Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 12, Number 1 (2016), pp. 1069-1075

http://www.ripublication.com

A Logical Thinking Analysis through the Euclidean Geometry

Akhsanul In'am
Mathematics Education Department,
University of Muhammadiyah Malang Indonesia

Abstract

The objective of this present research is to analyze the logical thinking of the students of mathematics education department through the solution to the Euclidean geometry problems. This research was made at the Mathematics Education Department, University of Muhammadiyah Malang where six students consisting of each two students under low, medium, and high abilities were taken as the research subject. The data were obtained through the Euclidean geometry test and then were analyzed through the solution results of the Euclidean geometry item tests by paying attention to the grouping of their learning results. The result showed that the students under the low category had a less logical thinking pattern since they did not pay attention to a proper order in solving the problems. Those under the medium category showed a relatively logical thinking order as suggested by the fact that some of the orders was not properly followed, but the students under the high category showed that their thinking is based on an awareness so that their thinking order is said to be logical.

Keywords: logical thinking, euclidean geometry, learning result

Introduction

Researches dealing with Polya approach to the solution of the Euclidean Geometry problems show that students under the low ability category had a less comprehension in solving the problems, and did not recheck their answers due to their less capability in managing time to solve the problems, meanwhile those under the high category had a good comprehension and rechecked their solution of Euclidean geometry problems [10].

One of the requirements to implement a logical thinking is the awareness of various mental hindrances and obstacles that may influence any efforts to have a creative

thinking. Based on some studies, it is shown that any creativity cannot grow well because there are some influences from the development of other abilities that does not require some critical and creative thinking [18]. Moreover, the awareness aspect concerning with the implemented thinking pattern turns out being related to the thinking process made in the problem solving [5].

Any steps developed to solve mathematical problems may grow some logical thinking ability [17], especially through the Euclidean Geometry that is said to develop one's logical thinking ability [9]. This ability is strongly related to the academic quality [2]. Based on other research results, it is shown that the logical thinking ability is the main factor in building a concept [12] and it may be acquired through a series of learning activities [15] and in mathematics, it may be obtained through the Euclidean Geometry. Viewed from the types of sexes, there are some differences in the logical thinking process [16] but some researches show that the types of sexes did not influence the logical thinking [4, 11]. But this present research would not study this logical thinking viewed from the types of sexes, but from the differences of the learning results.

A problem must have a solution, but this requires a series of processes. One's mental operation activities in solving a problem is considered as a logical thinking [13], but it is also said that a logical thinking is one's ability to solve a problem scientifically [1]. One of the logical thinking activities in solving a problem is made through the solutions of the Euclidean Geometry problems [19] and it may be done through learning aims at improving formal reasoning and thinking ability [3].

Meanwhile, thinking is a process of any activity through the existing fact as the basis for showing a new fact proved through a series of activities involving a mental operations, knowledge and attitudes [8]. The thinking orderliness may realize the implementation of ordered activities and this makes the activities more effective and efficient. A work may be done by thinking first of what to do and what aspects to leave in order to make the implementation of a program effective.

A systematic thinking pattern cannot be acquired without any a series of continuous learning activities and also some habituation and interaction which are always made. Learning may be done through various enabling subjects in line with the study. It is through mathematics that some trainings and some habituation of the thinking process may be done and one of the units that may lead to a logical and systematic thinking pattern is the Euclidean Geometry.

Through Geometry, students may learn how to solve problems using a series of ordered activities and good reasons in making statements by basing them on the definitions, theorems or postulates. Definition is a series of statements of which the truth may be obtained through a series of proofs, while postulate is a statement where its truth may be accepted without any proof [9, 14].

Based on the descriptions above, it is shown that the logical thinking ability may be developed through mathematics, especially the Euclidean Geometry. The research results showed that the logical thinking ability determines the success in the learning process, meaning that a logical thinking ability is different. Another research result also showed that in terms of sexes, different sexes showed different logical thinking

abilities, though another research in this matter suggested that there is no such a difference in the learning results in Euclidean Geometry.

