_j)$ is the degree of a region r_j.

Theorem 4 [8]. For any (p, q) graph G, $e_{vb}(G)$ is always nonseparable.

Theorem 5 [8]. For tree T, vertex semientire block graph $e_{vb}(T)$ is always planar.

Theorem 6 [7] For any tree T, pathos vertex semientire block graph $P_{vb}(T)$isplanar.

Theorem 7 [7]. For any (p, q) graph T with b blocks and r regions, pathos vertex semientire block graph $P_{vb}(T)$ has (2p +k)vertices and $4p-3+\sum_{i=1}^{k} v(p_i)$ edges, where $v(p_i)$ be the number of vertices lies on the path p_i.

Theorem 8 [2]. A connected graph G is eulerian if and only if each vertex in G has even degree.

Theorem 9 [2]. A nontrivial graph is bipartite if and only if all its cycles are even.

2. Total Vertex Semientire Block Graph

We now define the following graph valued function.
Definition 2.1. The total vertex semientire block graph $T_e(G)$ of a plane graph G is the graph whose vertex set is the union of the set of vertices, blocks and regions of a graph G in which two vertices are adjacent if the corresponding vertices or blocks of G are adjacent or one corresponding to the vertex v of G and other to a blocks b of G and v is

in b or one corresponds to the vertex v of G and other the region r of G and v lies on the region r.

In Figure 2.2, a graph G and its total vertex semientire block graph are shown.

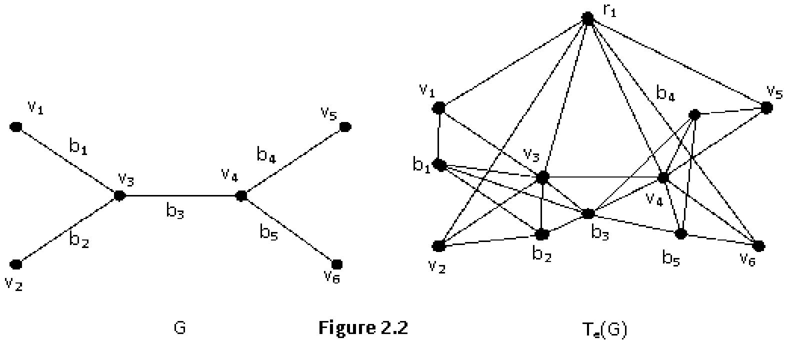


Figure 2.2

Remark 1. For any graph G, $G \subseteq e_{vb}(G) \subseteq T_e(G)$.

Remark 2. For any graph G,$T_e(G) = e_{vb}(G) \cup B(G)$.

Remark 3. For every edge e_i of a tree T, the corresponding sub graph in T_e(G) is a graph g_1 as shown in figure 2.3.

Figure 2.3

We now establish a result which determines the number of vertices and edges in total vertex semientire block graph.

Theorem 10. If G is a (p,q) connected graph whose vertices have degreed_i and if b_i is number of blocks to which vertex v_i belongs in G, then T_e(G) has$1 + r + \sum_{j=1}^{p} b_i$ vertices and $q + \sum_{i=1}^{p} \binom{b_i}{2} + \sum_{i=1}^{p} d(r_i) + \sum_{i=1}^{p} d(b_i)$ edges, where r be the number of regions in G, $d(r_i)$the number of vertices lies on the region r_i and $d(b_i)$ be the number of vertices lies on the blockb_i.

Proof. By Remark 1, $e_{vb}(G)$ *is a* spanning subgraph of T_e(G). Thus the number of vertices of T_e(G) equals to the number of vertices of$e_{vb}(G)$. By Theorem 3,$e_{vb}(G)$has $p + b + r = 1 + r + \sum_{i=1}^{p} d(b_i)$ vertices. Hence the number of vertices in total vertex semientire block graph is the number of vertices of vertex semientire block graph. ThusT_e(G) $= 1 + r + \sum_{j=1}^{p} d(b_j)$.

Further, by Remark 2, the number of edges in T_e(G) is the sum of the number of edges in $e_{vb}(G)$and number of edges in a block graph B(G) of a graph G. By Theorem 3,$e_{vb}(G)$ hasq + $\sum d(r_i)$ + $\sum d(b_j)$ edges, Also the number of edges in a block graph is $\sum_{i=1}^{p} \binom{b_i}{2}$.

