Global Journal of Pure and Applied Mathematics.
ISSN 0973-1768 Volume 12, Number 1 (2016), pp. 1003-1012

http://www.ripublication.com

Airline Revenue Management Under Number of No-Shows Uncertainty

Zuhri [1] and Liza Setyaning Pertiwi[2]

[1]STIM Sukma Medan, Kopertis Area 1, Medan, Indonesia.

[2]Polytechnic State Finances, Eka Warni Street, Medan, Indonesia.

Abstract

The total demand for a particular flight even at a given time of day fluctuates by day of week and season of the year. In addition to these more predictable or "cyclical" fluctuations in demand, there are also less predictable or "stochastic" variations in demand around the mean or expected value for a flight. This paper will show that, the company should find the optimal customer mix if it wants to achieve the highest revenue possible. They have to decide how many seats to sell to discount passengers, and how many to save for future passengers, who are willing to pay the full fare. The main problem of quantity based revenue management is to find an optimal allocation of our resource among the different fare classes. This paper will appear the comparison between the random of number ticket sold and not random using the simulation. Both simulation use no-shows uncertainty.

Keywords: revenue management; airline model; stochastic programming.

Introduction

Quantity based yield management is very useful for airline companies. Due to the fact that a plane has a fixed capacity of seats, the company should find the optimal customer mix if it wants to achieve the highest revenue possible. They have to decide how many seats to sell to discount passengers, and how many to save for future passengers, who are willing to pay the full fare. The main problem of quantity based revenue management is to find an optimal allocation of our resource among the different fare classes. In contrast with the price based revenue management, quantity base methods focus on our resource. How can we make the most of what we have, without changing the prices too much? We can achieve this with the help of booking limits, which we already mentioned, and *protection levels*. Besides booking limits protection levels are also important controls. Protection levels set the number of capacity units to reserve for a particular class. These can also be partitioned or nested, like booking limits.

Literature review

Recently, the revenue management strategies in other industries have been studied. Kimms and Muller-Bungart (2007) propose a planning problem at a broadcasting company. Mangani (2007) derives an optimal ratio between advertising and sales income when a publisher maximises its profits on advertising space and product price. A mobile TV service bundle problem-based pricing strategy is provided by Rautio, Anttila and Tuominen (2007). A number of human-related factors in the design of an efficient revenue management system in a local subsidiary of Multinational Corporation are highlighted by Zarraga-Oberty and Bonache (2007). And a comprehensive review of the recent development of the revenue management in different industries and some important areas that warrant further research are provided by Chiang, Chen and Xu (2007). More different revenue management strategies including pricing, auctions, capacity control, overbooking and forecasting are discussed in their research.

However, most of the above-mentioned models do not consider the risk of fluctuating revenues, except in Lai and Ng (2005). As we know, demand for hotel rooms is uncertain and, therefore, a decision maker may face several demand scenarios in the decision making process. He (or she) needs to consider the risk involved under different scenarios. In this paper, an optimal strategy for renting hotel rooms is provided for situations when a decision maker faces random customer arrivals. Because of uncertain demand, the decision maker may face different demand scenarios; a stochastic programming model with semi-absolute deviations to measure a hotel's revenue risk is formulated, and a stochastic programming model that considers cancellations and no-shows is also provided.

Airline Revenue Management Model

Terms and Definitions:

- *DEMAND: The total number of potential passengers wishing to make a reservation on a particular scheduled flight leg. In line with our definition of "demand" for an origin-destination market in Air Transportation Economics, the "demand" for a flight leg reflects a maximum potential, independent of the capacity being offered on the flight.*
- *LOAD: The total number of passengers who are actually carried on the flight leg. Because the demand for a flight can sometimes be greater than its capacity, it must be the case that load is always less than or equal to demand:*
  – *When demand is less than capacity, then load is equal to demand, as all potential passengers are accommodated and carried.*

– When demand exceeds capacity, then the load is equal to capacity, as some of the potential passengers cannot travel and must be rejected by the airline.

