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Symmetric Skew 4-Derivations on Semi Prime Rings

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Abstract

In this paper we introduce the notation of symmetric skew 4-derivation of Semiprime ring and we consider R be a non-commutative 2, 3-torsion free semi prime ring, I be a non zero two sided ideal of R, α be anautomorphism of R, and $D: R^4 \to R$be a symmetric skew 4-derivation associated with the automorphism α. If f is trace of D such that $[f(x), \alpha(x)] \in Z$ for all $x \in I$, then $[f(x), \alpha(x)] = 0$, for all $x \in I$.

Keywords: Semiprime ring, Derivation, Bi derivation, Symmetric Skew 3-derivation, Symmetric Skew 4-derivation and Auto orphism.

Introduction

In 1957, the study of centralizing and commuting mappings on aprime rings was initiated by the result of E. C. Posner [2] which states that the existence of a non-zero centralizing derivation on a prime ring implies that the ring has to be commutative. Further Vukman [4, 5] extended above result for bi derivations. Recently jung and park[6]considered permuting 3-derivations on prime and semi prime rings and obtained the following:Let R be a non-commutative3-torsion free semi prime ring and let I be a non-zero two sided ideal of R. Suppose that there exists a permuting 3-derivation $D: R^3 \to R$ such that f is centralizing on Ithen f is commuting on I. A. Fosner [1] extended the above results in symmetric skew 3-derivations with prime rings and semi prime rings. Recently Faiza Shujat, Abuzaid Ansari[3] Studied some results in symmetric skew 4-derivations in prime rings. In this Paper we proved that Symmetric skew 4-derivations in semi prime rings.

Preliminaries

Throughout this paper, R will be represent a ring with a center Z and α bean automorphism of R. Let $n \geq 2$ be an integer. A ring R is said to be n-torsion free if for $x \in R, nx = 0$implies $x = 0$. For all $x, y \in R$ the symbol $[x, y]$ will denote the commutator $xy - yx$. we make extensive use of basic commutator identities$[xy, z] = [x, z]y + x[y, z]$ and$[x, yz] = [x, y]z + y[x, z]$. Recall that a ring R is semi prime if $xRx = 0$ implies that $x = 0$. An additive map $d: R \rightarrow R$ is called derivation if $d(xy) = d(x)y + xd(y)$, for all $x, y \in R$, and it is called a skew derivation (α-derivation) of R associated with the automorphism α if $d(xy) = d(x)y + \alpha(x)d(y)$ for all $x, y \in R$, associated with automorphism α if $d(xy) = xd(y) + \alpha(y)d(x)$ for all $x, y \in R$.

Before starting our main theorem, let us gives some basic definitions and well known results which we will need in our further investigation.

Let D be a symmetric 4-additive map of R, then obviously

$$D(-p, q, r, s) = -D(p, q, r, s), \text{ for all} p, q, r, s \in R \quad (1)$$

Namely, for all $y, z \in R$, the map $D(.,.,y,z): R \rightarrow R$ is an endomorphism of the additive group of R.

The map $f: R \rightarrow R$ defined by $f(x) = D(x, x, x, x), x \in R$ is called trace of D.

Note that f is not additive on R. But for all $x, y \in R$, we have

$f(x + y) = [f(x) + 4D(x, x, x, y) + 6D(x, x, y, y) + 4D(x, y, y, y) + f(y)]$

Recall that by equation (1), f is even function.

More precisely, for all $p, q, r, s, u, v, w, x \in R$, we have

$D(pu, q, r, s) = D(p, q, r, s)u + \alpha(p)D(u, q, r, s)$,

$D(p, qv, r, s) = D(p, q, r, s)v + \alpha(q)D(p, v, r, s)$,

$D(p, q, rw, s) = D(p, q, r, s)w + \alpha(r)D(p, q, w, s)$,

$D(p, q, r, sx) = D(p, q, r, s)x + \alpha(s)D(p, q, r, x)$.

Of course, if D is symmetric, then the above four relations are equivalent to each other.

Lemma 1:

Let$R$ be a prime ring and $a, b \in R$. If $a[x, b] = 0$, for all $x \in R$, then either $a = 0$ or $b \in Z$.

Proof:

Note that

$0 = a[xy, b] = ax[y, b] + a[x, b]y = ax[y, b]$, for all $x, y \in R$.