Thus the number of edges in T_e(G) = $q + \sum_{i=1}^{p} \binom{b_i}{2} + \sum_{i=1}^{p} r_i + \sum_{j=1}^{p} d(b_j)$.

Theorem 11. For any graph G, T_e(G) is always nonseparable.

Proof. By Remark 1,$e_{vb}(G)$ is a spanning sub graph of T_e(G).Also by Remark 2, T_e(G) =$e_{vb}(G) \cup B(G)$,clearly it follows that T_e(G) is nonseparable.
We characterize the graph whose vertex semientire block graph and total vertex semientire block graphs are isomorphic.

Theorem 12. Let G be a nontrivial connected graph. The graphs $e_{vb}(G)$ and T_e(G) are isomorphic if and only if G is a block.

Proof. Let G be a (p, q) graph and be a block. The graphs $e_{vb}(G)$ and T_e(G)have the same number of vertices. Since G is a block and B(G) has no edges, it implies by definitions, that $e_{vb}(G)$ and T_e(G)are isomorphic.
Conversely suppose G is a nontrivial connected graph andT_e(G) =$e_{vb}(G)$. We now prove that G is a block. On contrary, assume that G has at least two blocks. By Remark 2, it is clear that the number of edges in T_e(G) is the sum of the number of edges in e_{vb}(G) and the number of edges in a block graph B(G). Since G has at least two blocks, it implies that B(G) has at least one edge. Thus the number of edges in e_{vb}(G) is less than the number edges in T_e(G). Hence T_e(G) and e_{vb}(G) are not isomorphic, which is a contradiction. Thus G has no two or more blocks and hence G is a block.

Theorem 13. If G is a graph without isolated vertices, then T_e(G) is not a bipartite graph.

Proof. Let G be a graph without isolated vertices then G has a block b. Let u and v be the vertices lies on the block b. Since the blockvertex b is incident with the vertices u and v, it follows that the corresponding vertices b, u, v form a cycle C_3 in T_e(G), By Theorem 9,T_e(G) is not a bipartite graph.

Theorem 14. Let $v_i \in G$ and $\deg(v_i) = n.Then\ in$ T_e(G),$\deg(v_i) = 2n + 1$.

Proof. By the definition of T_e(G), each vertex of G lies in T_e(G). Also each vertex is lies on the block and region of G. Hence if$v_i \in G$ and $\deg(v_i) = n, then\ in$ T_e(G),$\deg(v_i) = 2n + 1$.
We next give a characterization of graph whose total vertex semientire block graph is planar.

Theorem 15. For any graph G,T_e(G)Planar if and only if G is a path.

Proof. Suppose T_e(G) is planar. Assume that Gis not a path. We have following cases.

Case 1. Let G be the star $K_{1,3}$ such that deg(v)=3. Let $b_1=e_1$,$b_2=e_2$ and $b_3=e_3$ be the blocks of $K_{1,3}$, By Theorem 5,$e_{vb}(G)$ is planar and all these block vertices lies in the different regions. In T_e(G), joining of these three blockverticesto form a subgraph K_3 and the edges of K_3 crosses the edges already drawn. This gives a non-planargraph, a contradiction.

Case 2. Let G be any graph which contains at least one cycle. By Theorem 5, $e_{vb}(G)$is non planar and then T_{vb}(G) is also non-planar, a contradiction.
Conversely, suppose G be any path. By Theorem 5, $e_{vb}(G)$ is planar. Let$b_{1,}$, b_2,b_3……b_n be the blocks corresponds to the edges of G such that b_i is adjacent to b_j for all i $> j = 1,2,3, \dots n$ in T_e(G). Since all the vertices $b_{1,}$, b_2,b_3……$b_n$lies outside the region. The edges between b_i&$b_j$does not cross over the edges already drawn. Hence T_e(G) is planar.
We next give a characterization of graph whose total vertex semientire block graph is minimally nonouterplanar.

Theorem 16. For any graph G,T_{vb}(G) is minimallynonouterplanar if and only ifG is a path $P_{3.}$

Proof. Suppose T_e(G) is minimally nonouterplanar. Assume that T is not $P_{3.}$ Suppose T is $P_n \, for \, n \geq 4$.For sake of simplicity we consider T= P_4. By the definition of e_{vb}(G), each edge e_i of G corresponds K_4–x in $e_{vb}(G)$.For a path$P_{4,}$there is 3K_4 –x in $e_{vb}(G)$as shown in figure 2.4.