• *SPILL: The total number of potential passengers who cannot obtain a reservation and travel on a given flight due to insufficient capacity. "Spill" is also known as "rejected demand", since these passengers are rejected by the airline because the number of seats on the aircraft assigned to the flight is less than total potential demand.*

– Spill is by definition equal to total demand minus the total load of a flight.

– When demand is less than capacity, load is equal to demand, and spill is zero.

– When demand exceeds capacity, load is equal to capacity and spill is equal to demand minus capacity (load).

• *"Spill" occurs as the result of greater potential demand for a flight than the physical capacity of the aircraft assigned to operate on the flight leg in question. "Spill" has little direct relationship to overbooking, and must not be confused with "denied boardings". We will explore overbooking and denied boarding issues in much greater detail in Module 6. In the meantime, the most important differences between the two concepts can be summarized as follows:*

– "Spill" is the rejected demand resulting from operating two small an aircraft on a flight leg. It can occur whether or not the airline is using the practice of flight overbooking. For example, even if we assume no overbooking, an aircraft that departs with a load equal to capacity will likely have experienced spill.

– "Denied boardings" occur when the airline overbooks its flights, and more passengers show up than there are physical seats available on the aircraft. Denied boardings can occur even if no spill occurred during the booking process (i.e., all

Capacity Allocation:

The main idea behind capacity allocation is that customers, who are sensitive to price, are willing to lower their needs, and sacrifice comfort and time in order to get a cheaper discount ticket. On the other hand, higher fare customers are not sensitive to price, because they want to travel fast and comfortably, so they are willing to pay the full price in order to receive a service suitable for their needs. With these assumptions we can segment the market into customer groups, which we call *fare classes*, according to a customer's level of price sensitivity. We assume that the different fare classes are distinct, and they require the same resource. We also presume that we have homogeneous units of capacity, and that customers demand only a single unit of it. For now we do not allow no-shows or cancellations, we are going to deal with them later.

Capacity allocation with independent demand:

First we start with the less complex models, where demands are independent. This means that when we close a fare class, it does not affect the demand for other classes.

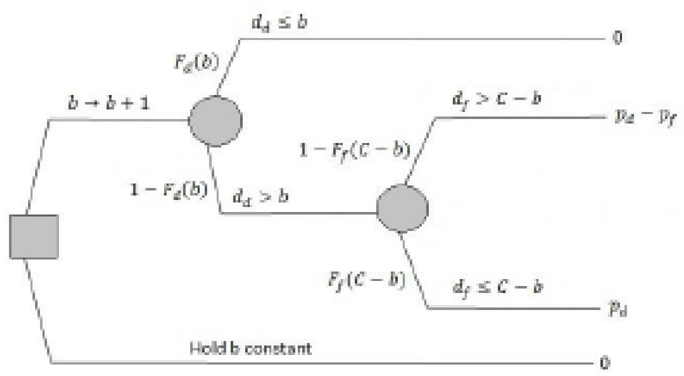
Two-class problem:

This is the simplest case, where we assume that we have a fixed capacity $C<\infty$ and two classes of customers: one is the *discount class passengers*, paying pd for a unit (or seat in case of airlines); the other is the *full fare passengers* paying pf. Obviously it is easy to see that $pd>0$ and $pf>pd$. Another important assumption we have to make is that all discount requests come before the full fare requests.

With these assumptions in mind the basic question of the two class capacity allocation problem is the following: How many seats of discount customers should we book, when there is a chance of future full fare demand? In other words: What is the optimal booking limit, for the discount class, for which we can gain the highest possible revenue? If we find the optimal booking limit, then it is easy to find the optimal protection level too. According to the above definition the protection level (y) can be calculated from the capacity and the booking limit: $y=C-b$.

When calculating the optimal booking limit we want to avoid two scenarios: setting the limit too high or setting it too low. If we set b too low then we talk about *spoilage*, because we turned away discount passengers for future high fare customers, but the number of high fare customers is smaller than the protection level. In this case we are spoiling our capacity. (E.g. in the airline business our plane is flying with empty seats). On the other hand if we set b too low we could have made more revenue by saving the seats for future high class customers, but we sold it to discount passengers. E.g. we have a plane with 100 seats out of which we sold 80 to discount passengers, and 20 to full fare passengers, but the actual demand for the higher fare class seats was 30. So we turned away 10 better paying customers, for discount customers. This obviously decreased our revenue. This scenario is called *dilution*. Our goal is to balance the risks of these too outcomes to maximize expected revenue.