Thus $aR[y, b] = 0, y \in R$, and, since R is prime, either $a = 0$or $b \in Z$.

Theorem 1:

Let R be a $2,3$ $-$torsion free non commutative semiprime ring and I be a nonzero ideal of R. Suppose α is anautomorphism of R and $D: R^4 \rightarrow R$ is a symmetric skew 4-derivation associated with α. If f is trace of D such that $[f(x), \alpha(x)] \in Z$ for all $x \in$

I, then $[f(x), \alpha(x)] = 0$.

Proof:

Let $[f(x), \alpha(x)] \in Z$, for all $x \in I$. (2)

Linearization of (2) yields that, we have

$[f(x+y), \alpha(x+y)] \in Z$

$[f(x+y), \alpha(x)] + [f(x+y), \alpha(y)] \in Z$

By skew 4-derivation, we have

$f(x+y) = [f(x) + 4D(x,x,x,y) + 6D(x,x,y,y) + 4D(x,y,y,y) + f(y)]$

$[f(x), \alpha(x)] + 4[D(x,x,x,y), \alpha(x)] + 6[D(x,x,y,y), \alpha(x)] + 4[D(x,y,y,y), \alpha(x)] + [f(y), \alpha(x)] + [f(x), \alpha(y)] + 4[D(x,x,x,y), \alpha(y)] + 6[D(x,x,y,y), \alpha(y)] + 4[D(x,y,y,y), \alpha(y)] + [f(y), \alpha(y)] \in Z$, for all $x \in I$. (3)

From (2) & (3), we get

$4[D(x,x,x,y), \alpha(x)] + 6[D(x,x,y,y), \alpha(x)] + 4[D(x,y,y,y), \alpha(x)] + [f(y), \alpha(x)] + [f(x), \alpha(y)] + 4[D(x,x,x,y), \alpha(y)] + 6[D(x,x,y,y), \alpha(y)] + 4[D(x,y,y,y), \alpha(y)] \in Z$,

for all $x \in I$. (4)

Replacing y by $-y$ in (4), we find

$$-4[D(x,x,x,y), \alpha(x)] + 6[D(x,x,y,y), \alpha(x)] - 4[D(x,y,y,y), \alpha(x)] + [f(y), \alpha(x)] - [f(x), \alpha(y)] + 4[D(x,x,x,y), \alpha(y)] - 6[D(x,x,y,y), \alpha(y)] + 4[D(x,y,y,y), \alpha(y)] \in Z,$$

for all $x \in I$. (5)

Comparing (4) and (5) and using 2-torsion freeness of R, wehave

$$4[D(x,x,x,y), \alpha(x)] + 4[D(x,y,y,y), \alpha(x)] + [f(x), \alpha(y)] + 6[D(x,x,y,y), \alpha(y)] \in Z,$$

for all $x \in I$. (6)

Substitute $y+z$ for y in (6) and use (6), we get

$$4[D(x,x,x,y+z), \alpha(x)] + 4[D(x,y+z,y+z,y+z), \alpha(x)] + [f(x), \alpha(y+z)] + 6[D(x,x,y+z,y+z), \alpha(y+z)] \in Z$$

$$\begin{aligned} 4[D(x,x,x,y), \alpha(x)] &+ 4[D(x,x,x,z), \alpha(x)] + 4[D(x,y,y,y), \alpha(x)] \\ &+ 4[D(x,y,y,z), \alpha(x)] + 4[D(x,y,z,y), \alpha(x)] \\ &+ 4[D(x,y,z,z), \alpha(x)] + 4[D(x,z,y,y), \alpha(x)] \\ &+ 4[D(x,z,y,z), \alpha(x)] + 4[D(x,z,z,y), \alpha(x)] \\ &+ 4[D(x,z,z,z), \alpha(x)] + [f(x), \alpha(y)] + [f(x), \alpha(z)] \\ &+ 6[D(x,x,y,y), \alpha(y)] + 6[D(x,x,y,z), \alpha(y)] \\ &+ 6[D(x,x,z,y), \alpha(y)] + 6[D(x,x,z,z), \alpha(y)] \\ &+ 6[D(x,x,y,y), \alpha(z)] + 6[D(x,x,y,z), \alpha(z)] \\ &+ 6[D(x,x,z,y), \alpha(z)] + 6[D(x,x,z,z), \alpha(z)] \in Z \end{aligned}$$