Figure 2.4

Let$b_{1,} = e_1, b_2 = e_2$,$b_3 = e_3$be the three blocks of G. By the definition of block graph, $B(P_n) = P_3$.By Remark2, T_e(G) is union of $e_{vb}(G)$ *and* $B(P_4)$. Hence T_e(G) contain two inner vertices, a contradiction.
Conversely suppose G be path P_3 and $b_{1,}$, b_2,v_1, v_2,v_3 be corresponding blocks and vertices of G. By the definition of $e_{vb}(G)$,$e_{vb}(P_3) = 2K_4 - x.$ In $e_{vb}(G)$ both block vertices adjacent to form a graph which contain one inner vertex v_2. Hence T_e(G) is minimally nonouter planar.

Theorem 17. For any graph G,T_e(G) always noneulerian.

Proof. Let G be any graph and let $v_i \in$ G with deg($v_i) = n$. By Theorem 3, the degree of v_i in T_{vb}(G)becomes 2n+1.For any n$\in N$, degree of $v_i$in T_e(G) is odd. Hence by Theorem 8, T_e(G) is noneulerian.
In the next Theorem we give graph whose total vertex semientire block graph is the Hamiltonian.

Theorem 18. For any graph G,T_e(G) is always Hamiltonian.

Proof. Consider a graph G. We have the following cases.
Case 1. Suppose G be path with$\{u_{1,}u_{2,}u_3 \dots \dots u_n\} \in V(G)$and, b_2,b_3…b_m be the blocks of G such that m = n-1 and r_1 be the region of G. Now the vertex set of T_{vb}(G), $V[T_e(G)] = \{u_{1,}u_{2,}u_3 \dots \dots u_n\} \cup \{b_{1,}, b_2, b_3 \dots \dots b_m\} \cup r_1$. Since given graph is a path, then inT_{vb}(G), $b_1 = e_1$, $b_2 = e_2, \dots \dots b_m = e_m$ such that$\{b_{1,}, b_2, b_3 \dots \dots b_m\} \subset V\,[T_e(G)]$. Then by the definition of T_e(G),$\{u_{1,}u_{2,}u_3 \dots u_{m-1}u_n\} \cup \{b_{1,}, b_2, b_3 \dots b_{m-1,}b_m\} \cup$
$\{b_1u_1, b_2u_2, \dots \dots b_{m-1}u_{n-1,,}b_mu_n\} \cup \{r_1u_1, r_1u_2 \dots \dots r_1u_n\}$ forms edge set ofT_e(G), as shown in the figure 2.5.

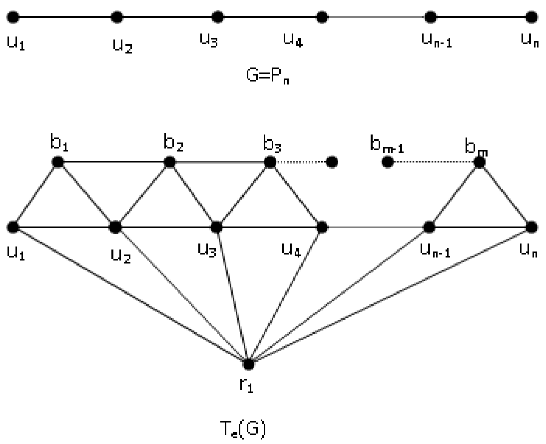


Figure2.5

In T_e(G), the Hamiltonian cycle $r_1u_1b_1u_2b_2u_3b_3 \dots \dots .. u_{n-1}b_mu_nr_1$ will exists. Clearly the total vertex semientire block graph of a path is Hamiltonian.

Case 2. Suppose G is a star $K_{1,n}$.
Let$[T_e(G)] = \{u_{1,}u_{2,}u_3 \dots \dots u_n\} \cup \{b_{1,}, b_2, b_3 \dots \dots b_{n-1}\} \cup r_1$.By the definition of T_e(G), the Hamiltanian cycle $r_1u_1b_1b_2u_2u_3b_3 \dots \dots \dots . u_nr_1$ will exists. Hence,T_e(G) is Hamiltonian.