With the help of a decision tree we can illustrate the different outcomes of increasing the booking level by one unit.



D_f = full fare demand

D_d = discount demand

D_d and d_f are independent random variables

$F_f(x)=P(d_f \le x)$

$F_d(x)=P(d_d \le x)$

The right side of the tree gives us the change in expected revenue. If we take the probability weighted sum of these outcomes we can calculate the *expected change in revenue* caused by the fact that we increased b by 1.

$$E\big(h(b)\big) = F_d(b) \cdot 0 +$$
$$[1 - F_d(b)]\big\{[1 - F_f(c-b)]\big(P_d - P_f\big) + F_f(c-b)P_d\big\}$$
$$= [1 - F_d(b)]\big\{\big[P_d - [1 - F_f(c-b)P_f\big]\big\}$$

Where $1 - F_f(c-b) = P(full\,fare\ demand\ > protection\ level)$. This number is small when protection level is high.

From the above equation we can determine whether or not we should increase the booking limit for low fare class. If the value of E(h(b))<0, then increasing the booking limit by one decreases our revenue, but if E(h(b))>0, then we should definitely increase the booking limit, because we have a chance for higher profit.

So if $P_d<[1-F_f(c-b)]P_f$ we should not allocate any more seats to discount passengers.
If $P_d>[1-F_f(c-b)]P_f$ we should allocate at least one seat for a discount customer.
We can easily see that we get the optimal booking limit (b^*) if

$$\big\{\big[P_d = [1 - F_f(c-b^*)P_f\big]\big\}$$

This equation can also be written as

$$\frac{P_d}{P_f} = 1 - F_f(c-b^*)$$

Note that the optimal protection level $y^*=c-b^*$. So the above equation is equivalent to

$$\frac{P_d}{P_f} = 1 - F_f(y^*)$$

This was first described by Kenneth Littlewood in 1972, and is known as Littlewood's rule. Littlewood's rule says that:"to maximi**ze** expected revenue, the probability that full fare demand will exceed the protection level should equal to the ratio $\frac{P_d}{P_f}$ (Robert L Philips, Pricing and Revenue Optimi**z**ation, 2005).
An interesting fact is that the optimal protection level only depends on the two fares and the distribution of expected full fare demand. The discount demand or the capacity is not in the equation, so the optimal protection level does not depend on them in the two class model.

Spill Model for Estimating Spill and Unconstrained Demand

- *Variations of the "spill model" for airline demand analysis, specifically for estimating spill and unconstrained demand, have been developed both at MIT and by Boeing. The basic spill model makes the following assumptions:*
– Total demand for a flight departure or series of flight departures can be represented by a Gaussian distribution.

- The demand distribution has a mean and standard deviation that is known or which can be estimated from a sample observed historical load data for the same or similar flights.
- The estimated demand distribution can represent the magnitude and variability of demand for future flight departures, if properly adjusted for trends and/or seasonal changes in demand.

- *The Boeing Spill Model approach relies on the properties of the Gaussian distribution to estimate spill and unconstrained demand, in one of several ways:*
- Normalized “spill tables” are the simplest way to estimate spill and unconstrained demand, and will be described here.
- Use of “normal probability paper” to plot observed loads and estimate the mean and standard deviation of total demand is more complicated and is described in the Boeing paper.
- Use of iterative statistical estimation methods is even more complicated, and well beyond the scope of this course.

- *To make use of the “Spill Table” approach to estimation, the following additional terms and definitions are required:*
- DEMAND FACTOR is the mean total (unconstrained) demand per flight divided by the aircraft capacity. Unlike average load factor, the demand factor can exceed 1.0, as it is possible for mean total demand to exceed the assigned aircraft capacity. It is the total mean demand and/or the demand factor that we are trying to estimate, given an observed average load factor.
- SPILL FACTOR is the average (or “expected”) spilled passengers per flight divided by the aircraft capacity. Again, estimation of mean spill and/or the spill factor is the goal of our estimation effort.
- SPILL RATE is the average (or “expected”) spilled passengers per flight divided by the mean total demand for the flight. Thus, it is a measure of the probability or likelihood that a random passenger wishing to make a reservation for a flight will not be able to due to inadequate aircraft capacity. It is also the proportion of total demand that is rejected.