Research Method

In this present research, a qualitative approach with the type of a case study was employed. The subjects were the first semester students of Mathematics Education Department, University of Muhammadiyah Malang, Indonesia. The data were obtained through observations during the final examination of the Euclidean Geometry subject in order to understand the patters of the problem solving made by the students. Moreover, an analysis of the document of the results of the tests was also made to analyze the series of stages in the problem solving and the reasons given in the problem solving and also interviews were done to sharpen the research results of the two previous techniques of data collection.

The obtained data were categorized according to the students' learning result abilities and the categories were made by grouping the students into the low, medium and high abilities. The data were analyzed by taking into account the students' answers to the Euclidean Geometry test items and also their stages in proving them.

Research Result

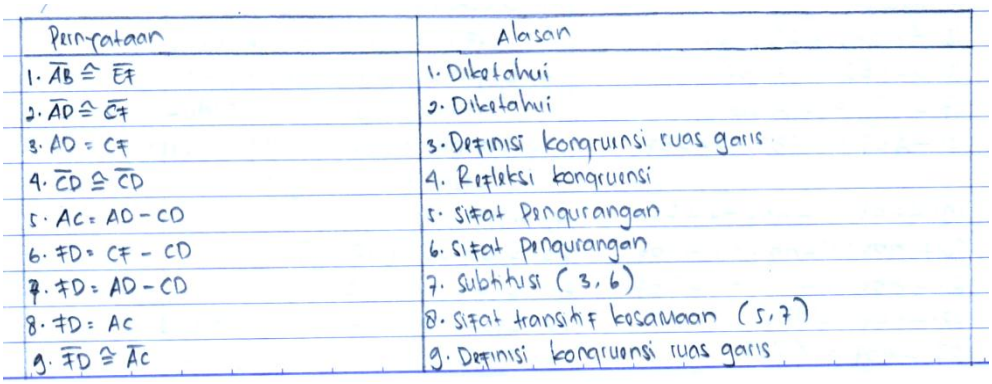
On the basis of the data from the document of the students' answers to the Euclidean Geometry problems, the students were grouped into three, namely those under the low, medium and high abilities.

Figure 1 shows a student's work under the low ability. It is seen that the steps of the solution is not orderly made such as in the step 1 $\overline{AB} \cong \overline{AC}$ with the given reason, but the continuation of the statement is not related to the stages of the solution. The Step 2 followed by step 3 may be said to be logical, but the step 4 continued to AC = AD – CD is jumped, and it should start with the statement of CD = CD. This shows that the solution to the problem does not follow the usual stages, some steps are left, though in the stages they are present.

The students' thinking pattern, from the stages in solving the Euclidean Geometry problems, is less illogical. Moreover, on the basis of the data on the document of the problem solving reinforced by the interviews with the respondents, it is caused by their less understanding, as shown by the transcript of the following interview:

...as far as I know the problem solving is ... yes like that ... because $\overline{CD} \cong \overline{CD}$ *must have an impact on or meaning of CD = CD, so I need no write the impact aroused.*

The transcript shows that according to the student's perception, a fixed statement should be written. Whereas, stages of a solution to an Euclidean Geometry problem requires an order statement as a learning process of a logical thinking.



Pernyataan	Alasan
1. $\overline{AB} \cong \overline{EF}$	1. Diketahui
2. $\overline{AD} \cong \overline{CF}$	2. Diketahui
3. $AD = CF$	3. Definisi kongruensi ruas garis
4. $\overline{CD} \cong \overline{CD}$	4. Refleksi kongruensi
5. $AC = AD - CD$	5. Sifat Pengurangan
6. $FD = CF - CD$	6. Sifat Pengurangan
7. $FD = AD - CD$	7. Subtitusi (3, 6)
8. $FD = AC$	8. Sifat transitif kesamaan (5, 7)
9. $\overline{FD} \cong \overline{AC}$	9. Definisi kongruensi ruas garis

Figure 1: An Answer to an Euclidean Geometry Problem from a Low Ability Student

The student's answer under the low category is shown in Figure 1. In general, the student's work from the document may be said to be right, but there are some solution principles that are not done according to the ordered stages. In the first step, $\overline{AB} \cong \overline{EF}$ should be written, and in the second step, in the statement $\angle 1 \cong \angle 2$ is written, although in the problem, the statement on the angle is written first. While from steps 3 to 11, it can be said that the stages have been in line with the proper steps in the problem solving. This shows that students under this group still possess an improper understanding in writing stages in the problem solving, but as a whole, they have used and had logical thinking in the problem solving.