Case 3. Suppose G be any graph which contains at least one cycle. Let $V[T_e(G)]$=$\{u_{1,}u_{2,}u_3 \dots \dots u_n\} \cup \{b_{1,}, b_2, b_3 \dots \dots b_{n-1}\} \cup \{r_1, r_2, \dots r_j\}$. For the vertex of T_e(G), it always exist cycle as $r_1, \mathrm{u_1}, \mathrm{u_2}, \mathrm{b_1}, \mathrm{u_3}, \mathrm{r_2}, , \dots \dots . . \mathrm{b_i}, \mathrm{u_{n-1}}, \mathrm{r_j}, \mathrm{u_1}, \mathrm{r_1}$.Hence$T_e$(G) is a Hamiltonian.

Theorem 19. For any graph G, T_e(G)never has crossing number one.

Proof. We consider the following cases.
Case 1. Suppose G is a tree. If G is a path, then by Theorem 15, T_e(G)has crossing number zero, while if it is not a path, then it contains $K_{1,3}$ as a subgraph and by Theorem 5, vertex semientire block graph e_{vb}(T) of a tree T is always planar. Let $b_{1,}$ b_2 and $b_3$be the block vertices in e_{vb}(T) and let b_1 and b_2 lies in the exterior region and b_3 lies in the interior region. In T_e(G), there are the edges between the vertices b_1 and b_2 with the vertex b_3, which gives crossing number at least two.

Case 2. Suppose G contains a cycle and it has at least two blocks. By Theorem 15, the vertex semientire block graph of a cycle C_3 is nonplanar and has crossing number one.Letb_1 and b_2 be the vertices lies in the different regions. In T_e(G), there is an edge between the block vertices b_1 and b_2, which gives T_e(G) with at least three crossings.
We now deduce a necessary and sufficient condition for entire graphs with crossing number two.

Theorem 20. For any graph G, T_e(G) has crossing number two if and only if the maximum degree$\Delta(G) \le 2$ and G is a tree with exactly one vertex of degree 3.

Proof. SupposeT_{vb}(G) has crossing number two. Then it is a nonplanar. By Theorem 6, we have deg(u) $\geq$ 3 for some u$\in G$. We now consider the following cases.

Case 1. Assume that G is a graph which contains at least one cycle and there exists a vertex v_i such that blockdegree of v_i is at least two. Let $b_1 = uu_1u_2 \dots u_{n-1}u$, b_2=$\{v_{1,}v_{2,}v_3 \dots \dots v_{n-1,}\ u\}$ be the two blocks incident to a cut vertex u_i.
By definition of vertex semientire block graph, for every cycle $C_{n,}$ there corresponds a crossing number one, shown in the figure2.6.

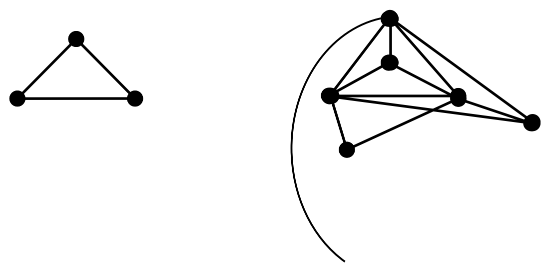


Figure 2.6

Since blockvertex b_2 is adjacent to the vertices v_1, v_2, u which lies in the internal region. These two block vertices lies on two different regions. Clearly the crossing number is at least two. Also in T_e(G) there is an edge between two vertices b_1 and b_2 and lies in different regions. Hence crossing number is at least 3, a contradiction.

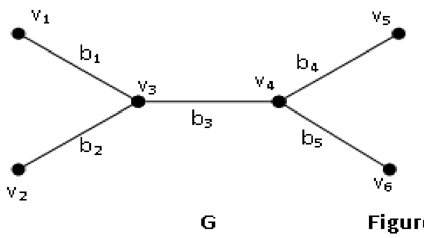
Case 2. Assume that G is a tree and there exist a vertex u of degree 4.SupposeG $=K_{1,4}$.By definition of$e_{vb}(G), e_{vb}(K_{1,4})$ is planar. In $T_e(G)$, all the blockvertices adjacent to each other to form a sub graph K_4 which crosses the edges of $e_{vb}(G)$ such that cr$[T_{vb}(G)] \geq 6$, a contradiction.