- *Given these definitions and those introduced earlier the following relationships must apply for a sample of flight departures with any degree of variability in demand (i.e., with a standard deviation greater than zero):*
- Mean demand is greater than or equal to the mean observed load. Mean demand is equal to mean load <u>only</u> if none of the flight departures in the sample experienced spill.
- Demand factor is greater than or equal to average load factor. Demand factor is equal to average load factor <u>only</u> if none of the flight departures in the sample experienced spill.
- Mean demand is always equal to mean observed load plus mean spill per flight.
- Demand factor is always equal to average load factor plus spill factor.

– Spill rate and spill factor must both be greater than zero if any of the flight departures in the sample experienced spill.

Illustrative examples

This paper will appear the comparison between the random of number ticket sold and not random using the simulation. Both simulation use no-shows uncertainty.
The price for a flight ticket from Medan to Batam is $50. Each plane can hold no more than 100 passengers. Usually, some passengers who have purchased a ticket are "no-shows". To protect against such no-shows, the airline would like to sell more than 100 tickets for each flight. Federal regulations require that any ticketed customer who is unable to board the plane due to overbooking is entitled to a compensation of 125% of the ticket value paid by the customer. Any no-show customer is refunded 50% of the ticket value paid by the customer. The number of no-shows is randomly distributed with a Lognormal distribution with mean of 10% of the Number of Tickets Sold and standard deviation of 6% of the Number of Tickets Sold.
In this problem, the only uncertainty, Number of No-Shows in cell H22, depends on the parameter Number of Tickets Sold (cell G27). (The ROUND function in cell G22 rounds the fractional value in cell H22 to a whole number.) The Number of Tickets Sold is set to 110. Total Revenue (cell G31) is a random quantity, since it depends on the number of no-shows. Cell G33 contains the PsiMean function which computes the Expected Total Revenue.