$$4[D(x,y,y,z),\alpha(x)]+4[D(x,y,z,y),\alpha(x)]+4[D(x,y,z,z),\alpha(x)] + 4[D(x,z,y,y),\alpha(x)]+4[D(x,z,y,z),\alpha(x)] + 4[D(x,z,z,y),\alpha(x)]+6[D(x,x,y,z),\alpha(y)] + 6[D(x,x,z,y),\alpha(y)]+6[D(x,x,z,z),\alpha(y)] + 6[D(x,x,y,y),\alpha(z)]+6[D(x,x,y,z),\alpha(z)] + 6[D(x,x,z,y),\alpha(z)]\in Z$$

$$12[D(x,y,y,z),\alpha(x)]+12[D(x,y,z,z),\alpha(x)]+12[D(x,x,y,z),\alpha(y)] + 6[D(x,x,z,z),\alpha(y)]+6[D(x,x,y,y),\alpha(z)] + 12[D(x,x,y,z),\alpha(z)]\in Z,$$

for all $x,y,z\in I$. (7)

Replacing z in $-z$ in (7) and compare with (7), we obtain

$$-12[D(x,y,y,z),\alpha(x)]+12[D(x,y,z,z),\alpha(x)]-12[D(x,x,y,z),\alpha(y)] + 6[D(x,x,z,z),\alpha(y)]-6[D(x,x,y,y),\alpha(z)] + 12[D(x,x,y,z),\alpha(z)]\in Z$$

$$2(12[D(x,z,y,y),\alpha(x)]+12[D(x,x,y,z),\alpha(y)]+6[D(x,x,y,y),\alpha(z)])\in Z$$

Using of two torsion free ring, we have

$$12[D(x,z,y,y),\alpha(x)]+12[D(x,x,y,z),\alpha(y)]+6[D(x,x,y,y),\alpha(z)]\in Z,$$

for all $x,y,z\in I$. (8)

Substitute $y+u$ for y in (8) and use (8) we get

$$12[D(x,z,y+u,y+u),\alpha(x)]+12[D(x,x,y+u,z),\alpha(y+u)] + 6[D(x,x,y+u,y+u),\alpha(z)]\in Z$$

$$12[D(x,z,y,y),\alpha(x)]+12[D(x,z,y,u),\alpha(x)]+12[D(x,z,u,y),\alpha(x)] + 12[D(x,z,u,u),\alpha(x)]+12[D(x,x,y,z),\alpha(y)] + 12[D(x,x,u,z),\alpha(y)]+12[D(x,x,y,z),\alpha(u)] + 12[D(x,x,u,z),\alpha(u)]+6[D(x,x,y,y),\alpha(z)] + 6[D(x,x,y,u),\alpha(z)]+6[D(x,x,u,y),\alpha(z)] + 6[D(x,x,u,u),\alpha(z)]\in Z$$

$24[D(x,z,y,u),\alpha(x)]+12[D(x,x,y,z),\alpha(u)]+12[D(x,x,u,z),\alpha(y)]+12[D(x,x,y,u),\alpha(z)]\in Z$, for all $x,y,z\in I$. (9)

Since R is 2 and 3-torsion free and replacing y,u by x in (9), we have

$$24[D(x,z,x,x),\alpha(x)]+12[D(x,x,x,z),\alpha(x)]+12[D(x,x,x,z),\alpha(x)]+12[D(x,x,x,x),\alpha(z)]\in Z$$

$$48[D(x,x,x,z),\alpha(x)]+12[D(x,x,x,x),\alpha(z)]\in Z$$

$4[D(x,x,x,z),\alpha(x)]+[f(x),\alpha(z)]\in Z$, for all $x,z\in I$. (10)

Again replaced z by xz in (10) and using (10) we obtain

$4[D(x,x,x,xz),\alpha(x)]+[f(x),\alpha(xz)]\in Z$, forall $x,z\in I$.

$4[D(x,x,x,xz),\alpha(x)]+[f(x),\alpha(x)\alpha(z)]\in Z$, forall $x,z\in I$.

$4[D(x,x,x,x)z+\alpha(x)D(x,x,x,z),\alpha(x)]+[f(x),\alpha(x)]\alpha(z)+\alpha(x)[f(x),\alpha(z)]\in Z$, for all $x,z\in I$.