The mistake in steps 1 and 2 is caused by to the subject's perception that the given statement is merely rewritten in the statement of the problem solution, as reflected in the following:

.... in my opinion, what is known may be merely rewritten in the statement and it should no follow the steps of solving a problem .... And in the statement of the proof, I have written the statement dealing with the proof of the congruence of the triangle using the orders of the second step followed by the first step and ended with the eleventh step.

The statement suggests that actually the student understood the proper stages, but since the statement stemmed from the statement of the angle, according to the subject, it would not cause any problem. Moreover, in the step 12, the orders of the statements in proving the congruence of the triangle have been stated. But in the thinking stages, the condition is less proper, namely the arrangement of the statements of the stages should follow the right orders.

The answer given by the student under the medium ability category is presented in Figure 2. The document of the student's answer shows that there is a good order of statements and reasons proving $\overline{AB} \cong \overline{EF}$ through a series of stages known from s cycle. It is stated that in the definition of the radius, four radiuses have been stated. Then in the proof stage, the statement of the lines AC and ED is needed, but since the figure has not existed yet, the student has given a statement dealing with the postulate stating that it is through two points one line and only one line may be made. However, there is one step that is actually not needed namely the step 7 which is exactly the

same as the step 8, $\overline{AC} \cong \overline{ED}$. This step is merely written at step 8 due to the definition of the congruence of the line segment.

Then to prove the congruence of the angle, based on the the known statement saying that there are two perpendicular lines, the subject has written a new statement that if two lines are perpendicular one another its angle is are a right angle. The reason given dealing with the congruence of the angle is rather improper, it should be the theorem saying that if there are two right angles, the two would be congruent, it is the mistake in solving this problem. But viewed from the stages, the students under the high ability category show a logical thinking pattern in solving the problem.

Pernyataan	Alasan
1. $\angle 1 \cong \angle 2$	1. Diketahui
2. $\overline{AB} \cong \overline{EF}$	2. Diketahui
3. $\overline{BD} \cong \overline{CE}$	3. Diketahui
4. BD = CE	4. Definisi kongruensi ruas garis
5. $\overline{CD} \cong \overline{CD}$	5. Refleksi kongruensi
6. CD = CD	6. Definisi kongruensi ruas garis
7. BC = BD − CD	7. sifat pengurangan
8. DE = CE − CD	8. sifat pengurangan
9. DE = BD − CD	9. subtitusi (4, 8)
10. BC = DE	10. sifat transitif kesamaan (7, 9)
11. $\overline{BC} \cong \overline{DE}$	11. Definisi kongruensi ruas garis
12. $\triangle ABC \cong \triangle FED$	12. Postulat sisi, sudut, sisi (2, 1, 11)

Figure 2: An Answer to an Euclidean Geometry Problem from a Medium Ability

Pernyataan	Alasan
1. ⊙O	1. Diketahui
2. OC, OD, OE, OA adalah jari-jari lingkaran	2. Definisi jari-jari lingkaran
3. $\overline{OC} \cong \overline{OD} \cong \overline{OE} \cong \overline{OA}$	3. Dalil: semua jari-jari lingkaran adalah kongruen.
4. $\overline{BC} \cong \overline{FD}$	4. Diketahui
5. ada garis AC melalui titik A dan C.	5. Postulat: melalui dua titik dapat dibuat satu dan hanya satu garis lurus.
6. Ada garis ED melalui titik E dan D	6. Seperti no. 5
7. AC ≅ ED	7. Melalui dua titik dpt dibuat satu garis
8. $\overline{AC} \cong \overline{ED}$	8. Definisi kongruensi ruas garis
9. $\overline{AB}$ dan $\overline{EF} \perp \overline{CD}$	9. Diketahui
10. ∠ABC, ∠ABD, ∠EFC, ∠EFD siku*-siku	10. Definisi garis tegak lurus
11. ∠ABC ≅ ∠ABD ≅ ∠EFC ≅ ∠EFD	11. Definisi sudut siku-siku
12. △ABC ≅ △EFD	12. Postulat sisi, sudut, sisi
13. $\overline{AB} \cong \overline{EF}$	13. Definisi poligon kongruensi

Figure 3: An Answer to an Euclidean Geometry Problem from a High Ability Student

Discussion

This research result shows that the logical thinking ability from the three categories of the abilities of the learning results suggest some differences, viewed from the stages of solving the Euclidean Geometry problems. This difference is in line with some studies on the differences in the logical thinking abilities considered from the sexes [6, 7, 16], while another research shows that there is not different in the logical thinking abilities due to the types of sexes [4, 11]. This shows that the in the logical thinking abilities, there are some differences viewed from some aspects, although no differences are found in terms of the sexes.