Case 3. Assume that G is a tree with two vertices u_1, u_2, such that deg(u_1) =deg$(u_2) =$ 3.So, G is a bi-star $K_{1,3}$. By the definition of e_{vb}(G), $e_{vb}(K_{1,3})$ is planar and the vertices $b_{1,}, b_2$ lies in any one interior region r_i. The vertices $b_{3,}, b_4$ lies in another interior region r_j. In $T_e(G)$, we join the vertices $b_{1,}, b_2, b_3$ through the edges $\{b_{1,}, b_{3,}\}$&$\{b_{2,}, b_{3,}\}$ which gives crossing number 4, a contradiction.
Conversely, suppose G is a tree with $\Delta(G) \leq 2$ and there exist a unique vertex of degree 3. Consider G be a $K_{1,3}$.By Theorem 5, $e_{vb}(G)$ is planar. In$T_e(G)$, the vertices $b_{1,}, b_{3,}$ and $b_{2,}, b_{3,}$ are adjacent. To join an edges$\{b_{1,}, b_3\}$ and$\{b_{2,}, b_3\}$this gives the crossing number two.Then crossing number in $T_{vb}(G)$ is two.

3. Total Pathos Vertex Semientire Block Graph

We now define the following graph valued function.

Definition 3.1. The total pathos vertex semientire graph of a tree T denoted by $T_{pv}(T)$is the graph whose vertex set is the union of the set of vertices, blocks, regions and the path of pathos of a tree T in whichtwo vertices are adjacentif the corresponding vertices of T are adjacent or one corresponding to the vertex v of T and other to a block b of T and v is in b or one corresponds to the vertex v of T and other the region r of T and v lies in the region r in T or one corresponds the vertex v of T and the other the path of pathos p of T and v lies on p or both are the path of pathos p_i and p_j and have a common vertex in T. The total pathos vertex semientire block graph is defined only when the tree contains at least two paths. In Figure 3.2, a graph T and its total pathos vertex semientire block graph$T_{pv}(T)$are shown. Since the path of pathos are not unique, the corresponding total pathos vertex semientire graph of a tree is also not unique.



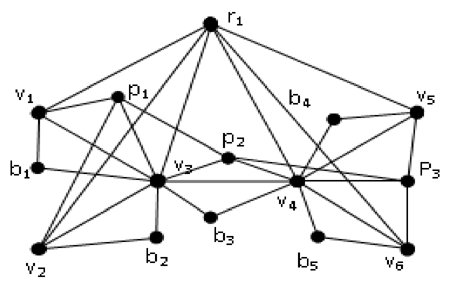


Figure 3.2.

Remark 4. For any graph G, $G \subseteq e_{vb}(G) \subseteq P_{vb}(T) \subseteq T_{pv}(T)$.

Remark 5. Let T be a tree with a vertex v_i, blockdegree B_{v_i}, path degree P_{v_i}, regiondegree R_{v_i} and the degree of a vertex deg(v_i), the degree of a corresponding vertex v_i in $T_{pv}(T)$isB_{v_i}+ P_{v_i}+ R_{v_i}+deg(v_i).

Remark 6. If P_{v_i} be the path degree of a vertex v_i, then the number of edges corresponds to P_{v_i}in $T_{pv}(T)$) is$\frac{P_{v_i(P_{v_i}-1)}}{2}$.

Remark 7. For any graph G, $T_{pv}(T) = P_{vb}(G) \cup P(G)$.
We now establish a result which determines the number of vertices and edges in total pathos vertex semientire block graph.

Theorem 21. For any (p, q) graph T with b-blocks and r-regions, the total pathos vertex semientire block graph $T_{pv}(T)$has $2p + k$ vertices and $4p - 3 + \Sigma v(p_i) + \frac{1}{2}\Sigma p_{v_i}(p_{v_i} - 1)$edges where $v(p_i)$ be the number of vertices lies on a path p_i and p_{v_i} be the pathdegree of the vertexv_i.

Proof. By the Remark 3, the total pathos vertex semientire block graph $T_{pv}(T)$is a spanning subgraph of pathos vertex semientire block graph. Thus the number of vertices in$T_{pv}(T)$ equals the number of vertices of $P_{vb}(T)$. By the Theorem 4, $T_{pv}(T)$has $2p + k$ vertices. Further by the Theorem 4, the number of edges in$P_{vb}(T)$ has$4p - 3 + \Sigma v(P_i)$.Also the number of edges in $T_{pv}(T)$ is the sum of the edges in $P_{vb}(T)$ and the edges obtained from paths which is equivalent to the pathdegree. By the Remark 5, it follows that the number of edges in $T_{pv}(T)$ is $4p - 3 + \Sigma V(p_i) + \frac{1}{2}\Sigma\Delta p_i(\Delta p_i - 1)$.