$4f(x)[z,\alpha(x)]+4[f(x),\alpha(x)]z+4\alpha(x)[D(x,x,x,z),\alpha(x)]+[f(x),\alpha(x)]\alpha(z)+\alpha(x)[f(x),\alpha(z)]\in Z$, for all $x,z\in I$.

$\alpha(x)([f(x),\alpha(z)]+4[D(x,x,x,z),\alpha(x)])+(\alpha(z)+4z)[f(x),\alpha(x)]+$

$4f(x)[z,\alpha(x)] \in Z$, for all $x, z \in I$. (11)
Therefore, from (11), we get
$[\alpha(x)([f(x),\alpha(z)] + 4[D(x,x,x,z),\alpha(x)]),\alpha(x)] + [(\alpha(z) + 4z)[f(x),\alpha(x)],\alpha(x)] + 4[f(x)[z,\alpha(x)],\alpha(x)] = 0$, for all$x, z \in I$. (12)
$\alpha(x)[([f(x),\alpha(z)] + 4[D(x,x,x,z),\alpha(x)]),\alpha(x)] + (\alpha(z) + 4z)[[f(x),\alpha(x)],\alpha(x)] + [\alpha(z) + 4z,\alpha(x)][f(x),\alpha(x)] + 4f(x)\big[[z,\alpha(x)],\alpha(x)\big] + 4[f(x),\alpha(x)][z,\alpha(x)] = 0$, for all $x, z \in I$.
$\alpha(x)\big[[f(x),\alpha(z)],\alpha(x)\big] + 4\alpha(x)\big[[D(x,x,x,z),\alpha(x)],\alpha(x)\big] + (\alpha(z) + 4z)\big[[f(x),\alpha(x)],\alpha(x)\big] + [\alpha(z),\alpha(x)][f(x),\alpha(x)] + 4[z,\alpha(x)][f(x),\alpha(x)] + 4f(x)\big[[z,\alpha(x)],\alpha(x)\big] + 4[f(x),\alpha(x)][z,\alpha(x)] = 0$, for all $x, z \in I$.
$\alpha(x)\big[[f(x),\alpha(z)],\alpha(x)\big] + [\alpha(z),\alpha(x)][f(x),\alpha(x)] + [4z,\alpha(x)][f(x),\alpha(x)] + 4f(x)\big[[z,\alpha(x)],\alpha(x)\big] + [f(x),\alpha(x)][4z,\alpha(x)] = 0$, for all $x, z \in I$.
$[(\alpha(z) + 8z),\alpha(x)][f(x),\alpha(x)] + 4f(x)\big[[z,\alpha(x)],\alpha(x)\big] = 0$,
for all $x, z \in I$. (13)
Replacing z by $f(x)[f(x),\alpha(x)]$in (13), we get
$[(\alpha(f(x)[f(x),\alpha(x)]) + 8f(x)[f(x),\alpha(x)],\alpha(x)][f(x),\alpha(x)] + 4f(x)\big[[f(x)[f(x),\alpha(x)],\alpha(x)],\alpha(x)\big] = 0$, for all $x \in I$.
$[(\alpha(f(x)\alpha([f(x),\alpha(x)]),\alpha(x)][f(x),\alpha(x)] + 8[f(x)[f(x),\alpha(x)],\alpha(x)][f(x),\alpha(x)] + 4f(x)\Big[[f(x),\alpha(x)][f(x),\alpha(x)] + f(x)\big[[f(x),\alpha(x)],\alpha(x)\big],\alpha(x)\Big] = 0$, for all $x \in I$.
$\alpha\big(f(x)\big)[\alpha([f(x),\alpha(x)]),\alpha(x)][f(x),\alpha(x)] + \big[\alpha\big(f(x)\big),\alpha(x)\big]\alpha([f(x),\alpha(x)])[f(x),\alpha(x)] + 8f(x)\big[[f(x),\alpha(x)],\alpha(x)\big][f(x),\alpha(x)] + 8[f(x),\alpha(x)][f(x),\alpha(x)][f(x),\alpha(x)] + 4f(x)\big[[f(x),\alpha(x)][f(x),\alpha(x)],\alpha(x)\big] = 0$, for all $x \in I$.
$\big[\alpha\big(f(x)\big),\alpha(x)\big]\alpha([f(x),\alpha(x)])[f(x),\alpha(x)] + 8[f(x),\alpha(x)][f(x),\alpha(x)][f(x),\alpha(x)] + 4f(x)[f(x),\alpha(x)]\big[[f(x),\alpha(x)],\alpha(x)\big] + 4f(x)\big[[f(x),\alpha(x)],\alpha(x)\big][f(x),\alpha(x)] = 0$,
forall $x \in I$.
$\big[\alpha\big(f(x)\big),\alpha(x)\big]\alpha([f(x),\alpha(x)])[f(x),\alpha(x)] + 8[f(x),\alpha(x)][f(x),\alpha(x)][f(x),\alpha(x)] = 0$, for all $x \in I$.
$\big[\alpha\big(f(x)\big),\alpha(x)\big]\alpha[f(x),\alpha(x)][f(x),\alpha(x)] + 8[f(x),\alpha(x)]^3 = 0$, for all$x \in I$.
Since f is commuties on $I$and we have 2, 3-torsion freeness,
$2[f(x),\alpha(x)]^3 = 0$.
It follows that $(2[f(x),\alpha(x)]^2)\ R\ 2([f(x),\alpha(x)]^2) = 0$.
Since R is semi prime, we have
$2[f(x),\alpha(x)]^2 = 0$, for all$x \in I$. (14)
On the other hand, taking$z = x^2$ in equation (10), we get
$4[D(x,x,x,x^2),\alpha(x)] + [f(x),\alpha(x^2)] \in Z$, for all $x \in I$.
$4[D(x,x,x,x)x + \alpha(x)D(x,x,x,x),\alpha(x)] + [f(x),\alpha(x)\alpha(x)] \in Z$, for all $x \in I$.