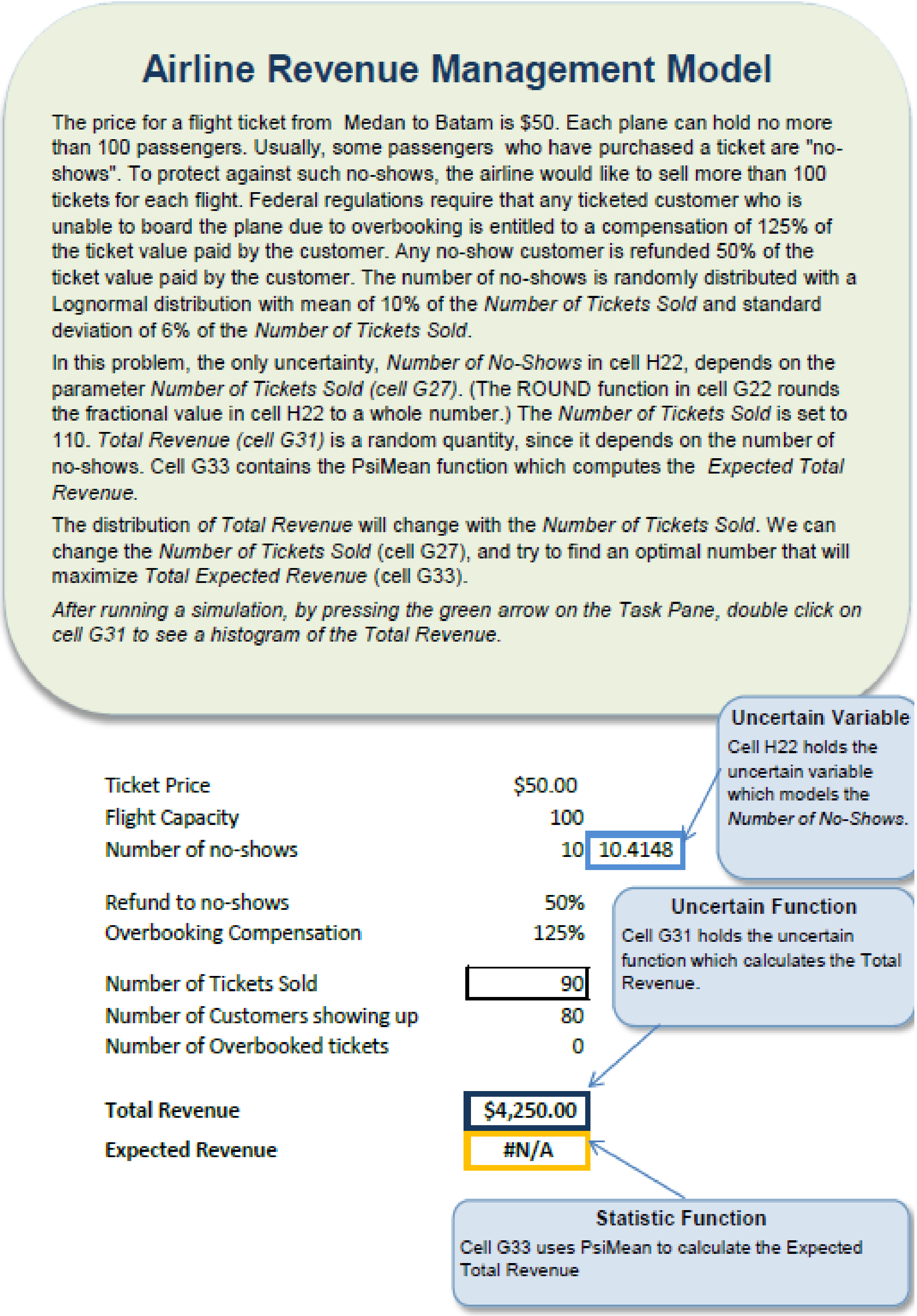
The distribution of Total Revenue will change with the Number of Tickets Sold. We can change the Number of Tickets Sold (cell G27), and try to find an optimal number that will maximize Total Expected Revenue (cell G33).

The data for second simulation is same with the first simulation.

As in the simpler example, AirlineManagementModel1, the only uncertainty is the *Number of No-Shows* in cell E26 which depends on the parameter *Number of Tickets Sold (cell D31)*. However, in this example, we use the PsiSimParam() function to observe how the *Expected Total Revenue* varies as the *Number of tickets Sold* varies from 100 to 150.

When you run multiple simulations at once, you can use the PsiSimParam() function to specify a list of values for a cell, one per simulation, where the value is held constant for all trials in a simulation. The PsiSimParam() function is located in cell D31. This function varies the *Number of Tickets Sold* from 100 to 150 (cells H25:H75). The number of simulations to perform is set to 51 in the Platform Model Pane which means that fifty-one simulations will be performed, each with 1000 trials. The Expected Revenue is computed using the *PsiMean* function, for each simulation, in cells I25:I75. Notice the second argument of PsiMean is passed, the Simulation number, in cells G25:G75.

When Interactive Simulation is turned on, we see that the *Expected Revenue* indeed varies with the *Number of Tickets Sold*; and the highest *Expected Revenue* is achieved when the *Number of Tickets Sold* is around 117.



Airline Revenue Management Model2

The price for a flight ticket from Medan to Batam is $50. Each plane can hold no more than 100 passengers. Usually, some passengers who have purchased a ticket are "no-shows". To protect against such no-shows, the airline would like to sell more than 100 tickets for each flight. Federal regulations require that any ticketed customer who is unable to board the plane due to overbooking is entitled to a compensation of 125% of the ticket value paid by the customer. Any no-show customer is refunded 50% of the ticket value paid by the customer. The number of no-shows is randomly distributed with a Lognormal distribution with mean of 10% of the *Number of Tickets Sold* and standard deviation of 6% of the *Number of Tickets Sold*.

As in the simpler example, AirlineManagementModel1, the only uncertainty is the *Number of No-Shows* in cell E26 which depends on the parameter *Number of Tickets Sold (cell D31)*. However, in this example, we use the PsiSimParam() function to observe how the *Expected Total Revenue* varies as the *Number of tickets Sold* varies from 100 to 150.