Theorem 22. For any tree T, total pathos vertex semientire block graph $T_{pv}(T)$ is always nonseparable.

Proof. Let T be any tree. We know that in a tree T, all the internal vertices are the cut vertices c_i and these cut vertices lies on the region as well as on at least two blocks. By the definition of$T_{pv}(T), c_i$ become non-cut vertex. Also the Pathos vertex P_i is adjacent to all the vertices of v_i which becomes non-cut vertex. Hence $T_{pv}(T)$is always nonseparable.
We characterize the graph whose pathos vertex semientire block graph and total pathos vertex semientire block graphs are isomorphic.

Theorem 23. Let Tbe a nontrivial connected tree. The graphs $P_{vb}(T)$ and T_{pv}(T) are isomorphic if and only if G is a path P_n.

Proof. Let Tbe path P_n. The graphs $P_{vb}(T)$ and T_{pv}(T) have the same number of vertices. Since T is a path, the path graph P(T) has no edges, it implies by definitions, that $P_{vb}(T)$ and T_{pv}(T) are isomorphic.
Conversely suppose T is a nontrivial connected tree and $P_{vb}(T)$ and T_{pv}(T). We now prove that T is a path. On contrary, assume that T is not a path and it contains a star $K_{1,n}$. It has at least two path of pathos. By Remark7, it is clear that the number of edges in T_{pv}(T) is the sum of the number of edges in $P_{vb}(T)$ and the number of edges in a path graph P(T). Since G has at least two path of pathos, it implies that P(T) has at least one edge. Thus the number of edges in $P_{vb}(T)$) is less than the number edges in T_{pv}(T). Hence $P_{vb}(T)$ and T_{pv}(T) are not isomorphic, which is a contradiction. Thus G has no two or more path of pathos and hence T is a path.

Theorem 24. If T is a tree without isolated vertices, then T_{pv}(T) is not a bipartite graph.

Proof. Let T is a tree without isolated vertices. Then T has a block b, Let u and v is the vertices lies on the block b. Since the blockvertex b is incident with the vertices u and v, it follows that the corresponding vertices b, u, v form a cycle C_3 in T_{pv}(T), By Theorem 9, T_e(G) is not a bipartite graph.

Theorem 25. For any tree T, the total pathos vertex semientire block graph$T_{pv}(T)$ always nonplanar.

Proof. Suppose a graph T be a tree. For each edge of a tree, there is $K_4 - e$ in $e_{vb}(T)$.In $P_{vb}(T)$ the pathos vertices are adjacent to the vertex$v_{1,}v_{2,}$ $\cdots$ v_k these lies on $p_{i.}$
Further in $T_{pv}(T)$, at least two pathos have a common vertex v_j these two vertices are adjacent to the pathos vertices p_i andp_j . Further in $T_{pv}(T), p_i$ and p_j have an edge with crossover the edges already drawn. Hence $T_{pv}(T)$ is nonplanar.

Theorem 26. For any tree T, $T_{pv}(T)$ is always noneulerian.

Proof. Consider a tree T. We have the following cases.

Case 1. Let T be a star $K_{1,n}$ if n is even, and then the degree of the region vertex in $T_{pv}(T)$ is odd. Hence $T_{pv}(T)$ is noneulerian.

Case 2. Let n be an odd, since in$K_{1,n}$, there are $\frac{n+1}{2}$ path of pathos, then there is exactly one pendent pathos with two vertices and each non pendent pathos contains three vertices.

Sub Case 2.1. Suppose the path of pathos$\frac{n+1}{2}$is odd. In T, the pendent pathos p_i is adjacent to two vertices and p_i is adjacent to both pathose vertex $p_i$and p_j. So degree of p_i is even. Further the nonpendent pathos vertex p_j is adjacent to three vertices $(u_{j_1}, u_{j_2}, u_{j_3})$ and also p_j is adjacent to $p_i$and p_k, clearely degree of p_j becomes odd and$T_{pv}(T)$ is noneulerian.

Sub Case 2.2. Suppose the number of pathos $\frac{n+1}{2}$ is even. Clearly it contains only one pendent pathos $p_i = \{v_{i_1,} v_{i_2}\}$ and odd number of nonpendent pathos $\{p_j, p_k \cdots\}$.By the definition of $T_{vb}(T), p_i$ is adjacent to v_{i_1}, v_{i_2} and all nonpendent pathos to form a graph with degree of P_i is odd. Hence $T_{pv}(T)$is noneulerian.