$4[f(x)x+\alpha(x)f(x),\alpha(x)]+\alpha(x)[f(x),\alpha(x)]+[f(x),\alpha(x)]\alpha(x)\in Z$, for all $x\in I$.
$4[f(x)x,\alpha(x)]+4[\alpha(x)f(x),\alpha(x)]+2\alpha(x)[f(x),\alpha(x)]\in Z$, for all $x\in I$.
$4f(x)[x,\alpha(x)]+4[f(x),\alpha(x)]x+4\alpha(x)[f(x),\alpha(x)]+4[\alpha(x),\alpha(x)]f(x)+2\alpha(x)[f(x),\alpha(x)]\in Z$, for all $x\in I$.
$6\alpha(x)[f(x),\alpha(x)]+4x[f(x),\alpha(x)]+4f(x)[x,\alpha(x)]\in Z$, for all $x\in I$. (15)
Therefore, from equation (15), we get
$\big[f(x),6\alpha(x)[f(x),\alpha(x)]+4x[f(x),\alpha(x)]+4f(x)[x,\alpha(x)]\big]=0$, for all $x\in I$.
$\big[f(x),6\alpha(x)[f(x),\alpha(x)]\big]+\big[f(x),4x[f(x),\alpha(x)]\big]+\big[f(x),4f(x)[x,\alpha(x)]\big]=0$.

$$6\alpha(x)[f(x),[f(x),\alpha(x)]+6[f(x),\alpha(x)][f(x),\alpha(x)]+4x\big[f(x),[f(x),\alpha(x)]\big] + 4[f(x),x][f(x),\alpha(x)]+4f(x)\big[f(x),[x,\alpha(x)]\big] + 4[f(x),f(x)][x,\alpha(x)]=0.$$

$6[f(x),\alpha(x)]^2+4f(x)\big[f(x),[x,\alpha(x)]\big]=0$, for all $x\in I$.
$6[f(x),\alpha(x)]^2+4f(x)[f(x),x],\alpha(x)]=0$, for all $x\in I$. (16)
Since f is commutative and using equation (16), we get
$6[f(x),\alpha(x)]^2=0$, for all $x\in I$.
We have 2-torsion freeness, we get
$3[f(x),\alpha(x)]^2=0$, for all$\in I$. (17)
Comparing (14) and (17) and we have 2-torsion freeness, we get
$[f(x),\alpha(x)]^2=0$, for all$\in I$.
Note that zero is the only nilpotent element in the center of semiprime ring.
Thus,$[f(x),\alpha(x)]=0$, for all$x\in I$.
This completes the proof.

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