When you run multiple simulations at once, you can use the PsiSimParam() function to specify a list of values for a cell, one per simulation, where the value is held constant for all trials in a simulation. The PsiSimParam() function is located in cell D31. This function varies the *Number of Tickets Sold* from 100 to 150 (cells H25:H75). The number of simulations to perform is set to 51 in the Platform Model Pane which means that fifty-one simulations will be performed, each with 1000 trials. The Expected Revenue is computed using the *PsiMean* function, for each simulation, in cells I25:I75. Notice the second argument of PsiMean is passed, the Simulation number, in cells G25:G75.

When Interactive Simulation is turned on, we see that the *Expected Revenue* indeed varies with the *Number of Tickets Sold*; and the highest *Expected Revenue* is achieved when the *Number of Tickets Sold* is around 117.

After running a simulation, by pressing the green arrow on the Task Pane, double click on cell D35 to see a histogram of the Total Revenue.

Ticket Price	$ 50.00	
Flight Capacity	100	
Number of no-shows	9	9
Refund to no-shows	50%	
Overbooking Compensation	125%	
Tickets Sold	*90*	
Number of Customers showing up	81	
Number of Overbooked tickets	0	
Total Revenue	**$ 4,275.00**	
Expected Revenue	**#N/A**	

Simulation	Tickets Sold	Exp Revenue
1	100	#N/A
2	101	#N/A
3	102	#N/A
4	103	#N/A
5	104	#N/A
6	105	#N/A
7	106	#N/A
8	107	#N/A
9	108	#N/A
10	109	#N/A
11	110	#N/A
12	111	#N/A
13	112	#N/A
14	113	#N/A
15	114	#N/A
16	115	#N/A
17	116	#N/A
18	117	#N/A
19	118	#N/A

Conclusion

Clearly, assignment of a bigger aircraft results in higher loads (but lower load factors), increased revenues, and reduced spill and spill rate, as expected. In this case, given our assumption for total unconstrained demand, the demand distribution reflects quite high (but not a typical) demand variability.

References

[1] Chiang, W.C., Chen, J.C.H. and Xu, X.J. (2007) ‘An overview of research on revenue management: current issues and future research’, *Int. J. Revenue Management*, Vol. 1, pp.97–128.

[2] Lai, K.K. and Ng, W.L. (2005) ‘A stochastic approach to hotel revenue optimization’, *Computers and Operations Research*, Vol. 32, pp.1059–1072.

[3] S. Liu, K.K. Lai, S.Y. Wang. (2008) ‘ Booking models for hotel revenue management considering multiple-day stays’, *Int. J. Revenue Management, Vol. 2, No. 1.*

[4] Kimms, A. and Muller-Bungart, M. (2007) ‘Revenue management for broadcasting commercials: the channel’s problem of selecting and scheduling the advertisements to be aired’, *Int. J. Revenue Management*, Vol. 1, pp.28–44.

[5] Mangani, A. (2007) ‘The optimal ratio between advertising and sales income’, *Int. J. Revenue Management,* Vol. 1, pp.65–78.

[6] Rautio, T., Anttila, M. and Tuominen, M. (2007) ‘Bundling of information goods: a value driver for new mobile TV services’, *Int. J. Revenue Management*, Vol. 1, pp.45–64.

[7] Zarraga-Oberty, C. and Bonache, J. (2007) ‘Human factors in the design of revenue management systems in multinational corporations’, *Int. J. Revenue Management*, Vol. 1, pp.141–153.

[8] Robert G. Cross, *Revenue Management* (1997)

[9] Robert L. Phillips, *Pricing and Revenue Optimization* (2005)

[10] Kalyan T. Talluri, Garrett J. van Ryzin, *The Theory and practice of Revenue Management* (2005)

[11] Janakiram Subramanian, Shaler Stidham Jr., Conrad J. Lautenbacher, *Airline Yield Management with Overbooking, Cancellations, and No Shows* (1999)

[12] Tres*zl, M. Yield Management.* Åbo Akademi University *(2012)*