This research is expected to enrich any knowledge, especially the knowledge that there are different logical thinking abilities considered from the categories of the learning results abilities. The result of this research also reinforces previous studies, that it is through mathematics that a logical thinking ability may be improved [17, 2]. In this research, steps of solving Euclidean Geometry problems may develop some logical thinking abilities, because the abilities may grow well through learning [15]. Moreover, this research may also reinforce the research result stating that this logical thinking ability may lead to the best way to develop the concept [12].

Conclusion

This research result shows that students under the low category possesses a low ability in logical reasoning, meaning that they cannot make use of logical thinking stages viewed from the solution to an Euclidean Geometry problem. Those under the medium category may be said that their logical thinking pattern is relatively logic, though some of their thinking is less logic, while those under the high ability category may said to have high logical thinking.

References

[1] Abdulkadir, Cagru, A. & Lutfi (2013) An Analysis of Mathematcis Teacher Candidates Logical Thinking Level: Case of Turkey, *Journal of Education and Instructional Studies in The World 3(1), 83-91*

[2] Atay, P.D.(2006). *Relative ınfluence of cognitive and motivational variables on genetic concepts in traditional and learning cycleclassrooms*, (Published Doctoral Dissertation). Medium East Technical University,Ankara.

[3] Cohen,H.G.(1980).Dilemma of the objective paper-and-pencil assessment within the Piagetian framework *Science Education, 64(5), 741-745*.

[4] Fah,L.Y.(2009).Logical thinking abilities among form 4 students in the ınterior division of Sabah, Malaysia, *Journal of Science and Mathematics Education in Southeast Asia, 32(2), 161-187*.

[5] Heidari, F & Bahrami, Z (2012). The Relationship between Thinking Styles and Metacognitive Awareness among Iranian EFL Learners. *International Journal of Linguistic, Vol 4 No.* 3

[6] Hernandez,L.De.,Marek,E.A.,& Renner,J.W.(1984).Relationships among gender, age, and intellectual development,*Journal of Research in ScienceTeaching,1(4), 365-375*.
[7] Howe,A.C.,& Shayer,M. (1981). Sex-related differences on task of volume and density, *Journal of Research in Science Teaching, 18(2), 169-175.*
[8] Hiang, P., S. (2006). *Kemahiran Berfikir.* Kualalumpur: Kumpulan Budiman SDN. BHD
[9] In'am, A (2003). *Geometri.* Malang: Bayu Media
[10] In'am, A (2015). *Menguak Penyelesaian Masalah Matematika: Analisis Pendekatan Matekognitif dan Model Polya,* Yogyakarta: Aditya Media Publishing
[11] Kıncal, R.Y.,& DenizYazgan,A.(2010). Investigating the formal operational thinking skills of 7th and 8th grade primary school students according to some variables, *Elementary Education Online, 9(2),723-733.*
[12] Lawson,A.E.(1992). The development of reasoning among college biology students a review of research, *Journal of College Science Teaching, 21, 338-344*.
[13] Piaget,J.(1969). *The origins of intelligence in children*, NewYork: International UniversityPress.
[14] Rich, B. (2004). *Gometri,* New York: McGraw Hill
[15] Savant, M.(1997). *The power of Logical Thinking*.NewYork: St.Martin's Press
[16] Shemesh,M.(1990). Gender-related differences in reasoning skills and learning interests of junior high school students, *Journal of Research in Science Teaching, 27(1), 27-34*.
[17] Valanides,N.C.(1996). Formal reasoning and science teaching, *School Science and Mathematics, 96(2),99-111.*
[18] Yazid, Z (2003). *Berfikir kreatif dan Cambah Fikiran.* Kualalumpur: Pustaka Al Shafa