Case 3. Suppose G be a tree which is non-star. If T contains odd number of vertices, then in $T_{pv}(T)$,the degree of region vertex becomes odd. So $T_{pv}(T)$is noneulerian. Consider Tbe tree with even number of vertices, we have the following cases

Case 3.1. If any one path of pathos p_i contains even degree and its path degree is even. Then in$T_{vb}(T)$ the degree of the corresponding pathos vertex is odd. Hence $T_{pv}(T)$is noneulerian.

Case 3.2. If path of pathos p_i contains odd degree and its path degree is even. By definition, the corresponding pathos vertex p_i in $T_{pv}(T)$is even, but in G at least one path of pathos is a other then p_j other thanp_i is even. In$T_{pv}(T)$, degree of p_j is odd, a noneulerian. Hence $T_{pv}(T)$is always noneulerian.
In the next theorem we characterize the graph whose $T_{pv}(T)$ is Hamiltonian.

Theorem 27. For any tree T, $T_{pv}(T)$ is always nonhamiltonian.

Proof. Since for each edge, there corresponds to a graph g_1 shown in figure 2.4. Clearly g_1 is nonhamiltonian. Hence$T_{pv}(T)$ is nonhamiltonian.

Theorem 28. For any tree T,$T_{pv}(T)$ has crossing number one if and only if T is $K_{1,3}$or $K_{1,3}(P_n)$.

Proof. Suppose that $T_{pv}(T)$has crossing number one.Then it is nonplanar. By Theorem 25, T be a tree. We now consider the following cases.

Case 1. Assume that T is a star $K_{1,4}$. Clearly $K_{1,4}$ contains two path of pathos and the corresponding vertices p_i and p_j lies in the interior region of $P_{vb}(T)$. In $T_{pv}(T)$, these two vertices p_i and p_j have joined by an edge and gives crossing number is two, which is a contradiction.

Case 2. Assume that T be a graph $\sigma=K_{1,3}(P_n, P_n)$. By Theorem6, $P_{vb}(T)$ is planar. Clearly a tree σ contains two path of pathos and the corresponding vertices lies in the interior region of $P_{vb}(T)$. In $T_{pv}(T)$, these two verticesp$_i$ and p_j have joined by an edge gives crossing number is two, which is a contradiction.
Conversely suppose T is $K_{1,3}(P_n)$. By the Theorem 25, $K_{1,3}(P_n)$ is planar. Clearly $K_{1,3}(P_n)$ contains two path of pathos which corresponds two vertices p_i and $p_j$in which one vertex p_i lies in the interior region and the other lies in exterior region. In $T_{pv}(T)$,these two vertices are joined by an edge and gives crossing number one. Hence $T_{pv}(T)$ has crossing number one.

Conclusion

In this paper, we introduced the concept of the total vertex semientire block graph and the total pathos vertex semientire block graph of a graph. We characterized the graphs whose total vertex semientire block graph and the total pathos vertex semientire block graphs are planar, Eulerian and Hamiltonian and crossing number one and two.

References

[1] Haray F., Annals of New York, Academy of Sciences, (1977),175, 198.
[2] Harrary, F., Graph Theory, Addison- Wesley Reading Mass, (1969).,p.72, 107.
[3] Kulli. V R., On minimally nonouterplanar graphs, proceedings of the Indian National Science Academy, (1975), 41A.
[4] Kulli. V. R. and Akka. D. G., Journal mathematical Science, Vol.14, No. 6, (1980),P. 585- 588.
[5] Rajanna N E and Venkanagouda. M. Goudar “Pathos vertex semientire block graph”, International Journal of Mathematics Trends and Technology (2014), Vol 7 (2), 103-105.

[6] Sedlacek J., Some properties of interchange graphs. The Graphs and the applications. Academic press, New York (1962).

[7] Venkanagouda. M. Goudar, “On Pathos vertex semientire graph of a tree", international journal of applied mathematical research (2012),Vol 1 (4), 666-670.

[8] Venkanagouda. M. Goudarand Rajanna N E “ Vertex semientire blockgraph ", Global Journal of Science Frontier Research (2014),Vol 14 (1) 1-4.